Mathematical Model of Complex Radio-Location Portrait of Aim with a Final Number of Bright Points

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Introduction. To form radio-location portrait (RLP) of aim – in the structures and algorithms of modern radar sets (RS), methods based on high identification signals are used. The hereinabove was established by the research analysis concerning the synthesis of RLP and existing approaches to the identification of unknown systems by adaptive methods. These methods imply high requirements for transmitters and prevent their implementation in pulse RS with low-frequency oscillators. This problem can be solved in another way, for use in RS with low-stability transmitters. In this case, the aim is regarded as a certain unknown system that brings certain known distortions in the deterministic signal that correspond to its transient response. At active location, the signal is fully known on both the transmitting and receiving sides (probing and echo signals), while spreading in a homogeneous medium, non-linear phase-frequency distortions are not introduced into it. When the adaptive filtering algorithm is applied, its transient feature is formed, to which the optimal weight vector of the synthesized adaptive filter will correspond. Thus, forming RLP in each probing period, it is possible to perform single-angle identification of aims and to realize coherent processing of echo signals, even when using incoherent sources of ultra-high frequencies of probe signals. This allowed us to formulate the purpose of the article, which is to increase the coherence of processing echo signals in pulsed RS with incoherent sources of probing signals. In order to achieve this research goal, the paper analyzes the existing methods of radio-location portrait of aim formation, on the basis of which mathematical models of signals reflected from aims with complex geometric surface shape are exploited, on which simulation work of the developed algorithms is carried out.

Theoretical results. The mathematical model of the complex of radio-location portrait of aim image with a finite number of «bright points» has been improved pursuant to the analysis of existing methods of formation of radio-location portrait of aim and identified inconsistencies of existing methods with the modern requirements regarding the use in incoherent pulse radar stations. The model differs from the existing ones as it allows to take into account the amplitude-phase transformations of a complex circumflex of probing signal when reflected from a aim with a complex geometric shape in the azimuthal and longitudinal plane.

Results. The simulation results, received with the help of the developed method, showed that RLPs enable to distinguish by visible range bright points on the surface of objects with great accuracy. It is 3-4 times greater than the potential for distinguishing the probing signal caused by the duration of the radio pulse. RLPs also enable to increase the coherence of inter-period signal processing by a value of coherence coefficient from 3 to 6 times (depending on the spatial shape of the aim surface).

Key words: radar station; radar signal; echo-signal; radio-location portrait of aim; complex circumflex; transient response

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Introduction

The development of radar in recent decades has been accompanied by a sharp increase of requirements for the main characteristics of radar stations (RSn). At the same time, requirements for the completeness and reliability of radar information have been growing, which is the basis for providing radar of aims recognition. Despite the considerable progress in the characteristics of the element base, the increasing requirements concerning the completeness and reliability of radar information, in many cases, it is not possible to satisfy in the framework of the traditional RSn construction. This is the evidence that one of the ways to providing radar of aims recognition can be considered the development of efficient algorithms of secondary information digital processing.
1 Analysis of recent research and publications

The analysis of scientific works in the field showed [1–6] that the basis of almost all methods of formation of radio-location portrait of aims is the use of complex sounding reference signals, such as linear-frequency modulated signals, frequency- and phase-manipulated signals, possessing optimized correlation properties and providing resolvability in the range of a few centimeters. Other known methods are based on the use of complex multifrequency signals and are implemented by gradually probing the surface of whole signals with discrete increasing or decreasing frequency. In doing so, an amplitude-frequency response of the aim, surface is formed, which, when performing the Fourier Reverse Transformation, makes it possible to obtain a long-range portrait. The implementation of these approaches in radar systems requires a significant complication of their transceiver part, which is not always economically justified.

Many scientific works have been devoted to the development of methods and means of radio-location aims recognition. Among them are the works of J. D. Shirman, Y. L. Barabash, E. L. Kazakova, A. E. Okhrimenko, S. A. Gorskov, S. P. Leschenko, O. I. Sukharevsky, V. M. Olenko and others [7–11]. The analysis of results of their scientific works has proved that almost all methods of forming radio-location portrait of aims, which are the source information for recognition, are based on the use of complex signals with frequency, phase modulation, which have optimized correlation properties required for high resolution in the range of the formation faraway portraits. There are also methods based on the use of multifrequency signals, which are implemented by gradually probing the surface of the aim with signals with gradually increasing or decreasing frequency, thus, spectral characteristics of the scattering function of the surface of the aim are determined, which, when performing the inverse Fourier transform, also allow to obtain faraway portraits. Given the complexity of the probe signals, the implementation of these approaches in radar requires a significant complication of the transceiver part, which is not always optimal in terms of efficiency/cost.

2 The purpose and objectives of the research

Thus, the question arises of finding effective methods of radar recognition based on the use of simple pulse signals. The formation of radio-location portrait of aims, in this case, can be accomplished by directly forming the transmitting characteristics of aims in the time and frequency planes using adaptive filtering algorithms. To achieve this goal, on the basis of performed mathematical modeling of the reflection functions of complex surfaces of radio-location aims, a mathematical model of the complex transient characteristic of an aim with a limited number of bright points when probing its surface with narrowband signals was developed.

3 Materials and methods of research

The research methods used in the work are: methods of mathematical analysis, theory of optimal signal reception, statistical theory of signals, theory of probability and mathematical statistics. Mathematical models are developed on the basis of spectral analysis theory and the theory of digital signal processing.

Methods of experimental research are: field experiment, simulation modeling. Simulation methods were implemented using computer aided design systems (MathCAD and MatLab software). The mathematical modeling method analyzes the potential accuracy of reproduction of radar characteristics of aims with different geometric surface shapes based on adaptive filtering algorithms.

4 Spectral method of forming radio-location portrait of aim using limited-spectrum signals

On the basis of the mathematical modeling of the reflection functions of radio-location aims complex surfaces, a mathematical model of the complex transient characteristic of the aim with a limited number of bright points was developed when probing its surface with narrowband signals.

The obtained model is designed to calculate and construct the spatial shape of a multipoint aim surface by phase, frequency and amplitude transformations of a radar signal with spatial elements of resolution the size of which is much smaller than the temporal and azimuthal element of the difference between aims portraits.

In order to obtain the model, at the first stage, the physical scattering processes of electromagnetic radiation upon reflection from the surface of the aim were analyzed. It was found that a complex radar image of a multipoint object could be developed using an approach based on a point model of effective radar scattering centers of objects.

In this case, it is assumed that the amplitude of the reflected wave of each «bright point» of the object in the frequency band of the sounding reference signal is constant, since the duration of the sounding reference signal is much shorter with respect to the minimum time of moving the aim.

Besides, considering that the main mechanism for the interaction of electromagnetic fields in the presence
of several centers of object scattering is interference, the received radar signal scattered by the radar object is suggested to be considered as the signals superposition from each «bright point» of the aim.

The complex circumflex of radar signal reflected by the i-th scattering center is introduced as:

\[ \hat{s}_i(t) = u_i e^{j\Psi_i} e^{j\omega t_i} = \hat{u}_i s(t - t_i), \]  

where \( u_i \) and \( \Psi_i \) - the amplitude and phase of the complex circumflex signal, scattered by the i-th scattering center; \( s(t) \) - the circumflex of the sounding radar pulse; \( t_i \) - the delay due to the time of the electromagnetic wave expansion from the aerial to the scattering center; \( \omega_0 \) - carrier frequency.

The complex circumflex of the signal reflected by a fixed scattering object of the complex shape is determined by the superposition of the complex circumflex of the signals reflected from the final number of this object scattering centers \([1]\). The total complex circumflex is the result of its components’ vector putting together. Thus, the interference of waves reflected from different elements of an object upon its irradiation by a radar signal can be described. In this approach, the complex radar image is the dependence on the range \( r \) and azimuth angle \( \varphi \) of the values of the echo-signal complex circumflex scattered by the i-th «bright points» of the object:

\[ \hat{\chi}(r, \varphi) = \sum_{i=1}^{N} \hat{u}_i \hat{s}(r - r_i) G_A^2(\varphi - \varphi_i) + \hat{\eta}(r, \varphi), \]  

where \( \hat{u}_i \) - is the complex amplitude; \( \varphi_i \) - the azimuth coordinate of the i-th center of the object scattering; \( r_i \) - the coordinate of the range of the i-th center of the object scattering; \( s(r) \) - the circumflex of radar signal; \( G_A^2(\varphi) \) - the characteristic of aerial direction; \( \hat{\eta}(r, \varphi) \) - is the complex circumflex of noise in the receiving frequency band.

As the frequency approach is applied, the noise and encumbrances introduce the significant effect on the reliability and accuracy of the model. It is suggested to determine the required signal/noise ratio according to:

\[ q = \frac{\int_0^{R_{max}} 2\pi \int_0^{2\pi} \left| \sum_{i=1}^{N} \hat{u}_i \hat{s}(r - r_i) G_A^2(\varphi - \varphi_i) \right|^2 dr d\varphi}{2\pi \sigma^2 R_{max}}, \]

where \( R_{max} \) - is the spatial length of the sounding reference signal; \( \sigma^2 \) - noise dispersion.

The mathematical model of the radar image of the object within the distance interval \([r_{min}, r_{max}]\) and the azimuth interval \([\varphi_{min}, \varphi_{max}]\) is presented in the following form:

\[ \hat{\chi}[n, m] = \sum_{i=1}^{N} \hat{u}_i \hat{s}(\rho_n - r_i) G_A^2(\theta_m - \varphi_i) + \hat{\eta}[n, m], \]

where the discrete results of distance and azimuth angle are determined by the expressions:

\[ \rho_n = r_{min} + \frac{n}{N-1}(r_{max} - r_{min}), \]

\[ n = 0, 1, ..., N - 1, \]

\[ \theta_m = \varphi_{min} + \frac{m}{M-1}(\varphi_{max} - \varphi_{min}), \]

\[ m = 0, 1, ..., M - 1. \]

The module and argument of the complex radar image are formed according to the amplitude and phase distributions of the radar signal reflected from the aim.

In order to construct a map of the scattering centers location on the surface of the aim, an algorithm of parametric identification is presented in the article. Parametric identification of the object is carried out sequentially by a series of processing procedures in accordance with the selected model of the received radar signal.

The spectrum of the complex circumflex of the radar signal reflected by the i-scattering center has the following form \([7]\):

\[ \hat{S}_i(f) = \hat{u}_i \hat{S}(f)e^{-j2\pi f t_i}, \]

where \( \hat{S}(f) \) - is the spectrum of the complex circumflex of the sounding reference signal.

For a discrete complex radar image of the object, a two-dimensional discrete Fourier transformation (hereinafter referred to as DFT) is calculated as:

\[ \hat{X}[\nu, \mu] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \hat{\chi}[n, m] e^{-j2\pi(\frac{\nu n}{N} + \frac{\mu m}{M})}, \]

\[ \nu = 0, 1, ..., N - 1; \mu = 0, 1, ..., M - 1. \]

The model of the scattering characteristic of the object’s «bright points» in the spectral region of a discrete radar image is described by the expression:

\[ \hat{Z}[\nu, \mu] = \sum_{i=1}^{N} \hat{u}_i e^{-j(\alpha_i \nu + \beta_i \mu)}, \]

where

\[ \alpha_i = 2\pi - \frac{r_{max} - r_{min}}{r_{max} - r_{min}}; \beta_i = 2\pi\varphi_i - \frac{\varphi_{min} + \varphi_{max}}{2}. \]

The shape of the two-dimensional spatial spectrum of a single radar response of the object’s i-th «bright point» is determined along one spatial frequency of the \( s(r) \) signal circumflex spectrum – independently, and along the other – by the spectrum of the known aerial directivity in \( G_A^2(\varphi) \) power.

Thus, the two-dimensional spatial spectrum of a single radar reflection is a two-dimensional weighting function \( \hat{D}[\nu, \mu] \) of spectral readings \( \hat{Z}[\nu, \mu] \) for a multi-point object. To obtain the characteristic of point centers of the object scattering in the spectral field of
a discrete radar image, it is necessary to compensate the function $\tilde{D}[\nu, \mu]$ for two-dimensional DFT $\tilde{X}[\nu, \mu]$:

$$\tilde{Y}[\nu, \mu] = \frac{\tilde{X}[\nu, \mu]}{\tilde{D}[\nu, \mu]} = \tilde{Z}[\nu, \mu] + \frac{\tilde{W}[\nu, \mu]}{\tilde{D}[\nu, \mu]},$$  \hfill (11)

where $R < N$ and $Q < M$ are determined by the effective length of the signal circumference spectrum $s(r)$ and the spectrum of the aerial directional characteristics in $G_2(\varphi)$ power, respectively; $\tilde{W}[\nu, \mu] -$ two-dimensional DFT noise readings $\tilde{\eta}[\nu, \mu]$.

To calculate the sample of two-dimensional DFT noise readings $\tilde{\eta}$, the following expressions are suggested to be used:

$$\tilde{y}_0[\nu] = \tilde{Y}[\nu, 0] = \sum_{n=0}^{N-1} (\sum_{m=0}^{M-1} \tilde{X}[n, m]) e^{-j2\pi(\frac{\varphi}{\lambda} n)}, \hfill (12)$$

$$\tilde{y}_0[\mu] = \tilde{Y}[0, \mu] = \sum_{n=0}^{N-1} (\sum_{m=0}^{M-1} \tilde{X}[n, m]) e^{-j2\pi(\frac{\varphi}{\lambda} m)}, \hfill (13)$$

From the vector of data $\tilde{y} = [\tilde{y}_0, \tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_K]$, obtained for each succession of spectral readings in the following form:

$$\tilde{y}_k = [\tilde{y}_0[k], \tilde{y}_1[k]], k = 0, 1, ..., K - 1,$$ \hfill (14)

it is necessary to create a data matrix that looks like a Hankel matrix:

$$\tilde{Y} = \begin{bmatrix} \tilde{y}_0 & \tilde{y}_1 & \ldots & \tilde{y}_L \\ \tilde{y}_1 & \tilde{y}_2 & \ldots & \tilde{y}_{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{y}_{K-L} & \tilde{y}_{K-L-1} & \ldots & \tilde{y}_{K-1} \end{bmatrix}, L < \frac{K}{2}, \hfill (15)$$

where $K = \{R, Q\}$ is the number of data readings contained in the data vector $\tilde{y}$.

Estimates of the distance and azimuth coordinates of $i$-th scattering center position can be obtained by the formulas:

$$r_i = \frac{\arg(\tilde{Z}_0)}{2\pi} (r_{\text{max}} - r_{\text{min}}) + r_{\text{min}}, \hfill (16)$$

$$\varphi_i = \frac{\arg(\tilde{Z}_0)}{2\pi} (\varphi_{\text{max}} - \varphi_{\text{min}}) + \varphi_{\text{min}}, \hfill (17)$$

where $\tilde{Z}_0$ and $\tilde{Z}_i$ are the indexes of the sequences of spectral readings poles $\tilde{y}_0[\nu]$ and $\tilde{y}_i[\mu]$, are accordingly obtained using the matrix beam method.

## 5 Simulation modelling of functioning formation method of radio-location portraits of aims in spectral sphere

To check the adequacy of designed models simulation modelling of the object surface shape assessment has been conducted in MathCAD environment by means of synthesis of model reflection from the object that consists of three bright spots. Discrete radar picture was modelled on the coordinate area range and azimuth in $50 \times 50 = 2500$ points. Allocations were accepted within the range interval under the duration of sounding impulse $0.1 \mu$s [15 m] beyond 500 m (distant zone) and the interval by bearing angle $0, 0^\circ, 10^\circ, 20^\circ, 40^\circ, 60^\circ, 80^\circ, 100^\circ$; magnitude factor $= 4, 2, 1.2$; phase shift $= 1.8^\circ, 0.9^\circ, 0.1^\circ, 0.01$ rad. Obtained radar reflection is presented in figure 1 in the form of some range and azimuth reflection sections. As we can see in the figure, there are strongly marked local peaks in sections, and their quantity can point out the presence of reflections in the received signal from both a single dissipation center and from two or three short-range dissipation centers.

The range estimation based on the received signal in such sections is complicated by insufficient resolution for localization of some bright objects of the object, noise and interference of reflected signals from several scattering centers.

According to Fig. 1, the range estimates of 504 m, 510 m, 513 m for scattering centers correspond to the location of the "bright" points within the element of difference, both by the range and azimuth of the radar image at a distance of 300 m. Estimates of the signal amplitudes reflected by the «bright points» of the object are indicated by the height of the arrow.

The invisibility of «bright points» with almost complete coincidence of their range are shown on the Fig. 1. However, the position of the azimuth scattering centers remains unknown. In addition, the estimation of the intensity level of «bright points» is not carried out without carrying out a similar stage of the identification algorithm for azimuthal sections of the radar image of the object.

Several azimuth cross sections of the object radar response are depicted by the dotted lines (Fig. 1). The signal obtained as a result of coherent radar image processing over a distance is shown in the figure as a solid line. The parameters of the «bright points» of the object were estimated by the quadrature components of the signal spectrum.
An estimate of the azimuth coordinates of the «bright points» shows that the minimum angular distance between the «bright points» is 0.3°, that is much less than the difference between the radar image and the azimuth almost three times. The amplitudes of the reflected signals by the «bright points» were obtained for all three scattering centers.

Fig. 2 shows the obtained positions of the scattering centers on the contour maps of the amplitude and phase distributions of the radar image. In comparing these provisions with the initial parameters of the scattering centers, deviations associated with the effect of noise and the limited number of discrete points of the radar image can be observed.

According to Fig. 3, the proposed algorithm allows to estimate the number and position of scattering centers within the total amplitude burst of a complex radar with three-dimensional graph interpolation.

Thus, using the developed model and performing parametric identification based on it, it is potentially possible to form two-dimensional radio-location portraits of aims with resolution elements that are several times smaller than the resolution of radar images both by azimuth and length.

The accuracy and reliability of the models obtained was evaluated by modeling the processes in different software environments and according to the convergence of their results with each other and the results of practical implementation.
6 Discussion of results

Analysis of the results of numerical simulation of the radar object identification shows the possibility of estimating the intensity, coordinates and azimuth of scattering centers located within the radar image distinguishing element at radial distances, which are 3-4 times greater than the resolution of the radar with the duration of the signal and sufficient signal/noise ratio (not less than 20dB).

Using the model and performing parametric identification based on it, it is possible to form two-dimensional radio-location portraits of aims with elements of resolution which dimensions are several times smaller than the radar and azimuth resolution. However, the implementation of this approach requires considerable computational cost, which may not always be implemented in pulse radars.

In addition, it is possible to provide such requirements in respect of signal/sound (over 20dB) in active radar only on small distances or during an observation of aims with a large effective scattering surface. Thus, in order to implement potential radar signals for distinction, it's necessary to elaborate the radar forming method, which is resistant to the sound and the instability of the transmitter.

Conclusions

1. The transitional characteristic of a radio-location aim reflective surface in its informational content may as well be used as the main characteristic of a coherent accumulation. In observing the movable aims, even with their minor movement during probing, it is necessary to consider or to compensate the homogeneous angular displacement of the echo signal concerning the sounding, as they significantly reduce the accuracy of the monoaspect portrait of the goal both in the current review period and beyond. However, this circumstance does not require additional efforts when used to determine the number and the coordinates of the algorithm object scattering centers for the parameters identification of the radar image model in the frequency domain. By means of mathematical methods, the use of this algorithm enables to detect additional signs of an effective scattering aim surface, which significantly increases the efficiency of detection algorithms and distinction. As a result of identification, the parameters of the model received by the radar signal can be considered as characteristics of the most powerful equivalent points of scattering. Consequently, observing the behaviour of the object scattering properties on a limited number of of its «bright points» can be virtually used for the recognition of radio-location aims.

2. The presented model can be applied to describe the radar signals of pulse radar, which has a high separation ability due to the small duration of the probing signals. In this case, the monitored radar signal can be described as a reviews superposition from separate «bright points» of aims.

3. The simulation technique results demonstrated, that obtained with the help of the developed method radio-location aims portraits allow to cut by the range the bright points on the surface of objects with an accuracy of 3-4 times greater than the potential distinction probing signal by duration in respect of signal/sound at least 20 dB at the detector inlet.

4. The mathematical model of the aim integrated radar image with a final number of bright points got its further development. The difference between this model and the existing ones, which defines its novelty, is that it allows to take into account the amplitude-phase transformation of a complex circumflex signal at reflection from a aim with a complex geometric form in azimuth-distant flat.

5. In the future, the results of this work can be used by the department of the State Border Guard Service of Ukraine in the development of technical requirements.
for prospective pulse radar stations in the system of engineering and technical security of the state border.

A forward-looking direction of further research is the use of adaptive algorithms not only for the formation of radar goals portraits, but also for the implementation of coherent impulse methods of selection aims in pulse radar means for their identification.

References


отримані за допомогою розробленого методу РЛПЦ дають змогу розрізняти по дальності яскраві точки на поверхні об’єктів з точністю, що в 3-4 рази перевищує потенційні можливості розрізняння зондуючого сигналу обумовлени тривалістю радіоімпульсу, а також підвищують когерентність міжперіодної обробки сигналів за значенням коефіцієнта когерентності від 3 до 6 разів (в залежності від просторової форми поверхні цілі).

Ключові слова: радіолокаційна станція; радіолокаційний сигнал; ехосигнал; радіолокаційний портрет цілі; комплексна обвідна; перехідна характеристика

Математична модель комплексного радіолокаційного изображення цілі з кінчним кількістю яскравих точок

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Вступлення. На основі проведенного аналізу робот в частини синтезу радіолокаційних портретів цілі та сучасних подходів до ідентифікації неизвестних систем адаптивними методами було установлена, що в структурах та методах роботи сучасних радіолокаційних зсувних систем (РЛС) для формування радіолокаційних портретів цілі (РЛПЦ) використовують методи на основі сигналів з високим розрізнянням, що обумовлює високі вимоги до передачі та безперервності ідентифікації целей.

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