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COUPLING COEFFICIENTS OF THE DISK DIELECTRIC MICRORESONATORS WITH WHISPERING GALLERY MODES[1]

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КОЕФІЦІЄНТИ ЗВ'ЯЗКУ ДИСКОВИХ ДІЕЛЕКТРИЧНИХ РЕЗОНАТОРІВ З КОЛИВАННЯМИ ШЕПОЧУЧЕЇ ГАЛЕРЕЇ

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Introduction

Disk dielectric microresonators with whispering gallery (WG) modes quite naturally inscribes in the planar integral circuits. Today ones are actively studying for purpose of their application in the different devices of the optical, infrared and terahertz wavelength ranges [1 – 7]. For calculation and optimization of the device parameters it's convenient to carried out on basis of electrodynamic modeling with using coupling coefficients [8]. For that it's necessary be able to calculates microresonators both coupling coefficients with various transmission lines, and mutual coupling coefficients between its. Eigenoscillations of two disk dielectric microresonators was considered in [4 – 7], but its coupling coefficients did not calculated and not studied in full measure. The goal of the present work is the calculation and analysis of the coupling coefficients of the disk microresonators with WG modes in the Open space.

Calculation eigenoscillation field of the disk microresonator

For the coupling coefficients calculation it's necessary information about microresonator eigenoscillation fields. Most simple analytical presentation the field within dielectric cylinder can be obtained in the form of so-called one-wave approximation [9]. Toward this end, writes an electromagnetic field in the cylindrical coordinate system (ρ, α, z) (see fig. 1,a), approximately in the form of hybrid standing wave of the circular dielectric waveguide section:

$$e_\rho = \left[e_1 \frac{\beta_z}{\beta} J'_m(\beta\rho) + h_1 m \frac{i\omega\mu_0}{\beta} \cdot \frac{J_m(\beta\rho)}{\beta\rho} \right] \begin{Bmatrix} \sin m\alpha \\ \cos m\alpha \end{Bmatrix} \begin{bmatrix} \cos\beta_z z \\ -\sin\beta_z z \end{bmatrix};$$

[1] Електронний варіант статті: http://radap.kpi.ua/radiotechnique/article/view/1064

$$e_\alpha = \left[e_1 m \frac{\beta_z}{\beta} \cdot \frac{J_m(\beta\rho)}{\beta\rho} + h_1 \frac{i\omega\mu_0}{\beta} \cdot J'_m(\beta\rho) \right] \begin{Bmatrix} \cos m\alpha \\ -\sin m\alpha \end{Bmatrix} \begin{bmatrix} \cos\beta_z z \\ -\sin\beta_z z \end{bmatrix};$$

$$e_z = e_1 J_m(\beta\rho) \begin{Bmatrix} \sin m\alpha \\ \cos m\alpha \end{Bmatrix} \begin{bmatrix} \sin\beta_z z \\ \cos\beta_z z \end{bmatrix}; \qquad (1)$$

$$h_\rho = \left[e_1 m \frac{i\omega\varepsilon_1}{\beta} \cdot \frac{J_m(\beta\rho)}{\beta\rho} - h_1 \frac{\beta_z}{\beta} \cdot J'_m(\beta\rho) \right] \begin{Bmatrix} \cos m\alpha \\ -\sin m\alpha \end{Bmatrix} \begin{bmatrix} \sin\beta_z z \\ \cos\beta_z z \end{bmatrix};$$

$$h_\alpha = \left[-e_1 \frac{i\omega\varepsilon_1}{\beta} \cdot J'_m(\beta\rho) + h_1 m \frac{\beta_z}{\beta} \cdot \frac{J_m(\beta\rho)}{\beta\rho} \right] \begin{Bmatrix} \sin m\alpha \\ \cos m\alpha \end{Bmatrix} \begin{bmatrix} \sin\beta_z z \\ \cos\beta_z z \end{bmatrix};$$

$$h_z = h_1 J_m(\beta\rho) \begin{Bmatrix} \cos m\alpha \\ -\sin m\alpha \end{Bmatrix} \begin{bmatrix} \cos\beta_z z \\ -\sin\beta_z z \end{bmatrix};$$

Here $\rho \le r_0$; $|z| \le L/2$, where L – is the height, and r_0 – is the radius of the dielectric cylinder (fig.1, a); $J'_m(x)$ – is the derivative of Bessel function of the first kind of the m-th order; e_1, h_1 – is the electrical and magnetic field amplitudes; β, β_z – is the wave numbers.

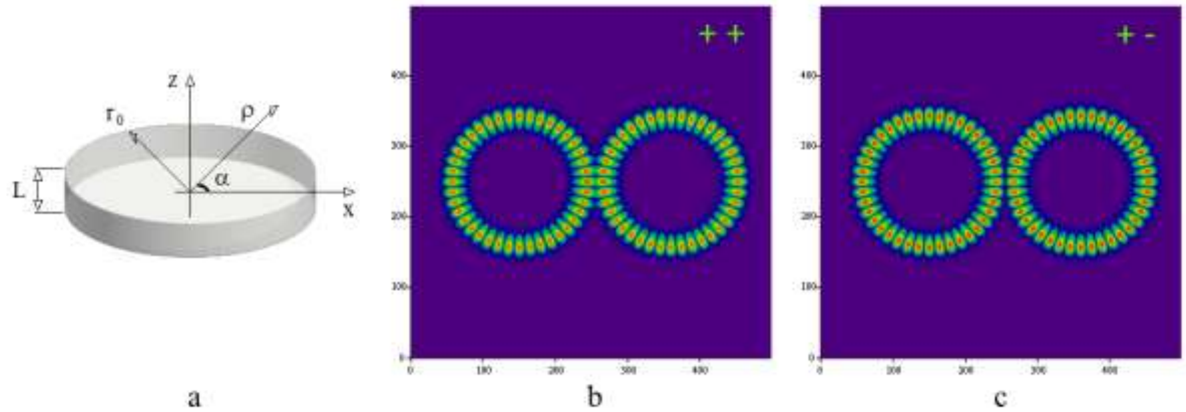

Fig. 1. A – sketch of the disk microresonator in the open space. The field of the coupling oscillations of two equal disk microresonators with even (b); odd (c) $EH^{+}_{1,20,1}$ modes; ($m = 20$): $\varepsilon_{1r} = 9,6$; $\Delta = L/2r_0 = 0,2$.

Approximate solution of the microresonator field in the external space can be made by various manner [1]. Depending on that, will be determined the constant values e_1, h_1, β, β_z. For a small dielectric permittivity most often uses a relationship, proportional to the Hankel function of the second kind: $H^{(2)}_m(\beta_0\rho)$. At the same time, for the receiving of real values of characteristic parameters, the Hankel functions should be replaced to the Neumann functions $H^{(2)}_m(\beta_0\rho) \approx Y_m(\beta_0\rho)$. At that, the amplitudes e_1, h_1 convenient to presents in the standardize form: $e_1 = ae_0$ and $h_1 = ah_0$, where the e_0, h_0 can be determined from more simple equations:

$$h_0 + e_0 = \frac{J_{m-1}(p_\perp)}{p_\perp J_m(p_\perp)} + \frac{Y_{m-1}(p_{0\perp})}{p_{0\perp} Y_m(p_{0\perp})};\; h_0 - e_0 = \frac{J_{m+1}(p_\perp)}{p_\perp J_m(p_\perp)} + \frac{Y_{m+1}(p_{0\perp})}{p_{0\perp} Y_m(p_{0\perp})}. \qquad (2)$$

Where dimensionless parameters: $p_\perp = \beta r_0$; $p_{0\perp} = \beta_0 r_0$, and also $p_z = \beta_z L/2$ and $p_{0z} = \beta_{0z} L/2$ can be calculated from combined equations (see, for example, [1]):

$$\left[\frac{\varepsilon_{1r}}{\beta}\frac{J'_m(\beta\, r_0)}{J_m(\beta\, r_0)} + \frac{1}{\beta_0}\frac{Y'_m(\beta_0\, r_0)}{Y_m(\beta_0\, r_0)}\right]\cdot\left[\frac{1}{\beta}\frac{J'_m(\beta\, r_0)}{J_m(\beta\, r_0)} + \frac{1}{\beta_0}\frac{Y'_m(\beta_0\, r_0)}{Y_m(\beta_0\, r_0)}\right] = \left(\frac{m\beta_z}{k_0 r_0}\right)^2\left(\frac{1}{\beta^2} + \frac{1}{\beta_0^2}\right)^2$$

;

$$\beta^2 + \beta_z^2 = k_1^2; \quad \beta_0^2 + \beta_z^2 = k_0^2; \quad \beta^2 - \beta_{0z}^2 = k_0^2; \qquad (3)$$

as well as for the $HE^{\pm}_{nml}$ modes:

$$\beta_{0z} = \beta_z \cdot \begin{bmatrix} -\mathrm{ctg}\beta_z L/2 \\ \mathrm{tg}\beta_z L/2 \end{bmatrix};$$

for the $EH^{\mp}_{nml}$ modes:

$$\varepsilon_{1r}\beta_{0z} = \beta_z \cdot \begin{bmatrix} \mathrm{tg}\beta_z L/2 \\ -\mathrm{ctg}\beta_z L/2 \end{bmatrix}.$$

Here n defines a number of half-waves, located in the radial direction inside dielectric cylinder, and l defines a half-waves number, located in the direction of z-axis in the microresonator material. Sign $+(-)$ in the cases of $HE^{\pm}_{nml}$ mode responds to an even (odd) distribution of z-component magnetic field; and the $-(+)$ for the $EH^{\mp}_{nml}$ mode responds to an even (odd) distribution of z-component electric field in the microresonator (see fig. 1) relative a plane of symmetry $z = 0$ (fig. 1, a).

Coupling coefficient calculating

Allocation microresonators side by side with each other leads to the coupling oscillations appearance. The fields and frequencies of this oscillations defines by values of the coupling coefficients. In the common case, the coupling coefficient we determined as a surface integral:

$$\kappa_{sn} = \frac{i}{2\omega_0 w_n (1+\delta_{sn})} \oint_{s_n} \left\{\left[\vec{e}_s, \vec{h}_n^{\,*}\right] + \left[\vec{e}_n^{\,*}, \vec{h}_s\right]\right\}\vec{n}ds, \qquad (4)$$

expressed via the eigenmode field $(\vec{e}_s, \vec{h}_s)$ of one (s-th) microresonator on the surface of another (n-th) microresonator. Here $s, n = 1, 2$; and $\vec{n}$ – is the normal to the surface s_n of n-th microresonator, ω_0 – is the resonance frequency; w_n – is the energy, stored in the dielectric.

In special case of one isolated microresonator: ($s = n$) $\kappa_{nn} = iQ_n^{-1}$, where

Q_n is the usual quality factor.

As follow from [8], eigenoscillations of two identical microresonators take on form of even and odd spatial field distributions with respect to symmetry plane, located between ones. In this case (4): $\kappa_{11} = \kappa_{22}$; $\kappa_{12} = \kappa_{21}$ and

$$b_2^1 = +b_1^1;\ b_2^2 = -b_1^2;\ \lambda_{1,2} = i\tilde{k}_1 \pm \kappa_{21},$$

responding to a cophased, or even, and antiphased, or odd, field distribution of the coupling oscillations (fig. 1, b, c). From here, using a definition [8], and determined the real and imaginary part of the frequency coupling oscillations:

$$\lambda = 2 \cdot \left(\frac{\delta\omega}{\omega_0} + i\frac{\omega''}{\omega_0}\right),$$

obtain:

$$\mathrm{Re}\left(\frac{\omega^{1,2}}{\omega_0}\right) = 1 \pm \frac{1}{2}\mathrm{Re}(\kappa_{21});\quad \mathrm{Im}\left(\frac{\omega^{1,2}}{\omega_0}\right) = \frac{1}{2}[\tilde{k}_1 \pm \mathrm{Im}(\kappa_{21})],$$

or

$$\mathrm{Re}(\kappa_{21}) = \frac{\delta\omega}{\omega_0} = \mathrm{Re}\left(\frac{\omega^1 - \omega^2}{\omega_0}\right);\ \mathrm{Im}(\kappa_{21}) = \mathrm{Im}\left(\frac{\omega^1 - \omega^2}{\omega_0}\right) = \frac{1}{2}\left(\frac{1}{Q^1} - \frac{1}{Q^2}\right). \qquad (5)$$

Thus it's seen, that the real part of the mutual coupling coefficient (5) now expressed via relative frequency difference, as well as an imaginary part of the coupling coefficient is proportional to the power radiation difference of even and odd coupling modes. As follows from (5), the real and imaginary part of the coupling coefficient can take as a positive well as a negative values.

Direct calculation of the integral (4) is sufficiently complicated problem, since we don't know external fields of the disk microresonators. The integral (4) can be calculated, on a base of early known analytical expression for the coupling coefficients, of the Cylindrical DRs in the Rectangular metal waveguide. In this case, required analytical expressions for mutual coupling coefficients κ_{12} , can be received by transferring the waveguide walls to the infinity. Using necessary expressions, for example, 3. of the [10], as well as [11, 12], after simplifications obtain:

A. In the case of two identical microresonators, with the same parity of each field, relatively a plane of symmetry: $y - y_s = 0$ ($s = 1,2$) (see fig. 2, a), the mutual coupling coefficients can be obtained in the form:

in the area: $\Delta z \geq 2r_0$; $\Delta x \leq L$:

$$\kappa_{1,2} = \frac{32i}{\varepsilon_{1r} w_{nml}} \cdot \frac{p_z}{q_z} \cdot \qquad (6)$$

$$\cdot \int_0^\infty F(\xi)^2 \left\{ H_0^{(2)}(k_0\Delta\rho\sqrt{1-\xi^2}) \mp \cos(2m\Delta\varphi) H_{2m}^{(2)}(k_0\Delta\rho\sqrt{1-\xi^2}) \right\} \cos(k_0\Delta x\xi)\, d\xi,$$

where the top sign of (6) responds to even field distribution, relatively the plane of symmetry and a bottom one responds to odd field distribution (see 1); $\sin\Delta\varphi = \Delta z / \Delta\rho$; $\Delta\rho = \sqrt{\Delta y^2 + \Delta z^2}$; $\Delta x = x_2 - x_1$; $\Delta y = y_2 - y_1$; $\Delta z = |z_2 - z_1|$.
Here

$$F(\xi)^2 = |F_1(\xi)|^2 + |F_2(\xi)|^2;$$

$$F_1(\xi) = \frac{e_0}{\beta_z}\{[k_1^2\sqrt{1-\xi^2}J'_m(p_\perp)J_m(q_\perp\sqrt{1-\xi^2}) - k_0\beta J_m(p_\perp)J'_m(q_\perp\sqrt{1-\xi^2})]\cdot$$
$$\cdot\frac{1}{\beta_z^2-(k_0\,\xi)^2}\begin{bmatrix}(\beta_z\cos p_z\sin q_z\xi - k_0\xi\sin p_z\cos q_z\xi)\\(\beta_z\sin p_z\cos q_z\xi - k_0\xi\cos p_z\sin q_z\xi)\end{bmatrix}+$$
$$+[\beta J_m(p_\perp)J'_m(q_\perp\sqrt{1-\xi^2}) - k_0\sqrt{1-\xi^2}J_m{}'(p_\perp)J_m(q_\perp\sqrt{1-\xi^2})]\cdot$$
$$\cdot\frac{1}{\beta^2-k_0^2(1-\xi^2)}\begin{bmatrix}(k_1^2\xi\sin p_z\cos q_z\xi - k_0\beta_z\cos p_z\sin q_z\xi)\\(k_1^2\xi\cos p_z\sin q_z\xi - k_0\beta_z\sin p_z\cos q_z\xi)\end{bmatrix}\}-$$
$$-h_0 m\frac{\xi(k_1^2-k_0^2)}{p_\perp k_0\sqrt{1-\xi^2}}J_m(p_\perp)J_m(q_\perp\sqrt{1-\xi^2})\cdot$$
$$\cdot\frac{1}{\beta_z^2-(k_0\,\xi)^2}\begin{bmatrix}(\beta_z\sin p_z\cos q_z\xi - k_0\xi\cos p_z\sin q_z\xi)\\(\beta_z\cos p_z\sin q_z\xi - k_0\xi\sin p_z\cos q_z\xi)\end{bmatrix}; \qquad (7)$$

$$F_2(\xi) = (k_1^2-k_0^2)\{e_0 m\frac{1}{p_\perp k_0\sqrt{1-\xi^2}}J_m(p_\perp)J_m(q_\perp\sqrt{1-\xi^2}) - h_0\frac{1}{\beta^2-k_0^2(1-\xi^2)}\cdot$$
$$\cdot[\beta J_m(p_\perp)J'_m(q_\perp\sqrt{1-\xi^2}) - k_0\sqrt{1-\xi^2}J_m{}'(p_\perp)J_m(q_\perp\sqrt{1-\xi^2})]\}\cdot$$
$$\cdot\frac{1}{\beta_z^2-(k_0\,\xi)^2}\begin{bmatrix}(\beta_z\sin p_z\cos q_z\xi - k_0\xi\cos p_z\sin q_z\xi)\\(\beta_z\cos p_z\sin q_z\xi - k_0\xi\sin p_z\cos q_z\xi)\end{bmatrix};$$

$e_0; h_0$ – is the normalized amplitudes, defined from (2);

$$w_{nml} = \Big[(e_0-h_0)^2[J_{m-1}^2(p_\perp) - J_m(p_\perp)J_{m-2}(p_\perp)]+$$
$$+(e_0+h_0)^2[J_{m+1}^2(p_\perp) - J_m(p_\perp)J_{m+2}(p_\perp)]+$$
$$+2(\frac{\beta}{k_1}h_0)^2[J_m^2(p_\perp) - J_{m-1}(p_\perp)J_{m+1}(p_\perp)]\Big]\begin{bmatrix}2p_z+\sin 2p_z\\2p_z-\sin 2p_z\end{bmatrix}+$$
$$+\Big[(\frac{k_1}{\beta_z}e_0 - \frac{\beta_z}{k_1}h_0)^2[J_{m-1}^2(p_\perp) - J_m(p_\perp)J_{m-2}(p_\perp)]+$$

$$+(\frac{k_1}{\beta_z}e_0+\frac{\beta_z}{k_1}h_0)^2[J_{m+1}^2(p_\perp)-J_m(p_\perp)J_{m+2}(p_\perp)]+$$

$$+2(\frac{\beta}{\beta_z}e_0)^2[J_m^2(p_\perp)-J_{m-1}(p_\perp)J_{m+1}(p_\perp)]\Bigg]\begin{bmatrix}2p_z-\sin 2p_z\\2p_z+\sin 2p_z\end{bmatrix};$$

$k_1=\sqrt{\varepsilon_{1r}}k_0$; $k_0=\omega_0/c$; ω_0 – is the circular resonance frequency; c – is the light velocity.

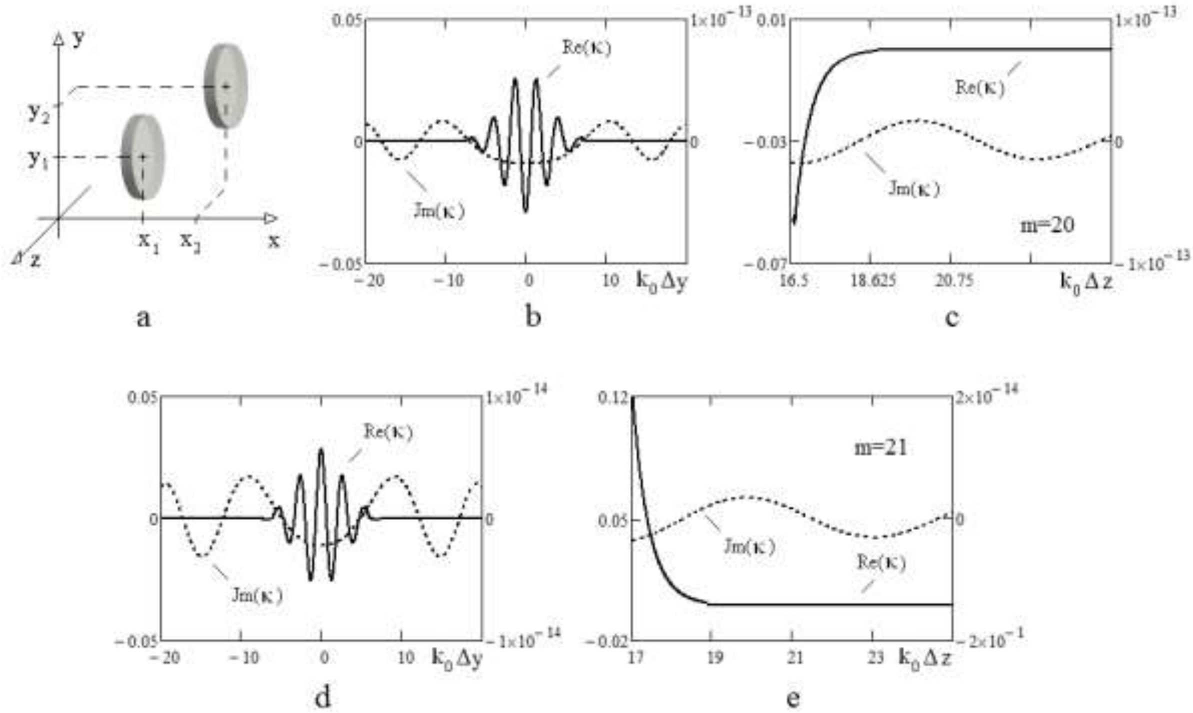

Fig. 2. Mutual coupling coefficients as a function coordinates of the microresonators with $\varepsilon_{1r}=9{,}6$; $\Delta=0{,}2$, for $EH^{+}_{1,20,1}$ modes; ($m=20$) (b, c); $EH^{+}_{1,21,1}$ (d, e).

B. In the case of identical disk microresonators with equal parity of the field distribution (see fig. 3, a) relatively the plane: $x-x_s=0$ ($s=1,2$),

in the area: $\Delta z\ge L$ obtains

$$\kappa_{1,2}=\frac{32i}{\varepsilon_{1r}w_{nml}}\cdot\frac{p_z}{q_z}\cdot \tag{8}$$

$$\cdot\int_0^\infty\frac{e^{-i\sqrt{1-\kappa^2}k_0\Delta z}}{\sqrt{1-\kappa^2}}\{[(k_1^2-k_0^2)^2\left|C(\kappa)\right|^2[J_0(k_0\Delta\rho\kappa)\pm(-1)^m\cos(2m\Delta\psi)J_{2m}(k_0\Delta\rho\kappa)]+$$

$$+\left|D(\kappa)\right|^2[J_0(k_0\Delta\rho\kappa)\mp(-1)^m\cos(2m\Delta\psi)J_{2m}(k_0\Delta\rho\kappa)]\}\kappa d\kappa,$$

where

$$C(\kappa)=[e_0 m\frac{1}{p_\perp k_0\kappa}J_m(p_\perp)J_m(q_\perp\kappa)-h_0\frac{1}{r_0(\beta^2-k_0^2\kappa^2)}\cdot$$
$$\cdot[p_\perp J_m(p_\perp)J'_m(q_\perp\kappa)-q_\perp\kappa J_m{}'(p_\perp)J_m(q_\perp\kappa)]\}\cdot$$
$$\cdot\frac{1}{\beta_z^2-k_0^2(1-\kappa^2)}\begin{bmatrix}\beta_z\sin p_z\cos q_z\sqrt{1-\kappa^2}-k_0\sqrt{1-\kappa^2}\cos p_z\sin q_z\sqrt{1-\kappa^2}\\ \beta_z\cos p_z\sin q_z\sqrt{1-\kappa^2}-k_0\sqrt{1-\kappa^2}\sin p_z\cos q_z\sqrt{1-\kappa^2}\end{bmatrix};\quad(9)$$

$$D(\kappa)=\frac{e_0}{\beta_z q_\perp}\{[k_0^2p_\perp J_m(p_\perp)J'_m(q_\perp\kappa)-k_1^2q_\perp\kappa J_m{}'(p_\perp)J_m(q_\perp\kappa)]\cdot$$
$$\cdot\frac{1}{\beta_z^2-k_0^2(1-\kappa^2)}\begin{bmatrix}k_0\sqrt{1-\kappa^2}\sin p_z\cos q_z\sqrt{1-\kappa^2}-\beta_z\cos p_z\sin q_z\sqrt{1-\kappa^2}\\ \beta_z\sin p_z\cos q_z\sqrt{1-\kappa^2}-k_0\sqrt{1-\kappa^2}\cos p_z\sin q_z\sqrt{1-\kappa^2}\end{bmatrix}-$$
$$-\frac{1}{\beta^2-(k_0\kappa)^2}[p_\perp J_m(p_\perp)J'_m(q_\perp\kappa)-q_\perp\kappa J_m{}'(p_\perp)J_m(q_\perp\kappa)]\cdot$$
$$\cdot\begin{bmatrix}k_0^2\beta_z\cos p_z\sin q_z\sqrt{1-\kappa^2}-k_1^2k_0\sqrt{1-\kappa^2}\sin p_z\cos q_z\sqrt{1-\kappa^2}\\ k_1^2k_0\sqrt{1-\kappa^2}\cos p_z\sin q_z\sqrt{1-\kappa^2}-k_0^2\beta_z\sin p_z\cos q_z\sqrt{1-\kappa^2}\end{bmatrix}\}-$$
$$-h_0m\frac{\left|\sqrt{1-\xi^2}\right|(k_1^2-k_0^2)}{p_\perp k_0\kappa}J_m(p_\perp)J_m(q_\perp\kappa)\cdot$$
$$\cdot\frac{1}{\beta_z^2-k_0^2(1-\kappa^2)}\begin{bmatrix}\beta_z\sin p_z\cos q_z\sqrt{1-\kappa^2}-k_0\sqrt{1-\kappa^2}\cos p_z\sin q_z\sqrt{1-\kappa^2}\\ k_0\sqrt{1-\kappa^2}\sin p_z\cos q_z\sqrt{1-\kappa^2}-\beta_z\cos p_z\sin q_z\sqrt{1-\kappa^2}\end{bmatrix}.$$

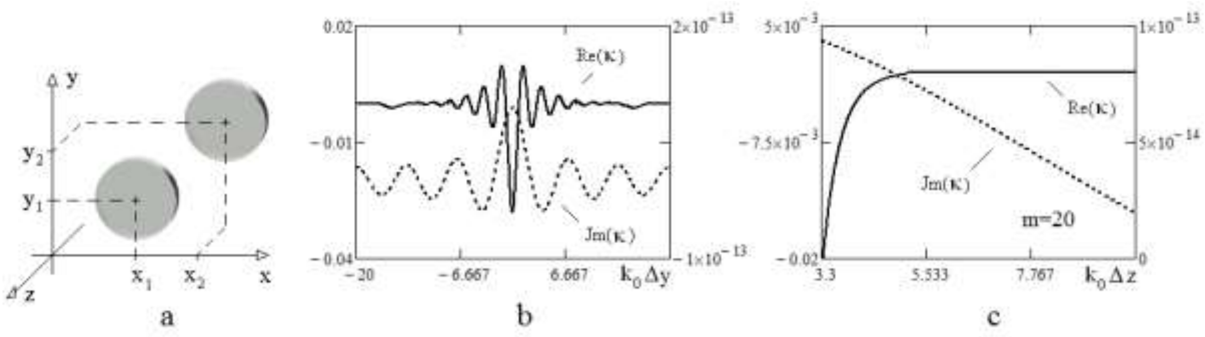

Fig. 3. Disk microresonator coupling coefficient dependencies for the $EH^{+}_{1,20,1}$ mode; ($m=20$); $\varepsilon_{1r}=9,6$; $\Delta=0,2$.

The integral convergence provides by choice of the radical signs for $\xi>1$: $\sqrt{\xi^2-1}=-i\sqrt{1-\xi^2}$ in the (6, 7), as well as for $\sqrt{\kappa^2-1}=-i\sqrt{1-\kappa^2}$ in the (8, 9).

The coupling between disk microresonator and the open space can be obtained from:

$$\kappa_{11} = \frac{32i}{\varepsilon_{1r} W_{nml}} \cdot \frac{p_z}{q_z} \cdot \int_0^1 \{(k_1^2 - k_0^2)^2 \left| C(\sqrt{1-\kappa^2}) \right|^2 + \left| D(\sqrt{1-\kappa^2}) \right|^2 \} d\kappa , \quad (9)$$

following from Helmholtz-Kirchhoff's integral theorem.

Coupling coefficient analysis

Discovered relationships valid for any eigenoscillations of the disk microresonators in the open space, but a greatest interest presents a WG modes, as is well known, possessed a highest possible quality. It's considering the coupling coefficients for the WG modes.

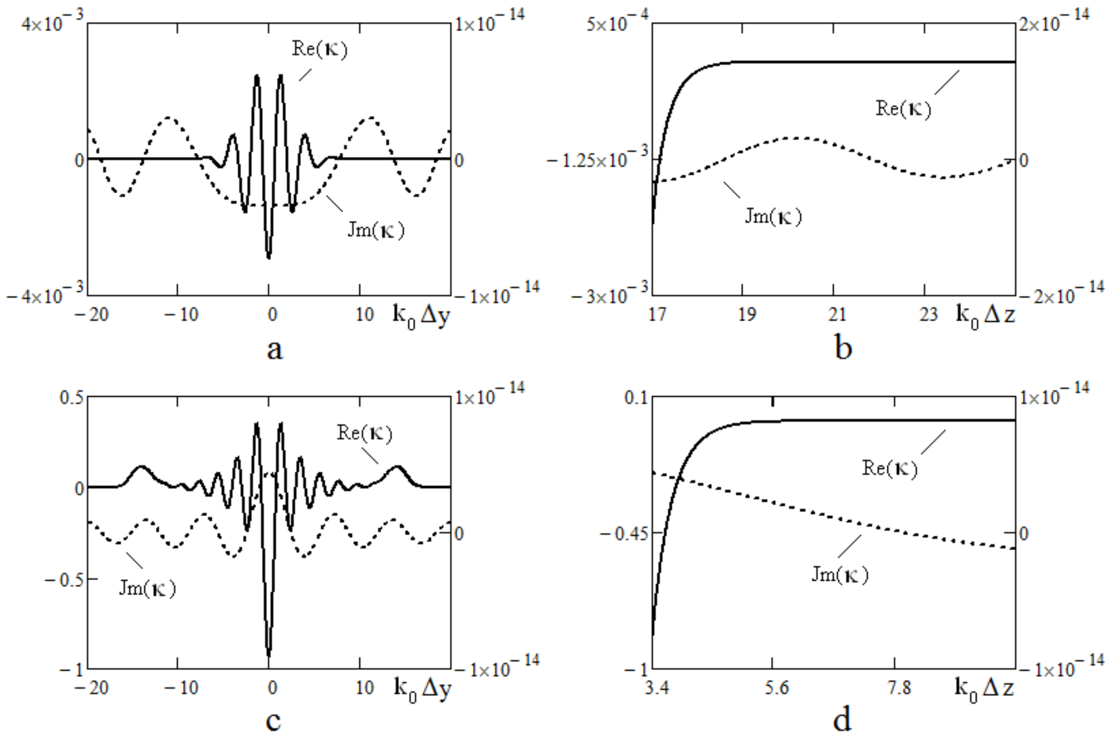

Fig. 4. Coupling coefficients as a functions of the microresonator coordinates: $\varepsilon_{1r} = 9,6$; $\Delta = 0,2$, for the $HE^{+}_{1,20,1}$ mode; a, b - for position, shown in fig. 2, a; c, d - for the position, shown in fig. 3.

Fig. 2 – 4 are showing a coupling coefficient dependences on Disk microresonator centers, composed of the dielectric with the relative permittivity $\varepsilon_{1r} = 9,6$ and the comparative dimensions $\Delta = L / 2r_0 = 0,2$. It's clear, that real part of the coupling coefficients has sufficiently visible values only in a small region, the resonator surfaces are closed with each other (fig. 2, b – e; fig. 3, b – c; fig. 4). Increasing distance between resonator centers accompanied by significant coupling coefficient decreasing. At that, relative motion in the tangent direction leads to complex interference (fig. 2, b, d; fig. 3, b; fig. 4, a, c) of their mutual influence, determining by significant eigenmode field variation nearby their sur-

face. The greatest amounts of coupling between microresonators appears on its coaxial arrangement (fig. 3, b, c). The signs of the coupling coefficients extreme values determines both by the azimuth numbers and mutual microresonators position (fig. 2, c, e).

Imaginary part of the coupling coefficients are more smooth functions on coordinates (fig. 2, b, d; fig. 3, b; fig. 4, a, c). For selected dielectric permittivity its values approximately one tenth degrees as many as real parts. The same order have a coupling coefficient of the disk microresonator in the open space (9).

Filter parameters calculation

Obtained results allow to us create an electrodynamic models of various filters in the millimeter, terahertz or infrared wavelength ranges [13].

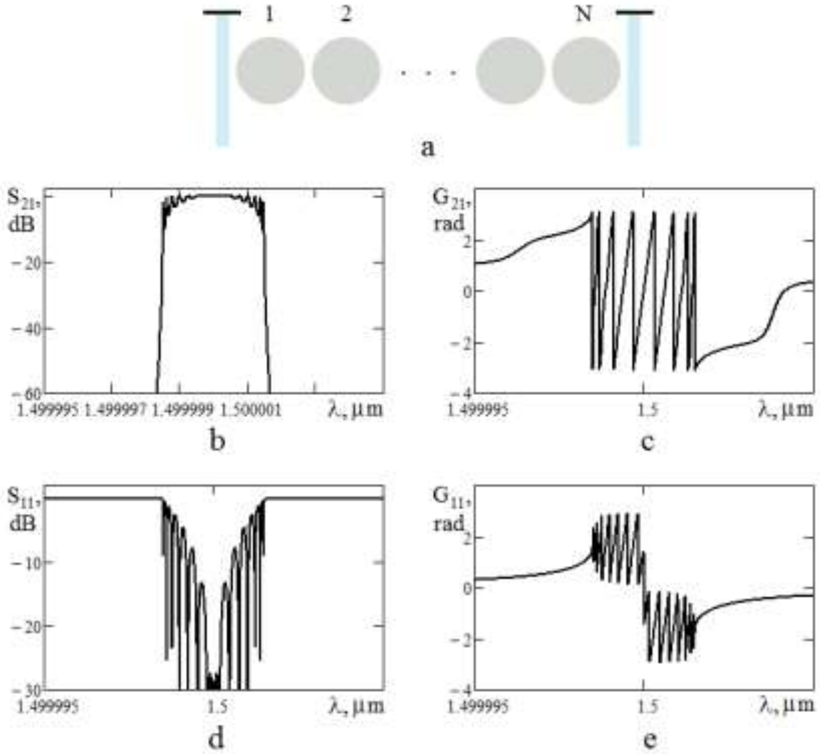

Fig. 5. Computed S-matrix parameters of the bandpass filter (a) on 15 Disk microresonators with $EH^{+}_{1,20,1}$ mode: $\varepsilon_{1r} = 9,6$; $Q^D = 10^9$; $\Delta = 0,2$; $f_0 = 200$ THz; $k_L = 10^{-6}$.

The fig. 5, 6 are showing results of the calculation of S-matrix bandpass filter parameters, that buildings up on a disk microresonators with $\mathrm{EH}^{+}_{1,20,1}$ mode. Proposed, that coupling coefficients $\mathrm{k_L}$ of the terminal microresonators with transmission lines are known. Mutual coupling coefficients was obtained from (6 – 9).

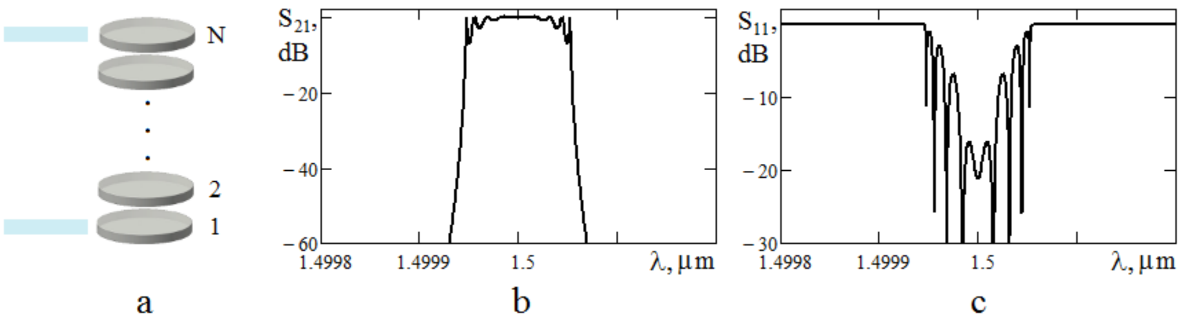

Fig. 6. Sketch of bandpass filter on vertically coupled disk microresonators (a). S-matrix responses of the 10-section bandpass filter with $EH^{+}_{1,20,1}$ mode as functions of the wavelength (b – c). The coupling coefficients of the terminal resonators with transmission lines: $k_L = 4\cdot10^{-5}$; $\varepsilon_{1r} = 9{,}6$; $Q^D = 10^9$.

It's seen, that in consequence of rapidly coupling coefficients decreasing, all S-matrix parameters are symmetrical functions on the wavelength. As we used a large number of resonators, the S_{21} squareness was obtained very well.

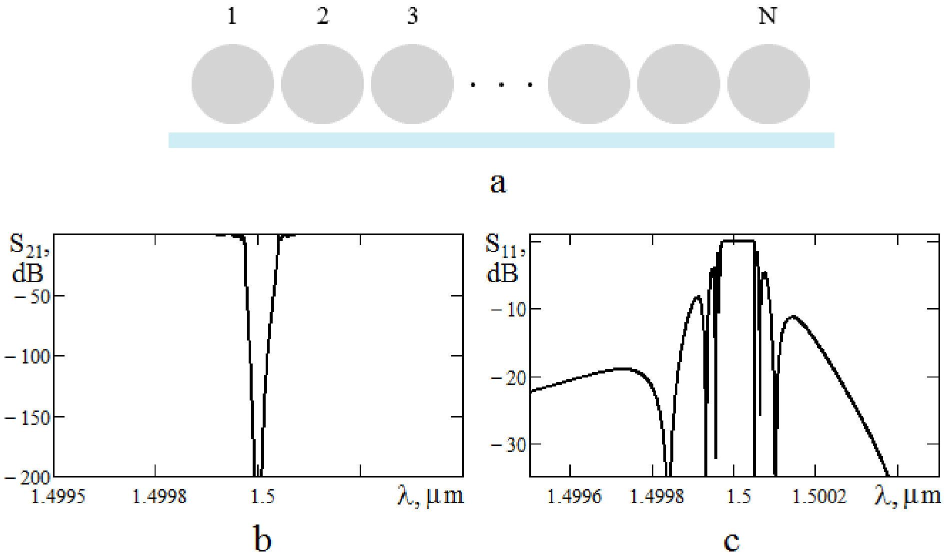

Fig. 7. Computed S-matrix parameters of the bandstop filter (a) on 10 disk microresonators with $EH^{+}_{1,29,1}$ mode: $\varepsilon_{1r} = 9{,}6$; $Q^D = 10^{10}$; $\Delta = 0{,}2$; $f_0 = 200$ THz; $k_L = 10^{-6}$.

Fig. 7 shows a scattering parameters, calculated for the bandstop filter on 10 disk microresonators with $EH^{+}_{1,29,1}$ mode. Obtained characteristics demonstrates capability of the receiving narrowly band filters in the infrared range, that allows to use ones in the optical integrated circuits of various communication systems.

Conclusions

Analytical relationships for the coupling coefficients for WG modes of the disk microresonators in the Open space has been obtained and investigated.

It stated, that WG mode coupling coefficients describes by complicated dependencies on the structure parameters.

The real and imaginary parts of the coupling coefficients for the WG modes can be differed more than one tenth degrees.

The coupling between not adjacent microresonators in the filters is small in comparison with coupling between adjacent ones, that allows simply to build filters with symmetrically parameters of the scattering.

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Трубін О. О. ***Коефіцієнти зв'язку дискових діелектричних резонаторів з коливаннями шепочучеї галереї.*** *Приведено результати розрахунків коефіцієнтів взаємного зв'язку дискових діелектричних резонаторів з коливаннями шепочучей галереї. Розглянуті основні закономірності зміни зв'язку при варіації параметрів мікрорезонаторів. Розраховані частотні залежності матриці розсіювання смугового та режекторного фільтрів на дискових микрорезонаторах інфрачервоного діапазону.*

Ключові слова: *мікрорезонатор, коефіцієнт зв'язку, коливання шепочучеї галереї,*

S-матриця, фільтр

Трубин А. А. ***Коэффициенты связи дисковых диэлектрических резонаторов с колебаниями шепчущей галереи.*** *Приведены результаты расчетов коэффициентов взаимной связи дисковых диэлектрических резонаторов с колебаниями шепчущей галереи. Рассмотрены основные закономерности изменения связи при вариации параметров микрорезонаторов. Рассчитаны частотные зависимости матрицы рассеяния полосовых и режекторного фильтров на дисковых микрорезонаторах инфракрасного диапазона.*

Ключевые слова: *микрорезонатор, коэффициент связи, колебания шепчущей галереи, S-матрица, фильтр*

Trubin A. A. ***Coupling coefficients of the disk dielectric microresonators with whispering gallery modes.*** *Calculation results of the Disk dielectric microresonators coupling coefficients are presented. Microresonator coupling coefficients as a functions of main parameters is considered. S-matrix frequency dependences of both bandstop and bandpass filters on Disk microresonators of infrared wavelength range is calculated.*

Keywords: *microresonator, coupling coefficient, whispering gallery mode, S-matrix, filter.*