

# Calculation Model for Optoelectronic Remote Sensing System's Radiometric Resolution at Arbitrary Viewing Angles

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**Introduction.** One of the urgent problems, which are facing developers of satellite optoelectronic remote sensing systems (ORSS), is to improve the images quality. Image quality is determined, above all, by its radiometric resolution, which means minimum difference between brightness or reflectivity of object and background, which can be detected by ORSS with a given probability. Modern ORSS make possible viewing angle deviation, which causes significant image distortions.

**Formulation of the problem.** The purpose of the paper is to develop physical and mathematical radiometric resolution model of satellite remote sensing optoelectronic systems at arbitrary sight angles.

**Video signal formation model and radiometric resolution study.** Solar radiation that is reflected from the Earth's surface on which the object of observation is placed, passes through atmosphere and enters into transmitting camera lens. The lens forms image of the object and the background radiation in the detector plane. Detector converts illuminance distribution to electric signal, which forms video signal after scanning. The object of observation has uniform spectral reflectance over its size and its angular size is much bigger than ORSS instantaneous field of view. The object is situated on Earth's surface with uniform spectral reflection coefficient. Both object and background reflect light on Lambert's law. An example of ORSS radiometric resolution calculation model was considered.

**Conclusions.** On the basis of proposed optoelectronic remote sensing system model there was developed method of determination its radiometric resolution at arbitrary angles of sight. Study of the model showed that, increasing of viewing angle significantly deteriorates optoelectronic system spatial resolution while the radiometric resolution is unchanged.

*Key words:* remote sensing; radiometric resolution; spaceborne optoelectronic imager

## Introduction

Optoelectronic remote sensing systems (ORSS) are widely used in various fields of human activity [1]. One of the urgent problems, which are facing developers of such systems, is to improve the quality of satellite images. Image quality is determined, above all, by its energy (radiometric) resolution. Energy resolution means minimum difference between brightness or reflectivity of object and background of large size, which can be detected by ORSS with a given probability. Modern ORSS allow you to change the viewing angle, which is determined by angle between optical axis of sensor and Nadir. However, there are significant image distortions which are caused by deviation of the optical axis. Considerable amount of researching works [2–5] were devoted to ORSS image quality. At the same time there are not enough studies of satellite ORSS radiometric resolution at arbitrary angles of sight in the scientific literature.

## 1 Formulation of the problem

The purpose of this paper is to develop physical and mathematical radiometric resolution model of satellite remote sensing optoelectronic systems at arbitrary sight angles.

## 2 Video signal formation model

Process of video signal formation in ORSS is as follows. Solar radiation that is reflected from the Earth's surface on which the object of observation is placed, passes through atmosphere and enters into transmitting camera lens. The lens forms image of the object and the background radiation in the detector plane. Detector converts illuminance distribution to electric signal, which forms video signal after scanning. Let's consider elements of the model in detail. The object of observation is placed on a uniform background of Earth's surface, which is characteri-

zed by its albedo  $\rho_b$ . The object has a reflection coefficient  $\rho_t$ , that is greater than the albedo by  $\Delta\rho$ . Sometimes is  $\Delta\rho$  determined as follows:

$$\Delta\rho = \frac{\rho_t - \rho_b}{\rho_b}. \quad (1)$$

If the object and background reflect solar radiation according to Lambert's law and Sun creates on the Earth surface spectral illumination  $E_{0\lambda}$  then brightness of the object and the background in the working spectral range  $\lambda_1 \dots \lambda_2$  will be [2]

$$\begin{aligned} L_t &= \frac{1}{\pi} \int_{\lambda_1}^{\lambda_2} \rho_t(\lambda) E_{0\lambda}(\lambda) d\lambda; \\ L_b &= \frac{1}{\pi} \int_{\lambda_1}^{\lambda_2} \rho_b(\lambda) E_{0\lambda}(\lambda) d\lambda \end{aligned} \quad (2)$$

That is useful signal is generated due to object and background reflectance difference according to absolute brightness contrast

$$\Delta L_e = \frac{1}{\pi} \int_{\lambda_1}^{\lambda_2} [\rho_t(\lambda) - \rho_b(\lambda)] E_{0\lambda}(\lambda) d\lambda. \quad (3)$$

The atmosphere transforms the radiation, which propagates from the object and background to the sensor, due to absorption and scattering. The atmosphere is characterized by the spectral transmittance  $\tau_A(\lambda)$  and integral transmittance  $\tau_A$ :

$$\tau_A = E_0/E_5,$$

where  $E_5$  is integrated illumination, which is created by the Sun on top of atmosphere in the working spectral range and  $E_0$  is integrated illumination of the Earth's surface. In certain cases  $E_5 = 4,8 \cdot 10^{-2} \text{ W/cm}^2$  and  $E_0 = 2,4 \cdot 10^{-2} \text{ W/cm}^2$  so for further calculations we assume  $\tau_A = 0,5$ . Analysis of the atmosphere influence on the ORSS operation considering spectral range, celestial latitude, the Sun height, time of day and year, cloud cover and atmospheric conditions requires additional research. The optical system consists usually of three principal elements: the main lens, spectral beam splitter and flat mirror, which changes pointing direction (in some cases there is no mirror or beam splitter). The most important element of ORSS the lens will be modeled using these parameters [6]:

- focal length  $f'_0$ ;
- relative aperture  $D_p/f'_0$  or effective f-number  $k_e f f = f'_0/D_p$ , where  $D_p$  is diameter of entrance pupil;
- field of view:
  - o angular  $2\omega_0$ ;

- o linear  $X \times Y$  in the object space (on the Earth's surface);
- o linear  $X' \times Y'$  in the images space (in detector plane);

- spectral range  $\lambda_1 \dots \lambda_2$ , taking into account respective spectral channel;
- spectral transmission coefficient  $\tau_0(\lambda)$  and integral transmission coefficient  $\tau_0$  for corresponding spectral channel.

The lens forms an image of object and background in detector plane with illumination contrast

$$\begin{aligned} \Delta E' &= \frac{1}{4} \left( \frac{D_p}{f'_0} \right)^2 \int_{\lambda_1}^{\lambda_2} \tau_A(\lambda) \tau_0(\lambda) \cdot \\ &\quad \cdot E_{0\lambda}(\lambda) [\rho_t(\lambda) - \rho_b(\lambda)] d\lambda. \end{aligned} \quad (4)$$

CCD array detectors are commonly used in ORSS. Their features are:

- scanning frequency  $f_d$ ;
- noise equivalent exposure  $H_n$ ;
- pixel size  $V_D \times W_D$ ;
- distance between pixels centers  $\Delta l_D$ ;
- number of pixels  $N_D$ ;
- sensitivity  $R_D$ .

### 3 Radiometric resolution

To determine the ORSS radiometric resolution let's consider geometric diagram, which is shown in Fig. 1. The Cartesian coordinate system is located on the Earth's surface so that  $y$  axis coincides with satellite motion direction and  $x$  axis forms angle of  $90^\circ - \theta_{\nu x}$  with the optical axis. Satellite flight altitude is  $h_f$ .

Let the object of observation has uniform spectral reflectance  $\rho_t(\lambda)$  over its size and its angular size is much bigger than ORSS instantaneous field of view. The object is situated on Earth's surface with uniform spectral reflection coefficient  $\rho_b(\lambda)$ . Both object and background reflect light on Lambert's law.

Then the spectral brightness of object and background surfaces is defined as [2]

$$L_t(\lambda) = \rho_t(\lambda) \frac{E_0(\lambda)}{\pi}, L_b(\lambda) = \rho_b(\lambda) \frac{E_0(\lambda)}{\pi}, \quad (5)$$

where  $E_0(\lambda)$  is spectral illumination of the Earth's surface.

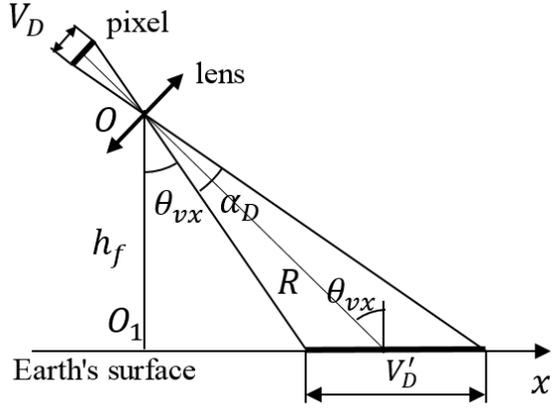


Fig. 1. Layout for ORSS spatial resolution determination

If the ORSS optical axis is inclined to object's surface normal at angle  $\theta_{vx}$ , then spectral radiant flux which enters the lens is

$$\Phi_t(\lambda) = \tau_A(\lambda) L_t(\lambda) A_t \Omega_0 \cos \theta_{vx}, \quad (6)$$

where  $\tau_A(\lambda)$  is atmosphere spectral transmission coefficient and  $A_t$  is object size, which is within ORSS instantaneous field of view, and  $\Omega_0 = A_p/R^2$  is solid angle within which light enters the entrance pupil of the lens.

Integral illumination of detector by the object of observation is

$$\begin{aligned} E_t &= \frac{1}{A_D} \int_{\lambda_1}^{\lambda_2} \Phi_t(\lambda) \tau_0(\lambda) d\lambda = \\ &= \frac{A_t \Omega \cos \theta_{vx}}{A_D} \int_{\lambda_1}^{\lambda_2} \tau_A(\lambda) L_t(\lambda) \tau_0(\lambda) d\lambda, \end{aligned} \quad (7)$$

where  $A_D = V_D W_D$  is detector pixel area.

For large object its area  $A_t$  that forms the radiation flux on detector pixel is determined by projection of this pixel on the Earth's surface. In [6] it was found that this area is equal to

$$A_t = \frac{h_f V_D}{f'_0 \cos^2 \theta_{vx}} \frac{W_D h_f}{f'_0 \cos \theta_{vx}} = \left( \frac{h_f}{f'_0} \right)^2 \frac{A_D}{\cos^3 \theta_{vx}}, \quad (8)$$

where  $f'_0$  is lens focal length.

From (8) it is clear that with increasing of viewing angle  $\theta_{vx}$  the area  $A_t$  increases as well. It causes deterioration of ORSS spatial resolution. At the same time, this should lead to radiometric resolution reduction. To analyze this claim, let's consider Fig. 1, from which the solid angle  $\Omega_0$  can be found

$$\Omega_0 = \frac{A_p}{R^2} = \frac{\pi D_p^2 \cos^2 \theta_{vx}}{4h_f^2}, \quad (9)$$

where  $D_p$  is entrance pupil diameter of the lens.

Substitution (5), (8) and (9) into (7) gives

$$\begin{aligned} E_t &= \int_{\lambda_1}^{\lambda_2} \tau_A(\lambda) L_t(\lambda) \tau_0(\lambda) d\lambda = \left( \frac{h_f}{f'_0} \right)^2 \frac{A_D}{\cos^3 \theta_{vx}} \times \\ &\times \frac{\pi D_p^2 \cos^2 \theta_{vx} \cos \theta_{vx}}{4h_f^2} \frac{1}{A_D} \int_{\lambda_1}^{\lambda_2} \tau_A(\lambda) \tau_0(\lambda) \rho_t(\lambda) \frac{E_0(\lambda)}{\pi} d\lambda = \\ &= \frac{1}{4} \left( \frac{D_p}{f'_0} \right)^2 \int_{\lambda_1}^{\lambda_2} \tau_A(\lambda) \tau_0(\lambda) E_0(\lambda) \rho_t(\lambda) d\lambda. \end{aligned} \quad (10)$$

Detector integral illumination, which generates by Earth's surface, is determined by same equation

$$E_b = \frac{1}{4} \left( \frac{D_p}{f'_0} \right)^2 \int_{\lambda_1}^{\lambda_2} \tau_a(\lambda) \tau_0(\lambda) E_0(\lambda) \rho_b(\lambda) d\lambda. \quad (11)$$

Detector detects object and Earth's surface illumination difference

$$\begin{aligned} \Delta E &= E_t - E_b = \frac{1}{4} \left( \frac{D - p}{f'_0} \right)^2 \times \\ &\times \int_{\lambda_1}^{\lambda_2} \tau_A(\lambda) \tau_0(\lambda) E_0(\lambda) [\rho_t(\lambda) - \rho_b(\lambda)] d\lambda. \end{aligned} \quad (12)$$

ORSS radiometric resolution is determined by minimum difference  $\Delta E_n$ , which in turn depends on detector exposure threshold  $H_n$

$$\Delta E_n = \frac{H_n}{t_i}, \quad (13)$$

where  $t_i$  is detector integration time.

For integrated parameters which are dependent on the wavelength  $\lambda$ , equation (12) turns into

$$\Delta E_n = \frac{1}{4} \left( \frac{D_p}{f'_0} \right)^2 \tau_A \tau_0 E_0 \Delta \rho_n, \quad (14)$$

where  $\Delta \rho_n = \rho_t - \rho_b$  is reflectance contrast threshold i.e. radiometric resolution.

From the system of equations (13) and (14) radiometric resolution is

$$\Delta \rho_n = \frac{4H_n}{\tau_A \tau_0 E_0 t_i} \left( \frac{f'_0}{D_p} \right)^2. \quad (15)$$

Analysis of formula (15) shows that ORSS radiometric resolution can be reduced by:

- decreasing of f-number  $F = f'_0/D_p$  or relative aperture  $D_p/f'_0$  growth. This is most efficient way, since the reduction of  $F$  twice leads to improved radiometric resolution in four times;

- using of low noise detector, which is equivalent to exposure threshold  $H_n$  reduction;
- increasing integration time  $t_i$ , which is limited by detector array readout period;
- increasing lens transmittance  $\tau_0$ , which is always less than unity.

## 4 Example of ORSS radiometric resolution calculations

As an example of proposed radiometric (energy) resolution calculation model application let's consider ORSS with following parameters:

- lens: focal length is  $f'_0 = 850$  mm, entrance pupil diameter is  $D_p = 200$  mm, lens transmittance is  $\tau_0 = 0,8$  in spectral range  $(\lambda_1 - \lambda_2) = (0,5 - 0,76) \mu m$ ;
- detector – silicon CCD array (CCD 151): number of pixels are  $N_D = 5000$ , pixel size is  $V_D \times W_D = 7 \times 7 \mu m^2$ ; scanning frequency is  $f_d = 5$  MHz, noise equivalent exposure is  $H_n = 2 \cdot 10^{-10} J/m^{-2}$ .

ORSS radiometric resolution  $\Delta\rho_n = \rho_t - \rho_b$  is defined by formula (15) where  $H_n = 2 \cdot 10^{-6} J/m^{-2}$  is noise equivalent exposure,  $t_i = N_D/f_d = 10^{-3}$  s is detector integration time and  $\tau_0 = 0,8$  is average transmittance of the lens.

From Table. 3.10, which is given in monograph [2], with method of numerical integration we find the average illumination, which is created by the Sun at sea level for the solar zenith angle  $60^\circ$  in spectral range  $(\lambda_1 - \lambda_2) = (0,5 - 0,76)$ . It is  $E_0 = 295,3 W/m^2$ .

For a vertical path average atmospheric transmission coefficient in cloudless weather is approximately 0.5 in the spectral range  $(\lambda_1 - \lambda_2) = (0,5 - 0,76)$  [2]. After substituting all the initial parameters in the formula (15) we obtain ORSS radiometric resolution:

$$\Delta\rho_n = \frac{4 \cdot 2 \cdot 10^{-6} \cdot 4,25^2}{0,5 \cdot 0,8 \cdot 295,3 \cdot 10^{-3}} = 1,2 \cdot 10^{-3}$$

Lens focal length is chosen on condition to ensure ORSS spatial resolution [5]. Therefore, specified radiometric resolution can be achieved by matching the lens entrance pupil diameter  $D_p$ . Fig. 2 shows dependence of the ORSS radiometric resolution  $\Delta\rho_n$  on the diameter of entrance pupil  $D_p$ .

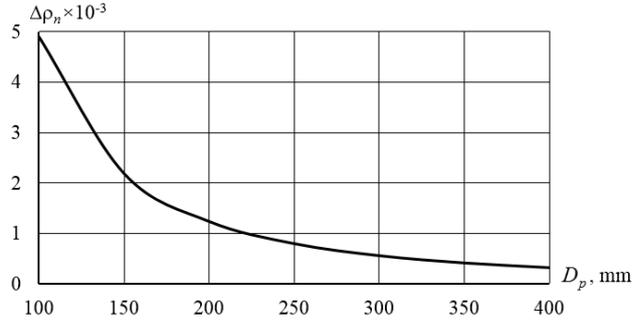


Fig. 2. Dependence of ORSS radiometric resolution on the lens entrance pupil diameter

## Conclusions

1. Modern optoelectronic remote sensing systems allow you to change the viewing angle of the Earth's surface. However, there appear significant image distortions which are caused by deviations of viewing axis from nadir. In the scientific literature there are not enough studies of satellite remote sensing optoelectronic systems radiometric resolution at arbitrary angles of sight.
2. On the basis of proposed optoelectronic remote sensing system physico-mathematical model there was developed method of determination its radiometric resolution at arbitrary angles of sight. Study of the model showed that:
  - (a) Increasing of viewing angle significantly deteriorates optoelectronic system spatial resolution while the radiometric resolution is unchanged;
  - (b) Radiometric resolution  $\Delta\rho_n$  can be reduced by reducing the lens f-number  $F = f'_0/D_p$ . This is the most effective way, since  $\Delta\rho_n \sim F^2$ .
  - (c) Radiometric resolution  $\Delta\rho_n$  is also reduced if the system uses a lens with high transmittance and low noise detector.
3. It is advisable to direct further studies on determination how lens aberrations effect on optoelectronic remote sensing system radiometric resolution at arbitrary angles of sight.

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### Фізико-математична модель для визначення радіометричного розділення космічних оптико-електронних систем дистанційного зондування Землі при довільних кутах візування

*Колобродов В. Г., Лихоліт М. І., Микитенко В. І., Тягур В. М., Добровольська К. В.*

У статті досліджується радіометричне (енергетичне) розділення космічної оптико-електронної зображуючої системи видимого діапазону спектра. Енергетичне розділення представлено як пороговий контраст коефіцієнта відбиття земної поверхні при заданій величині порогової експозиції. Розглянуто зміну енергетичного

розділення при нахилі візирної осі від надира. Показано, що зі збільшенням кута візування енергетичне розділення залишається незмінним.

*Ключові слова:* дистанційне зондування; радіометричне розділення; космічний оптико-електронний сканер

### Физико-математическая модель для определения радиометрического разрешения космических оптико-электронных систем дистанционного зондирования Земли при произвольных углах визирования

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В статье исследуется радиометрическое (энергетическое) разрешение космической оптико-электронной изображающей системы видимого диапазона спектра. Разрешение представлено как пороговый контраст коэффициента отражения земной поверхности при заданной величине пороговой экспозиции. Рассмотрено изменение разрешения при наклоне визирной оси от надира. Показано, что с увеличением угла визирования энергетическое разрешение остается неизменным.

*Ключевые слова:* дистанционное зондирование; радиометрическое разрешение; космический оптико-электронный сканер