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Rotations of Cylindrical Dielectric Resonators in a Rectangular Waveguide

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The coupling coefficients of cylindrical dielectric resonators with a regular rectangular waveguide under the condition their axes rotation are calculated. The dependences of the coupling coefficients on the angles of rotation and transverse coordinates of the resonator in the case of excitation of the main magnetic types of natural oscillations in them are considered. The dependence of the coupling value on the angles of rotation at the points of circular polarization of the fundamental wave of a rectangular waveguide is shown. The condition for the angle of rotation of the axis of the resonator, determined by the dimensions of the cross section of the waveguide and the frequency of the main type of natural oscillations, is also established, when fulfilled, the coupling coefficient becomes constant in the transverse plane of symmetry of the waveguide. New analytical expressions are derived for the mutual coupling coefficients of identical cylindrical dielectric resonators when their axes rotate relative to a rectangular waveguide. The dependences of the mutual coupling coefficients on the angles of rotation and coordinates of the resonators are investigated. Conditions are found under which the mutual coupling coefficients of two cylindrical resonators are independent of their transverse coordinate in the plane of symmetry of the waveguide. The reasons for the change in the sign of the coupling coefficients of the resonators during their rotation are discussed. The effect of the emergence of coupling extrema for different relative orientations of dielectric resonators is noted. In particular cases of parallelism of the resonator axes of one of the coordinate axes of the waveguide, the analytical expressions found in the work coincide with those obtained earlier. The obtained analytical results make it possible to construct analytical models of bandpass and notch filters, significantly reduce the computation time and optimize complex multi-cavity structures of microwave and optical communication systems.

Key words: coupling coefficient; mutual coupling coefficient; rotation; cylindrical dielectric resonator; waveguide

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Introduction

The rotation of dielectric resonators (DRs) can be used to reduce the size of filters [1–3], improve the radiation parameters of antennas [4–9, 11], and control the scattering characteristics of various metasurfaces [10, 12–14]. To calculate and optimize the scattering characteristics of these devices, it is necessary to study the coupling coefficients of dielectric resonators in various waveguides and open space. For rotating dielectric resonators, such a calculation of the coupling coefficients usually leads to extremely cumbersome analytical expressions. However, in some cases, obtaining analytical expressions is possible. The presence of the specified analytical formulas allows to establish previously unknown properties of the interaction of resonators with each other and the transmission line, as well as to build an electromagnetic scattering models on a large number of coupled elements.

1 Statement of the problem

The purpose of this article is to calculation and analysis of coupling coefficients of the cylindrical dielectric resonators in a rectangular propagating as well as a cut-off waveguide in the case of rotation of the resonator axes. Derivation of analytical formulas for the coupling coefficients of resonators with their possible rotation relative to one of the coordinate axes of a rectangular waveguide. Search for new patterns of change in the coupling of cylindrical dielectric resonators with the main magnetic types of resonances when they rotate relative to each other and a rectangular waveguide.

2 Coupling coefficients of rotation Cylindrical DRs with rectangular waveguide

To calculate the coupling coefficient of a resonator with a regular waveguide, we need to find the projection $c_t^{\pm}$ of the resonator field $(\vec{e}, \vec{h})$ onto a propagating waves $\left(\vec{E}_t^{\pm}, \vec{H}_t^{\pm}\right)$. In the case of the main magnetic type of natural oscillations H_{101}, the field of a cylindrical dielectric resonator in the local coordinate system (ρ', φ', z') associated with the axis of the dielectric cylinder (Fig. 1) is represented as:

$$e_{\rho'} = e_{z'} = 0;$$

$$e_{\varphi'} = -h_1 \frac{i\omega\mu_0}{\beta} J_1(\beta\rho') \begin{Bmatrix} \sin\beta_z z' \\ \cos\beta_z z' \end{Bmatrix}, \qquad (1)$$

here h_1 is the amplitude; ω is the circular frequency; μ_0 is the magnetic permeability; (β, β_z) are the wave numbers $\beta^2 + \beta_z^2 = k_1^2$; $k_1 = \omega\sqrt{\mu_0\varepsilon_1} k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, and; ε_0, ε_1 are the dielectric permittivity of the external space and resonator, respectively.

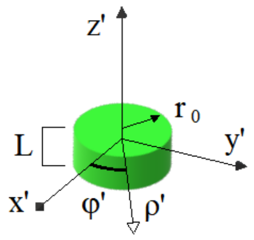

Fig. 1. Cylindrical dielectric resonator in the local cylindrical coordinate system (ρ', φ', z')

In the general case of rotations,

$$c_t^{\pm} = i/2\omega(\varepsilon_1 - \varepsilon_0) \int_V (\vec{e}, (\vec{E}_t^{\pm})^*) dv. \qquad (2)$$

The integral (2) takes on a very cumbersome form, therefore, in this work, we will consider only rotations relatively one selected coordinate axis of a rectangular waveguide.

In the case of propagating waves, the coupling coefficient of the resonator with the waveguide, taking into account the normalization, takes the form:

$$\tilde{k} = \frac{\left|c_t^{\pm}\right|^2}{\omega W}, \qquad (3)$$

where W- is the energy, stored in the DR material.

The results of calculating the coupling coefficients (3) are most conveniently presented in the form:

a) in the case of rotation of the resonator axis about the x axis of the waveguide (Fig. 2, a)

$$\tilde{k} = 4\tilde{k}_0 \chi_{1x}^2 \cos^2\beta_2 \cos^2(\chi_{1x}x_0) |\upsilon(\chi_{1x}, \Gamma\sin\beta_2, \Gamma\cos\beta_2)|^2; \qquad (4)$$

b) rotation of the resonator relative to the y axis of the waveguide (Fig. 2, b)

$$\begin{aligned} \tilde{k} = \tilde{k}_0 \Big| &(\chi_{1x}\cos\alpha_1 \pm \Gamma\sin\alpha_1)\upsilon((\chi_{1x}\cos\alpha_1 \pm \Gamma\sin\alpha_1), 0, (\chi_{1x}\sin\alpha_1 \mp \Gamma\cos\alpha_1))e^{-i\chi_{1x}x_0} + \\ &+ (\chi_{1x}\cos\alpha_1 \mp \Gamma\sin\alpha_1)\upsilon((\chi_{1x}\cos\alpha_1 \mp \Gamma\sin\alpha_1), 0, (\chi_{1x}\sin\alpha_1 \pm \Gamma\cos\alpha_1))e^{i\chi_{1x}x_0}\Big|^2; \end{aligned} \qquad (5)$$

c) rotation of the resonator relative to the z axis of the waveguide (Fig. 2, c)

$$\tilde{k} = 4\tilde{k}_0 |\Gamma|^2 \cos^2\alpha_1 \sin^2(\chi_{1x}x_0) |\upsilon(\Gamma, \chi_{1x}\sin\alpha_1, \chi_{1x}\cos\alpha_1)|^2, \qquad (6)$$

where

$$\upsilon(s_x, s_y, s_z) = \frac{\left[p_\perp J_1\left(\sqrt{s_x^2 + s_y^2}\, r_0\right) J_0(p_\perp) - \sqrt{s_x^2 + s_y^2}\, r_0 J_0\left(\sqrt{s_x^2 + s_y^2}\, r_0\right) J_1(p_\perp)\right]}{(s_x^2 + s_y^2) r_0^2 - p_\perp^2} \frac{\omega_z^*(s_z)}{\sqrt{s_x^2 + s_y^2}}; \qquad (7)$$

* – complex conjugate symbol; $J_n(z)$ – Bessel functions.

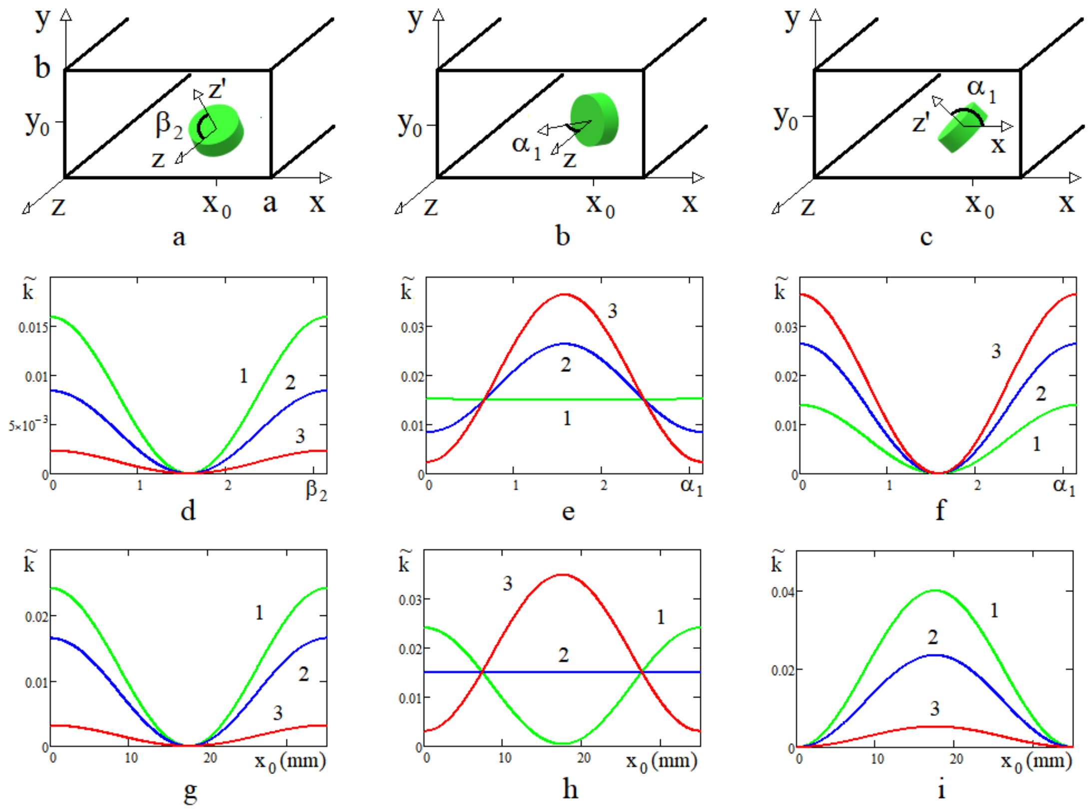

Fig. 2. Rotation of the dielectric resonator relatively the x-axis (a); y-axis (b); z-axis (c). Dependence of the DR coupling coefficient on the rotation angle β_2 (d): ($y_0 = 0,5b$; 1 – $x_0 = 0,2a$; 2– $x_0 = 0,3a$; 3 – $x_0 = 0,4a$). Dependence of the DR coupling coefficient on the rotation angle α_1 relatively the y-axis (e): ($y_0 = 0,5b$; 1 – $x_0 = 7,341$mm; 2 – $x_0 = 0,3a$; 3 – $x_0 = 0,4a$). The dependence of the coupling coefficient of the DR on the angle of rotation α_1 for the initial position $z' \parallel x$ (f): ($y_0 = 0,5b$; 1 – $x_0 = 0,2a$; 2 – $x_0 = 0,3a$; 3 – $x_0 = 0,4a$). Dependence of the coupling coefficient on the DR coordinate $x_0(y_0 = 0,5b)$ (g): (1 – $\beta_2 = 0,1$; 2 – $\beta_2 = 0,6$; 3 – $\beta_2 = 1,2$); (h): (1 – $\alpha_1 = 0,1$; 2 – $\alpha_1 = 0,659$; 3 – $\alpha_1 = 1,2$); (i): (1 – $\alpha_1 = 0,1$; 2 – $\alpha_1 = 0,7$; 3 – $\alpha_1 = 1,2$).

As can be seen from (7)

$$\upsilon(s_x, s_y, s_z) = \upsilon(s_y, s_x, s_z). \quad (8)$$

For basic magnetic oscillations H_{101} (1), ($e_{\varphi'} = -h_1 \frac{i\omega\mu_0}{\beta} J_1(\beta\rho') \cos\beta_z z'$) the function (7) also is even in all arguments:

$$\upsilon(-s_x, s_y, s_z) = \upsilon(s_x, -s_y, s_z) = \\ = \upsilon(s_x, s_y, -s_z) = \upsilon(s_x, s_y, s_z). \quad (9)$$

Here and below $\chi_{sx} = s\pi/a$; $\chi_{uy} = u\pi/b$; $\Gamma = \sqrt{(\chi_{sx})^2 + (\chi_{uy})^2 - k_0^2}$ is the wave numbers of a rectangular waveguide with a cross section $a \times b$ (Fig. 2, a); $\varepsilon_{1r} = \varepsilon_1/\varepsilon_0$; $p_\perp = \beta r_0$; $p_z = \beta_z L/2$ and $q_\perp = k_0 r_0$; $q_z = k_0 L/2$ is the characteristic parameters; r_0 – radius; L – height of the resonator (Fig. 1);

$$\omega_z(s_z) = \frac{1}{p_z^2 - (s_z L/2)^2} \times \\ \times \left\{ \begin{matrix} -i\,[p_z \cos p_z \sin(s_z L/2) - s_z L/2 \sin p_z \cos(s_z L/2)] \\ [p_z \sin p_z \cos(s_z L/2) - s_z L/2 \cos p_z \sin(s_z L/2)] \end{matrix} \right\};$$

$$\tilde{k}_0 = 4\pi q_\perp^2 p_z \frac{(\varepsilon_{1r} - 1)^2}{\varepsilon_{1r}} \frac{L}{|\Gamma|\, ab \mathrm{v}_0};$$

$$\mathrm{v}_0 = \left[J_1^2(p_\perp) - J_0(p_\perp) J_2(p_\perp)\right](2p_z + \sin 2p_z).$$

The results of calculating the dependences of the coupling coefficients on the coordinates and the angle of rotation of the resonator are shown in Fig. 2 for H_{101} oscillations and $\varepsilon_{1r} = 36$; $L/2r_0 = 0,4$; $f_0 = 7$ GHz; $a \times b = 35 \times 15\,\mathrm{mm}^2$. In Fig. 2, d, f shows the possibility of changing the coupling coefficient by rotating the resonator about the x and y axes, as well as by varying the coordinate of the resonator center along the x-axis.

But the most nontrivial results are obtained by the rotation of the resonator about the y-axis (Fig. 2, e, h). In the first case (Fig. 2, h; e curve 1), we see that the coupling coefficient does not depend on the orientation of the DR axis at the points of "circular polarization" of the magnetic field of the waveguide:

$$x_{O1} = \frac{1}{\chi_{1x}} arcctg \frac{|\Gamma|}{\chi_{1x}}, \\ \text{and} \quad x_{O2} = a - x_{O1}. \quad (10)$$

In the second case (Fig. 2, e), the coupling coefficient does not depend on the transverse coordinates

(Fig. 2, h curve 2) when the axis of the DR is rotated by an angle:

$$\alpha_C = arcctg\frac{|\Gamma|}{\chi_{1x}}. \tag{11}$$

In limiting cases $\alpha_1, \beta_2 = 0, \pm\pi/2$, expressions (4-6) coincide with the coupling coefficients for standard positions of the resonator in the waveguide.

3 Calculating mutual coupling coefficients of rotation Cylindrical DRs in the rectangular waveguide

Calculation of the mutual coupling coefficients of identical cylindrical DRs with oscillations (1) in a cut-off rectangular waveguide leads to the expressions in the waveguide coordinate system (x, y, z) (Fig. 3, a-c):

$$k_{12} = k_{12}^0 \sum_{t \geq t_M} f_t^1(\mp i\Gamma)(f_t^2(\pm i\Gamma))^* e^{-\Gamma|z_2 - z_1|}, \tag{12}$$

here

$$k_{12}^0 = -2\pi\frac{q_\perp^2 p_z}{k_0 \mathrm{w}_0}\frac{(\varepsilon_{1r}-1)^2}{\varepsilon_{1r}}\frac{L}{\mathrm{v}_0};$$

a) in the case of rotation of the resonator axis about the x axis (Fig. 3, a)

$$\begin{aligned} f_t^n(\mp i\Gamma) = &\cos(\chi_{1x}x_n)\left\{[E_{x0}^*(\chi_{uy}\cos\beta_n \mp i\Gamma\sin\beta_n) - E_{y0}^*\chi_{sx}\cos\beta_n + iE_{z0}^*\chi_{sx}\sin\beta_n]\times\right.\\ &\times e^{-i\chi_{uy}y_n}\upsilon(\chi_{sx}, (\chi_{uy}\cos\beta_n \mp i\Gamma\sin\beta_n), (\chi_{uy}\sin\beta_n \pm i\Gamma\cos\beta_n)) +\\ &+ [E_{x0}^*(\chi_{uy}\cos\beta_n \pm i\Gamma\sin\beta_n) - E_{y0}^*\chi_{sx}\cos\beta_n + iE_{z0}^*\chi_{sx}\sin\beta_n]\times\\ &\left.\times e^{i\chi_{uy}y_n}\upsilon(\chi_{sx}, (\chi_{uy}\cos\beta_n \pm i\Gamma\sin\beta_n), -(\chi_{uy}\sin\beta_n \mp i\Gamma\cos\beta_n))\right\}; \end{aligned} \tag{13}$$

b) rotation of the resonator relative to the y axis of the waveguide (Fig. 3, b)

$$\begin{aligned} f_t^n(\mp i\Gamma) = &\cos(\chi_{uy}y_n)\left\{[E_{x0}^*\chi_{uy}\cos\alpha_n - E_{y0}^*(\chi_{sx}\cos\alpha_n \mp i\Gamma\sin\alpha_n) - iE_{z0}^*\chi_{uy}\sin\alpha_n]\times\right.\\ &\times e^{-i\chi_{sx}x_n}\upsilon((\chi_{sx}\cos\alpha_n \mp i\Gamma\sin\alpha_n), \chi_{uy}, (\chi_{sx}\sin\alpha_n \pm i\Gamma\cos\alpha_n)) +\\ &+ [E_{x0}^*\chi_{uy}\cos\alpha_n - E_{y0}^*(\chi_{sx}\cos\alpha_n \pm i\Gamma\sin\alpha_n) + iE_{z0}^*\chi_{uy}\sin\alpha_n]\times\\ &\left.\times e^{i\chi_{sx}x_n}\upsilon((\chi_{sx}\cos\alpha_n \pm i\Gamma\sin\alpha_n), \chi_{uy}, -(\chi_{sx}\sin\alpha_n \mp i\Gamma\cos\alpha_n))\right\}; \end{aligned} \tag{14}$$

c) rotation of the resonator relative to the z axis of the waveguide (Fig. 3, c)

$$\begin{aligned} f_t^n(\mp i\Gamma) = &[\pm i\Gamma(E_{x0}^*\sin\alpha_n + E_{y0}^*\cos\alpha_n) - i(\chi_{sx}\sin\alpha_n + \chi_{uy}\cos\alpha_n)E_{z0}^*]\times\\ &\times \sin(\chi_{sx}x_n + \chi_{uy}y_n)\upsilon(-i\Gamma, (\chi_{sx}\sin\alpha_n + \chi_{uy}\cos\alpha_n), (\chi_{sx}\cos\alpha_n - \chi_{uy}sin\alpha_n)) -\\ &- [\pm i\Gamma(E_{x0}^*\sin\alpha_n - E_{y0}^*\cos\alpha_n) - i(\chi_{sx}\sin\alpha_n - \chi_{uy}\cos\alpha_n)E_{z0}^*]\times\\ &\times \sin(\chi_{sx}x_n - \chi_{uy}y_n)\upsilon(-i\Gamma, (\chi_{sx}\sin\alpha_n - \chi_{uy}\cos\alpha_n), (\chi_{sx}\cos\alpha_n + \chi_{uy}sin\alpha_n)), \end{aligned} \tag{15}$$

where

$$E_{x0} = i\chi_{uy}\left|\mathrm{w}_{su}^H\right| h_{su}^0; \; E_{y0} = -i\chi_{sx}\left|\mathrm{w}_{su}^H\right| h_{su}^0$$

for H-waves;

$$E_{x0} = \mp\chi_{sx}e_{su}^0; \; E_{y0} = \mp\chi_{uy}e_{su}^0; \; E_{z0} = \chi^2/|\Gamma|\, e_{su}^0$$

for E-waves of the waveguide;

$$h_{su}^0 = \frac{2}{\chi}\left[\frac{|\Gamma|}{\omega\mu_0 ab}\right]^{1/2}\frac{1}{(1+\delta_{s0}+\delta_{u0})^{1/2}};$$

$$e_{su}^0 = \frac{2}{\chi}\left[\frac{|\Gamma|}{\omega\varepsilon_0 ab}\right]^{1/2}(1-\delta_{s0}-\delta_{u0})^{1/2};$$

$$\mathrm{w}_{su}^H = \omega\mu_0/\Gamma; \; \chi = [\chi_{sx}^2 + \chi_{uy}^2]^{1/2};$$

(x_n, y_n, z_n) is the coordinates of the center of the n-th resonator in the waveguide coordinate system; β_n, α_n is the angles between the z'-axes of the local coordinate system of the resonator and the z-axes waveguide (Fig. 3, a, b), or x-axes waveguide (Fig. 3, c) $(n = 1, 2)$.

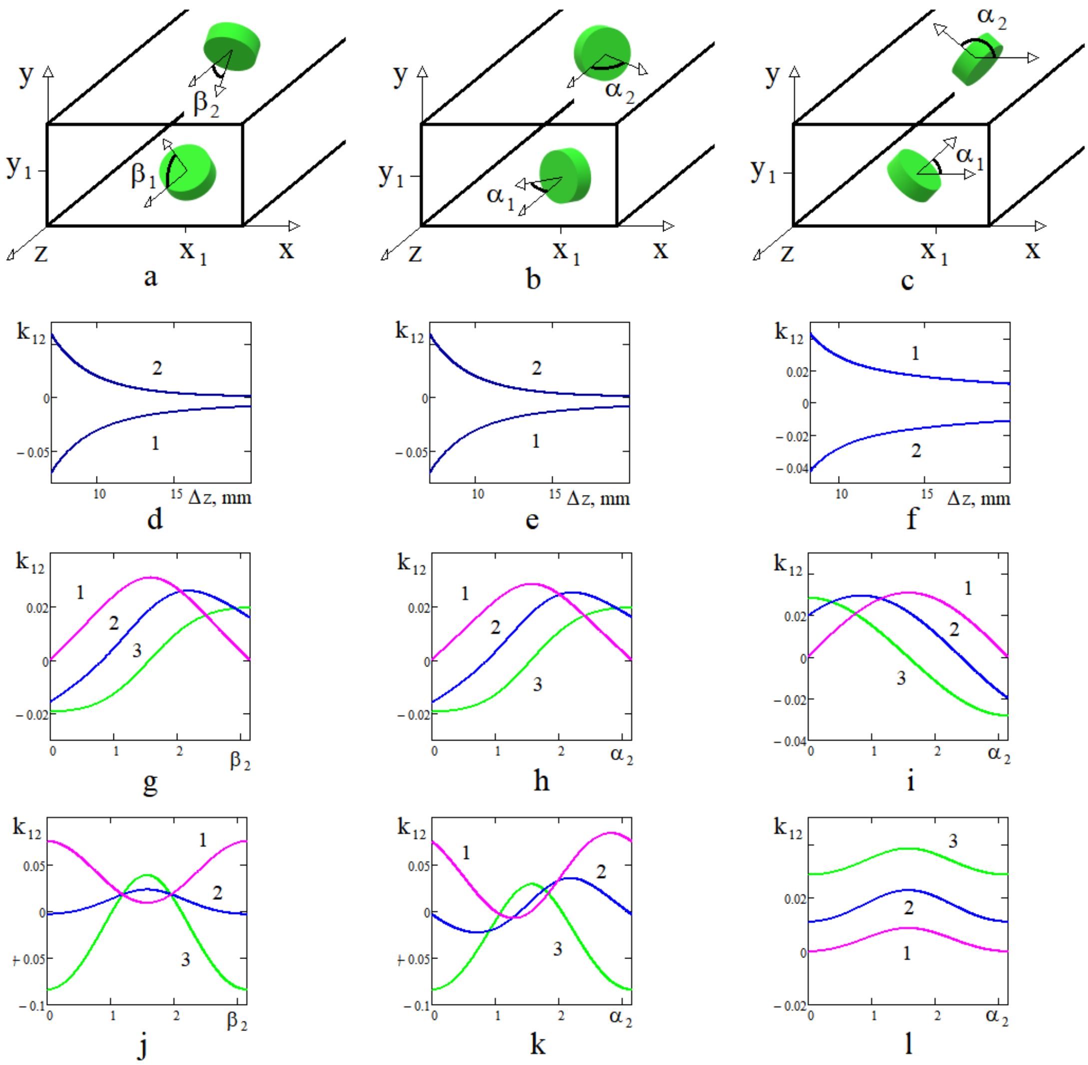

Fig. 3. Simple rotations of the dielectric resonator relatively the x-axis (a); y-axis (b); z-axis (c) of the rectangular waveguide. Dependence of mutual coupling coefficients on the DR longitudinal coordinate $\Delta z = |z_1 - z_2|$ for $y_1 = y_2 = 0,5b$; (d) $\beta_1 = 0$; 1 – $\beta_2 = 0$; 2 – $\beta_2 = \pi$: (e, f) $\alpha_1 = 0$; 1 – $\alpha_2 = 0$; 2 – $\alpha_2 = \pi$. Dependence of the coupling coefficients on the rotation angle of the 2 DR for $x_1 = x_2 = 0,5a$; $y_1 = y_2 = 0,5b$; $\Delta z = 10\,\text{mm}$: (g-i) 1 – $\beta_1, \alpha_1 = 0$; 2 – $\beta_1, \alpha_1 = \pi/4$; 3 – $\beta_1, \alpha_1 = \pi/2$. Dependence of the coupling coefficients on the rotation angle ((j) $\beta_1 = \beta_2$; (k-l) $\alpha_1 = \alpha_2$) of the DRs for $x_1 = 0,25a$; $y_1 = y_2 = 0,5b$; $\Delta z = 10\,\text{mm}$, and (j-l) 1 – $x_2 = 0,25a$; 2 – $x_2 = 0,5a$; 3 – $x_2 = 0,75a$.

4 Analysis of mutual coupling coefficients

Relations (12-15) make it possible to calculate the mutual coupling coefficients of the DRs in the cases of the considered rotations of their axes. The found expressions in particular cases when the axes of the resonators become parallel to one of the coordinate axes of the waveguide coincide with those obtained earlier.

In Fig. 3 shows the dependences of the coupling coefficients on the coordinates and angles of rotation of the resonators. As follows from the results obtained, the sign of the coupling coefficient can change with a smooth change in the structure parameters. This reason has a twofold nature: purely mathematical – due to a change in the direction of the field in one of the resonators and physical, due to different values of the coupling coefficients for different relative positions of the resonators in the waveguide. An example of a purely mathematical reason for a change in the sign of a link is shown in Fig. 3, d-f. If the z'-axis of one of the resonators changes its direction by an angle π, the direction of the field changes sign with respect to the direction of the local coordinate system. In this case, the structure remains the same, but the sign of the coupling coefficient changes to the opposite (see for example curves 2 in Fig. 3, d-f).

In the second case, as is known, the sign of the coupling is determined by the mutual position of the resonators. For coaxial arrangement of resonators if $\Delta z > L$ and for example $\Delta\rho = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} < 2r_0$, then $k_{12} < 0$ (see curves 1 in Fig. 3, d-e).

If the axes of the resonators are parallel, for example, the x axis and $\Delta z > 2r_0$, then $k_{12} > 0$ (see curve 1 in Fig. 3, f). This is a purely physical property of coupled oscillations, determined by the nature of the spatial distribution of the resonator fields.

In Fig. 3, g-l shows the calculated dependences of the coupling on the mutual orientations of the axes of the DR and waveguide. The effect of the independence of the value of the mutual coupling from the transverse coordinate of the second resonator at a certain angle of inclination of their axes, shown in Fig. 3, j, in the case of rotation relative the x axis.

As follows from the data shown in Fig. 3, g-k, for each resonator 1 position, there is an optimal resonator 2 position at which the coupling coefficient between them reaches a maximum.

Discussion and Conclusion

In this paper, we obtained analytical expressions for the coupling coefficients, as well as the mutual coupling coefficients of cylindrical DRs in the case of their rotation about one of the axes of a rectangular waveguide.

1) The conditions are established under which the coupling coefficient with the main propagating wave H_{10} of a rectangular waveguide at the point of its "circular polarization" of the magnetic field does not depend on the angle of rotation of the resonator (10).

2) It is shown that the coupling coefficient with the fundamental wave H_{10} does not depend on the transverse coordinates of the DR if the resonator axis is rotated relative to the axis of the waveguide in a plane parallel to the wide wall by an angle determined by the dimensions of the waveguide and the frequency of the basic magnetic type of oscillations H_{101}. A simple formula is given for determining the angle of rotation of the resonator (11).

3) The existence of angles of rotation is established at which the coefficients of their mutual coupling do not depend on the transverse coordinates of cylindrical DGs in a rectangular waveguide (Fig. 3, j).

4) For each resonator 1 position, there is an optimal resonator 2 position at which the mutual coupling coefficient between them reaches a maximum.

The proposed theory can be used to calculate and analyze complex band-pass or band-stop filters, multiplexers, and other communication devices built on dielectric resonators in the microwave, infrared and optical wavelength ranges.

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Обертання циліндричних діелектричних резонаторів в прямокутному хвилеводі

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Розраховані коефіцієнти зв'язку діелектричних резонаторів циліндричної форми з регулярним прямокутним хвилеводом за умови обертання їх осей. Розглянуто залежності коефіцієнтів зв'язку від кутів обертання і поперечних координат резонатора в разі збудження в них основних магнітних типів власних коливань. Показана незалежність величини зв'язку від кутів обертання в точках кругової поляризації основної хвилі прямокутного хвилеводу. Також встановлено умови для кута обертання осі резонатора, визначаємого розмірами поперечного перерізу хвилеводу і частотою основного типу власних коливань, при виконанні якого, коефіцієнт зв'язку стає постійним в поперечній площині симетрії хвилеводу. Виведено нові аналітичні вирази для коефіцієнтів взаємного зв'язку однакових діелектричних резонаторів циліндричної форми при обертанні їх осей щодо прямокутного хвилеводу. Досліджено залежності коефіцієнтів взаємного зв'язку від кутів обернення та координат резонаторів. Встановлено умови при виконанні яких коефіцієнти взаємного зв'язку двох циліндричних резонаторів не залежать від їх поперечної координати у площини симетрії хвилеводу. Обговорюються причини зміни знаків коефіцієнтів зв'язку резонаторів при їх обертанні. Відзначається ефект виникнення екстремумів зв'язку для різних відносних орієнтацій діелектричних резонаторів. В окремих випадках паралельності осей резонаторів однієї з координатних осей хвилеводу, знайдені в роботі аналітичні вирази збігаються з отриманими раніше. Знайдені аналітичні результати дозволяють будувати аналітичні моделі смугових і режекторних фільтрів, що значно скорочує час розрахунків та оптимізації складних багаторезонаторних структур мікрохвильових і оптичних систем зв'язку.

Ключові слова: коефіцієнт зв'язку; коефіцієнт взаємного зв'язку; обертання; циліндричний діелектричний резонатор; хвилевід

Вращения цилиндрических диэлектрических резонаторов в прямоугольном волноводе

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Рассчитаны коэффициенты связи диэлектрических резонаторов цилиндрической формы с регулярным прямоугольным волноводом при условии вращения их осей. Рассмотрены зависимости коэффициентов связи от углов вращения и поперечных координат резонатора в случае возбуждения в них основных магнитных типов собственных колебаний. Показана независимость величины связи от углов вращения в точках круговой поляризации основной волны прямоугольного волновода. Также установлено условие для угла вращения оси резонатора, определяемое размерами поперечного сечения волновода и частотой основного типа собственных колебаний, при выполнении которого, коэффициент связи становится постоянным в поперечной плоскости симметрии волновода. Выведены новые аналитические выражения для коэффициентов взаимной связи одинаковых диэлектрических резонаторов цилиндрической формы при вращении их осей относительно прямоугольного волновода. Исследованы зависимости коэффициентов взаимной связи от углов вращения и координат резонаторов. Установлены условия при выполнении которых коэффициенты взаимной связи двух цилиндрических резонаторов не зависят от их поперечной координаты в плоскости симметрии волновода. Обсуждаются причины изменения знака коэффициентов связи резонаторов при их вращении. Отмечается эффект возникновения экстремумов связи для различных относительных ориентаций диэлектрических резонаторов. В частных случаях параллельности осей резонаторов одной из координатных осей волновода найденные в работе аналитические выражения совпадают с полученными ранее. Полученные аналитические результаты позволяют строить аналитические модели полосовых и режекторных фильтров, значительно сокращать время вычислений и оптимизировать сложные многорезонаторные структуры микроволновых и оптических систем связи.

Ключевые слова: коэффициент связи; коэффициент взаимной связи; вращение; цилиндрический диэлектрический резонатор; волновод