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COUPLING COEFFICIENTS OF THE SPHERICAL DIELECTRIC MICRORESONATORS WITH WHISPERING GALLERY MODES[1]

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КОЭФФИЦИЕНТЫ СВЯЗИ СФЕРИЧЕСКИХ ДИЭЛЕКТРИЧЕСКИХ МИКРОРЕЗОНАТОРОВ С КОЛЕБАНИЯМИ ШЕПЧУЩЕЙ ГАЛЕРЕИ

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Introduction

Today the Spherical dielectric microresonators with whispering gallery (WG) modes are actively investigated for purpose of their application in different optical devices [1 - 10]. For calculation a scattering of electromagnetic waves on the microresonators in various structures it may be necessary a knowledge of mutual coupling coefficients between its. Mutual coupling coefficients of the Spherical dielectric microresonators are not studied in full detail.

The goal of the present work is the calculation and analysis of the coupling coefficients of the Spherical microresonators with WG modes in the free space.

Calculation of the eigenoscillation fields of the disk microresonator

For coupling coefficients calculation it's required information about fields of isolated microresonators. The eigenmode's electromagnetic field of the Spherical microresonator is well known [11].

In local spherical coordinate system, which is associated with microresonator center, the field of the magnetic type is described by equality to zero of the radial component of the electric field. Usually it is denoted as H_{nml}

$$e_r = 0;$$

$$e_\theta = -i\omega\mu_0 f_n^{(s)}(k_s r)\,\mathrm{m}\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{bmatrix}\cos\\-\sin\end{bmatrix} m\varphi;$$

$$e_\varphi = i\omega\mu_0 f_n^{(s)}(k_s r)\frac{dP_n^m(\cos\theta)}{d\theta}\begin{bmatrix}\sin\\\cos\end{bmatrix} m\varphi;$$

$$h_r = \frac{n(n+1)}{r} f_n^{(s)}(k_s r) P_n^m(\cos\theta)\begin{bmatrix}\sin\\\cos\end{bmatrix} m\varphi; \quad (1)$$

[1] Електронний варіант статті: http://radap.kpi.ua/radiotechnique/article/view/1074

$$h_\theta = \frac{1}{r}\frac{d}{dr}\{rf_n^{(s)}(k_s r)\}\frac{dP_n^m(\cos\theta)}{d\theta}\begin{bmatrix}\sin \\ \cos\end{bmatrix} m\varphi;$$

$$h_\varphi = \frac{1}{r}\frac{d}{dr}\{rf_n^{(s)}(k_s r)\}\,\mathrm{m}\frac{P_n^m(\cos\theta)}{\sin\theta}\begin{bmatrix}\cos \\ -\sin\end{bmatrix} m\varphi.$$

The second type of the field is described by the radial magnetic component $h_r = 0$. This type, named electrical, is denoted as E_{nml}. Corresponding field components for the electric type can be find from (1) with help of the known replacement $\vec{e} \rightleftarrows \vec{h}$ and $\mu_0 \rightleftarrows -\varepsilon_s$.

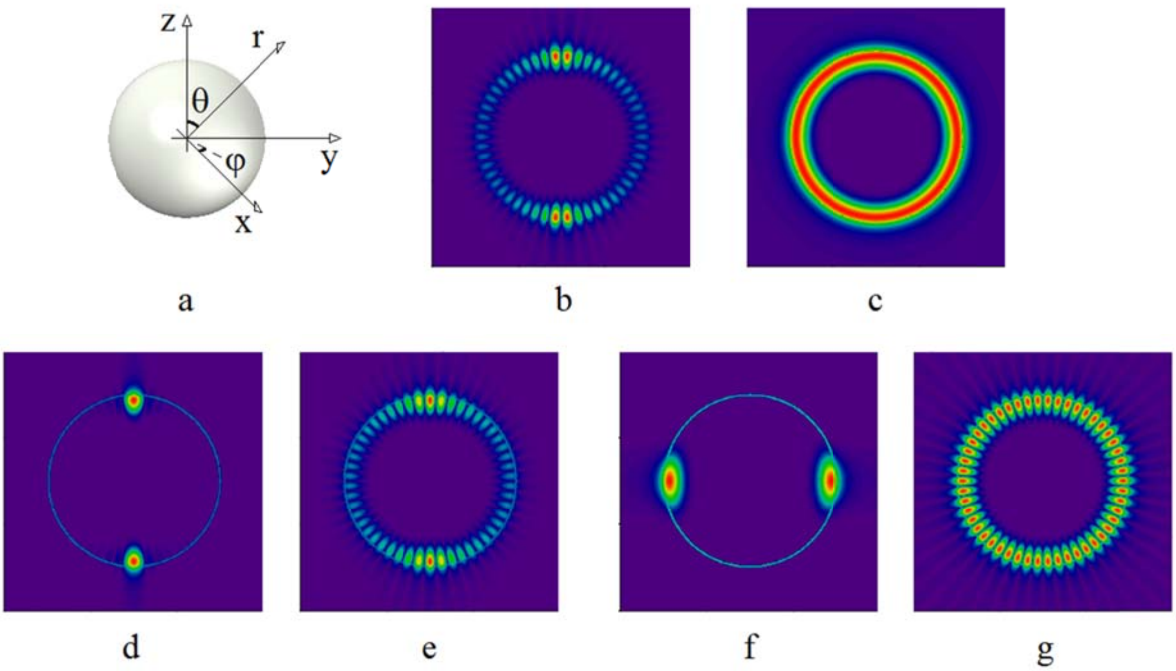

Fig. 1. A - the Spherical microresonator in the free space. The field distribution of the Spherical microresonator eigenoscillations ($\varepsilon_{1r} = 2,2$) for the WG mode: $H_{25,0,1}$ (b, c); $H_{25,1,1}$ (d, e); $H_{25,25,1}$ (f, g) in the plane: $\varphi = 0;\pi$ (b, f); $\varphi = 0$ (d); $\varphi = \pi$ (e); $\theta = \pi/2$ (c, g).

Here $P_n^m(\cos\theta)$ is the associated Legendre polynomial; $f_n^{(s)}(z)$ – one of the spherical Bessel functions: $j_n(z) = (\pi/2z)^{1/2} J_{n+1/2}(z)$; $y_n(z) = (\pi/2z)^{1/2} Y_{n+1/2}(z)$; $\mathrm{h}_n^{(2)}(z) = (\pi/2z)^{1/2}\,\mathrm{H}_{n+1/2}^{(2)}(z)$ [12], or their linear combinations.

The indexes n; $|m| \le n$ are integers; ones determines a number of the field's variations in the meridional plane: $\varphi = const$ and on the surface: $\theta = const$, respectively. In the dielectric volume $(r \le r_0)$: $f_n^{(1)}(k_1 r) = a_1 j_n(k_1 r)$; in the open space: $(r \ge r_0)$: $f_n^{(0)}(k_0 r) = a_0\,\mathrm{h}_n^{(2)}(k_0 r)$. Satisfaction to the boundary conditions leads to determination of the unknown constants (a_0; a_1). At that, the characteristic parameters p, q would satisfy to the equations [11]

$$\varepsilon_{1r}^{-1/2} j_n(p) h_{n-1}^{(2)}(q) = j_{n-1}(p) h_n^{(2)}(q) \tag{2}$$

for the magnetic modes H_{nml}, and

$$\varepsilon_{1r}^{-1} \frac{d}{dp}\{p j_n(p)\} h_n^{(2)}(q) = j_n(p) \frac{d}{dq}\{q h_n^{(2)}(q)\} \tag{3}$$

for the electrical modes E_{nml}.

The characteristic equations (2), (3) , are not depended on m; it have complex solutions, obtained numerically. Via l we shall denote a number of the root on the equation (2), or (3). The l defined a half-waves number, situated in the radial direction inside microresonator. The real part of the characteristic parameter: $p_R = \operatorname{Re} p$ defined the frequency of the mode, and the imaginary part: $p_I = \operatorname{Im} p$ defined loses, associated with radiation and dielectric dissipation. Q-factor of the mode can be calculated by the formula: $Q = p_R / 2p_I$. We shall interest to the high-Q WG modes of the Spherical DR.

Fig. 1 is showing an example of the field distribution of several degenerated WG modes H_{25m1} of the Spherical microresonator, consisting of silica ($\varepsilon_{1r} = 2,2$). This examples are showing the difference in spatial field distributions, corresponding to various values of the m numbers.

Coupling coefficient calculating

The coupling coefficients of the Spherical microresonators can be obtained from already known expressions. We used analytical expressions for the coupling coefficients of the Spherical DRs, situated in the metal Rectangular waveguide [13]. At rush the waveguide walls to the infinity, the sums on the waveguide waves numbers transformed to the integrals on nondimensional parameters.

As a result of integration, after simplifications, has been obtained next relationships for two equal Spherical microresonators with magnetic modes H_{nml}:

$$\kappa_{12} = \frac{2 \cdot i}{1+\delta_{m0}} \cdot \frac{(2n+1)}{n(n+1)} \cdot \frac{(n-m)!}{(n+m)!} \alpha_n^H(p,q) \cdot \tag{4}$$

$$\cdot \{ \int_0^\infty \left[(1-\gamma^2) \frac{dP_n^m(\gamma)}{d\gamma} \right]^2 \cdot \begin{bmatrix} J_0(\eta k_0 \Delta\rho) \mp (-1)^m \cos(2m\Delta\varphi) J_{2m}(\eta k_0 \Delta\rho) \\ (-1)^{m+1} \sin(2m\Delta\varphi) J_{2m}(\eta k_0 \Delta\rho) \end{bmatrix} \frac{e^{-i\gamma k_0 \Delta z}}{\eta\gamma} d\eta +$$

$$+ \int_0^\infty \left[m P_n^m(\gamma) \right]^2 \cdot \begin{bmatrix} J_0(\eta k_0 \Delta\rho) \pm (-1)^m \cos(2m\Delta\varphi) J_{2m}(\eta k_0 \Delta\rho) \\ (-1)^{m+1} \sin(2m\Delta\varphi) J_{2m}(\eta k_0 \Delta\rho) \end{bmatrix} \frac{e^{-i\gamma k_0 \Delta z}}{\eta\gamma} d\eta \},$$

where $\gamma = \sqrt{1-\eta^2}$; $\Delta\rho = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; $\sin(\Delta\varphi) = (y_2 - y_1) / \Delta\rho$; $\Delta z = |z_2 - z_1|$ (see fig. 2, a).

In the special cases of the H_{101}; H_{111} modes, the obtained expression (4) coincides with known early [14].

Top relationship (4) in the square brackets corresponds to odd - odd (higher), or even-even (lower) modes that radial magnetic field proportional to the $\sin(m\varphi)$ or $\cos(m\varphi)$ in the local spherical coordinate system (1). The bottom relationship in the square brackets corresponds to the odd - even, or even - odd modes. It's intended, that the direction of the axis at all local coordinate systems, associated with resonator centers, are coincides.

The relationship (4) is superfluous. Most often, the coupling oscillation field in such structure forms in the view of even, or odd spatial distributions relatively symmetry plane, passing to the DR centers. It's examines the case, when the DR centers situated in the plane, that parallel to the xz: $y_1 = y_2$.

In this case $\Delta\varphi = 0$ and the relationship (4) can be simplified:

$$\kappa_{12} = \frac{2\cdot i}{1+\delta_{m0}} \cdot \frac{(2n+1)}{n(n+1)} \cdot \frac{(n-m)!}{(n+m)!} \alpha_n^H(p,q) \cdot \tag{5}$$

$$\cdot \{ \int_0^\infty \left[(1-\gamma^2) \frac{dP_n^m(\gamma)}{d\gamma} \right]^2 \cdot [J_0(\eta k_0 \Delta\rho) \mp (-1)^m J_{2m}(\eta k_0 \Delta\rho)] \frac{e^{-i\gamma k_0 \Delta z}}{\eta\gamma} d\eta +$$

$$+ \int_0^\infty \left[m P_n^m(\gamma) \right]^2 \cdot [J_0(\eta k_0 \Delta\rho) \pm (-1)^m J_{2m}(\eta k_0 \Delta\rho)] \frac{e^{-i\gamma k_0 \Delta z}}{\eta\gamma} d\eta \}$$

If all resonator centers are disposed on the z-axis $\Delta\rho = 0$ (fig. 2, a), then (5) takes more simple form:

$$\kappa_{12} = 2\cdot i \cdot \frac{(2n+1)}{n(n+1)} \cdot \frac{(n-m)!}{(n+m)!} \alpha_n^H(p,q) \cdot \int_0^\infty \{ \left[(1-\gamma^2) \frac{dP_n^m(\gamma)}{d\gamma} \right]^2 + \left[m P_n^m(\gamma) \right]^2 \} \frac{e^{-i\gamma k_0 \Delta z}}{\eta\gamma} d\eta$$

The integrals (4 - 6) can be calculated, if used relationships, following from the non tabular integral [14]:

$$\int_0^\infty e^{-ib\sqrt{1-t^2}} J_m(ct) P_n^m\left(\sqrt{1-t^2}\right) t\, dt / \sqrt{1-t^2} = i^{(m-n)} P_n^m\left(b / \sqrt{b^2+c^2}\right) h_n^{(2)}\left(\sqrt{b^2+c^2}\right), \tag{7}$$

where b, c – is the real constants.

In the particular case of $n = m = 0$ the (7) takes more simple form, known as Sommerfeld's integral:

$$\int_0^\infty e^{-ib\sqrt{1-t^2}} J_0(ct) t\, dt / \sqrt{1-t^2} = h_0^{(2)}\left(\sqrt{b^2+c^2}\right). \tag{8}$$

Differentiates (8) of m times on the parameter b, obtain:

$$\int_0^\infty (\sqrt{1-t^2})^m J_0(ct) e^{-ib\sqrt{1-t^2}} t\, dt / \sqrt{1-t^2} = i^m \frac{d^m}{db^m} h_0^{(2)}\left(\sqrt{b^2+c^2}\right), \text{ for } (b>0) \tag{9}$$

Since, using the expansion:

$$t^{2m} = \sum_{s=0}^{m} (-1)^s \frac{m!}{(m-s)!s!} (\sqrt{1-t^2})^{2s}$$

finds one necessary integral:

$$\int_0^{\infty} t^{2m} J_0(ct) e^{-ib\sqrt{1-t^2}} t\, dt / \sqrt{1-t^2} = \sum_{s=0}^{m} \frac{m!}{(m-s)!s!} \cdot \frac{\partial^{2s}}{\partial b^{2s}} h_0^{(2)}\left(\sqrt{b^2+c^2}\right). \quad (10)$$

Convergence of the (7) – (10), in the area $t \geq 1$ provided by choice of the radical sign $-i\sqrt{t^2-1}$.

In the limit of $b=0$ finds from (9) for the $m=2n$:

$$\lim_{b\to 0} \int_0^{\infty} (\sqrt{1-t^2})^{2n} J_0(ct) e^{-ib\sqrt{1-t^2}} t\, dt / \sqrt{1-t^2} = (-1)^n \frac{d^{2n}}{db^{2n}} h_0^{(2)}\left(\sqrt{b^2+c^2}\right)\Bigg|_{b=0}. \quad (11)$$

One put in the (11) $c=0$, obtain:

$$\int_0^{\infty} t^{2m} e^{-ib\sqrt{1-t^2}} t\, dt / \sqrt{1-t^2} = \sum_{s=0}^{m} \frac{m!}{(m-s)!s!} \cdot \frac{d^{2s}}{db^{2s}} h_0^{(2)}(b). \quad (12)$$

Expressions (9); (11) do not comfortable for calculating if they contain derivatives of higher degree. In the cases of $m \gg 1$, the derivatives can be calculated with help of the decompositions:

$$\frac{d^n}{db^n} h_0^{(2)}(b) = h_0^{(2)}(b) \cdot n! \sum_{s=0}^{n} \frac{(-i)^{n+s}}{(n-s)!} \cdot \frac{1}{b^s}, \quad (13)$$

and

$$\frac{d^{2n}}{db^{2n}} h_0^{(2)}\left(\sqrt{b^2+c^2}\right)\Bigg|_{b=0} = (-1)^n \cdot (2n)! \sum_{s=0}^{n} \frac{(4s+1)(2s)!}{2^{n+s}(n-s)!(s!)^2} \cdot \frac{1}{\prod_{u=0}^{n+s}(2u+1)} h_{2s}^{(2)}(c). \quad (14)$$

The relation (13) was obtained as a result of the spherical Hankel function presentation via elementary functions as well as using the Leibniz's theorem to the product's of differencing. The decomposition (14) follows from the summation theorem for the spherical Hankel functions.

Obtained integral relations (9) - (14) allow to calculate from (4) - (6) new coupling coefficients for the high modes.

For example, for the eigenoscillations with $n=m$, taking into account the presentation of the Legendre polynomials [12]:

$$P_m^m(z) = \frac{(2m)!}{2^m m!}(1-z^2)^{m/2},$$

obtains the integrals (4), (6) in the form:

$$\kappa_{12} = i \cdot \frac{(2m+1)!}{2^{2m-1}(m+1)!} \alpha_n^H(p,q) \cdot \sum_{s=0}^{m} \frac{m-2s}{(m-s)!s!} \cdot \frac{d^{2s}}{d(k_0\Delta z)^{2s}} h_0^{(2)}(k_0\Delta z) \quad (15)$$

and

$$\kappa_{12} = 2i \cdot \frac{m}{(m+1)} \cdot \frac{(2m+1)!}{2^{2m}(m!)^2} \alpha_n^H(p,q) \cdot \tag{16}$$

$$\cdot \begin{bmatrix} (m-1)! \sum_{s=0}^{m} \frac{m-2s}{(m-s)!s!} \cdot \frac{d^{2s}}{d(\mathrm{k}_0 \Delta z)^{2s}} h_0^{(2)}(\mathrm{k}_0 \Delta r) \mp (-1)^m \cos(2m\Delta\varphi)(\sin\Delta\theta)^{2m} h_{2m}^{(2)}(\mathrm{k}_0 \Delta r) \\ (-1)^{m+1} \sin(2m\Delta\varphi)(\sin\Delta\theta)^{2m} h_{2m}^{(2)}(\mathrm{k}_0 \Delta \mathrm{r}) \end{bmatrix}.$$

Here $\Delta r = \sqrt{(x_2 - x_1)^2 + (\mathrm{y}_2 - y_1)^2 + (\mathrm{z}_2 - z_1)^2}$; $\sin(\Delta\theta) = \Delta\rho / \Delta \mathrm{r}$.

In the common case, the integral (4) can be calculated, if it is considered, that presented items, contained derivatives as well as the Legendre polynomials, itself are the polynomials and can be expanded into the series:

$$\left[(\sqrt{1-x^2})\frac{dP_n^m(\mathrm{x})}{dx}\right]^2 + \left[\frac{mP_n^m(\mathrm{x})}{\sqrt{1-x^2}}\right]^2 = \sum_{s=0}^{n} a_{n,s}^m P_{2s}(x);$$

$$\left[(\sqrt{1-x^2})\frac{dP_n^m(\mathrm{x})}{dx}\right]^2 - \left[\frac{mP_n^m(\mathrm{x})}{\sqrt{1-x^2}}\right]^2 = \sum_{s=m}^{\infty} b_{n,s}^m P_{2s}^{2m}(x); \tag{17}$$

$$\left[(\sqrt{1-x^2})\frac{dP_n^m(\mathrm{x})}{dx}\right]^2 + \left[\frac{mP_n^m(\mathrm{x})}{\sqrt{1-x^2}}\right]^2 = \sum_{s=m}^{\infty} c_{n,s}^m P_{2s}^{2m}(x).$$

where

$$a_{n,s}^m = \frac{(4s+1)}{2}\int_{-1}^{1}\{\left[(\sqrt{1-x^2})\frac{dP_n^m(\mathrm{x})}{dx}\right]^2 + \left[\frac{mP_n^m(\mathrm{x})}{\sqrt{1-x^2}}\right]^2\} P_{2s}(x)dx;$$

$$b_{n,s}^m = \frac{(4s+1)}{2} \cdot \frac{(2(s-m))!}{(2(s+m))!}\int_{-1}^{1}\{\left[(\sqrt{1-x^2})\frac{dP_n^m(\mathrm{x})}{dx}\right]^2 - \left[\frac{mP_n^m(\mathrm{x})}{\sqrt{1-x^2}}\right]^2\} P_{2s}^{2m}(x)dx;$$

$$c_{n,s}^m = \frac{(4s+1)}{2} \cdot \frac{(2(s-m))!}{(2(s+m))!}\int_{-1}^{1}\{\left[(\sqrt{1-x^2})\frac{dP_n^m(\mathrm{x})}{dx}\right]^2 + \left[\frac{mP_n^m(\mathrm{x})}{\sqrt{1-x^2}}\right]^2\} P_{2s}^{2m}(x)dx.$$

Substituting (17) in the (4) and using integral (7), obtains finally for equal parity each resonator oscillations:

$$\kappa_{12} = \frac{2 \cdot i}{1+\delta_{m0}} \cdot \frac{(2n+1)}{n(n+1)} \cdot \frac{(n-m)!}{(n+m)!} \alpha_n^H(p,q) \cdot \tag{18}$$

$$\{\sum_{s=0}^{n} (-1)^s a_{n,s}^m P_{2s}(\cos\Delta\theta)\,\mathrm{h}_{2s}^{(2)}(k_0\Delta r) \mp \cos(2m\Delta\varphi)\sum_{s=m}^{\infty}(-1)^s b_{n,s}^m P_{2s}^{2m}(\cos\Delta\theta)\,\mathrm{h}_{2s}^{(2)}(k_0\Delta r)\};$$

for the oscillations of different parity:

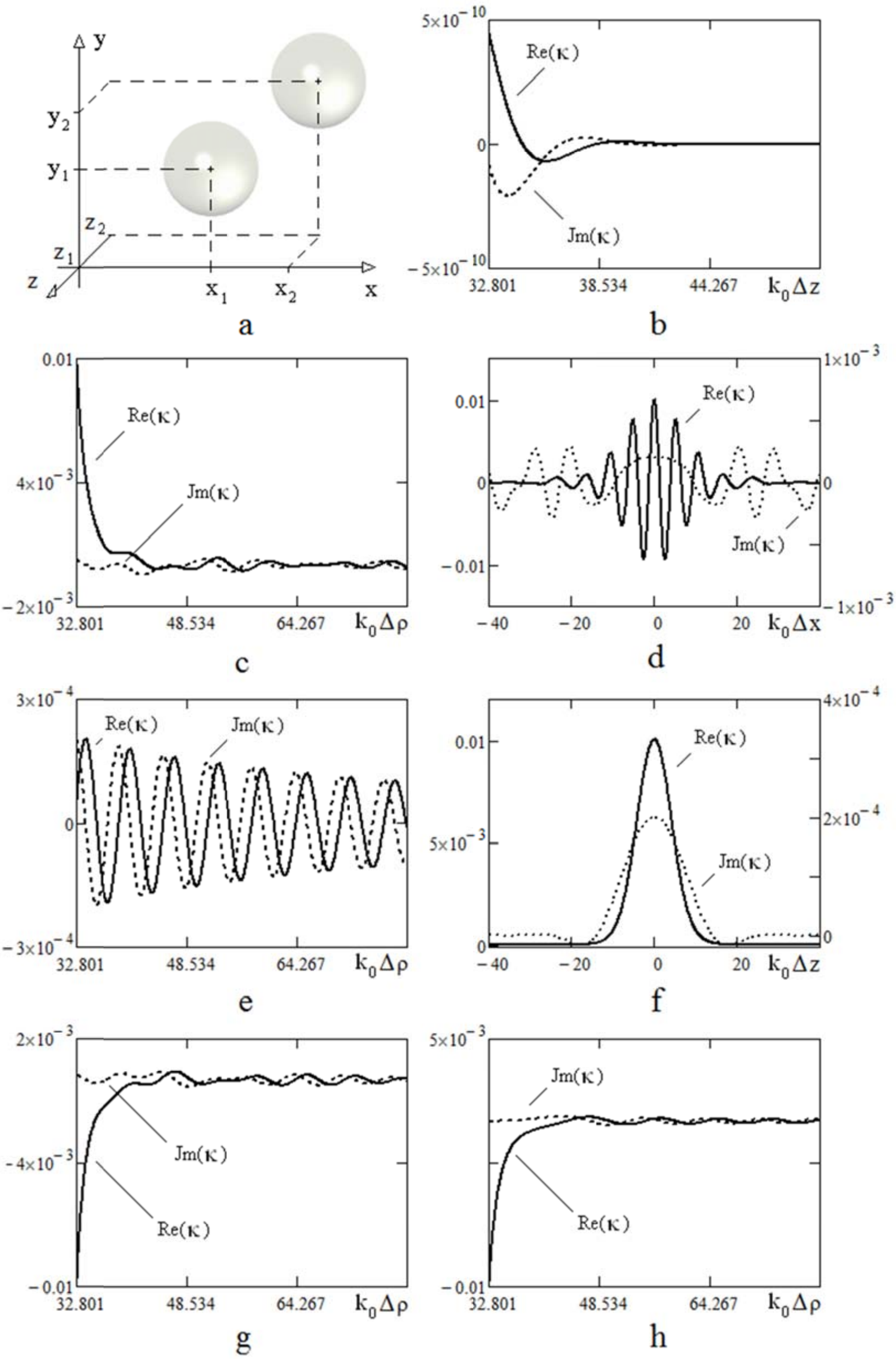

Fig. 2. Two Spherical microresonators (a). The complex coupling coefficients as functions of the distance between microresonator centers for the WG modes H_{mm1} ($m = 20$; $\varepsilon_{1r} = 2,2$). For the even-even modes (c - f); for the odd - odd mode (g); for the odd - even modes (h); $\Delta\rho = 0$ (b); $\Delta z = 0$ (c, e); $\Delta y = 2r_0, \Delta z = 0$ (d); $\Delta x = 0, \Delta y = 2r_0$ (f); $\Delta\varphi = 0$ (c, g); $\Delta\varphi = \pi / 4m$ (e).

$$\kappa_{12} = \frac{2 \cdot i}{1+\delta_{m0}} \cdot \frac{(2n+1)}{n(n+1)} \cdot \frac{(n-m)!}{(n+m)!} \alpha_n^H(p,q) \cdot \qquad (19)$$

$$\{-\sin(2m\Delta\varphi) \sum_{s=m}^{\infty} (-1)^s c_{n,s}^m P_{2s}^{2m}(\cos\Delta\theta)\, \mathrm{h}_{2s}^{(2)}(k_0 \Delta r)\}\,.$$

For the electrical modes E_{nml}, the coupling coefficient relationships nominally coincides with (4) - (19), however the multiplier $\alpha_n^H(p,q)$, determining coupling dependence on the dielectric parameters, should be replaced by $\alpha_n^E(p,q)$ [13]:

$$\alpha_n^E(p,q) = \frac{\varepsilon_{1r}}{q} \left| \frac{j_n(p)}{h_n^{(2)}(q)} \right|^2 / \left\{ \left[p^2 - n(n+1) \right] j_n^2(p) + \left[n j_n(p) - p j_{n-1}(p) \right]^2 \right\},$$

where also $p = k_1 r_0$ и $q = k_0 r_0$ - is the characteristic parameters of the Spherical microresonator for the electrical modes (3).

Coupling coefficient analysis

Obtained above relationships were used for the calculating of the coupling coefficients for WG modes Spherical microresonators. As can be seen from the calculation results, presented in fig. 2, 3, the coupling coefficients of the WG modes obtain sufficiently large values only in the near-field region in which the microresonators are touching by surfaces to each other. These regions have a largest field concentration on the corresponding modes (see fig. 1). Increasing distance between resonator centers accompanies by significant coupling decreasing. At that, the relative motion in the tangent directions leads to a complex interference of their mutual influence, determining by significant eigenmode field variation nearby their surfaces (fig. 2, d - f; fig. 3, c, f).

Different orientation of fields of the microresonators relatively it movement in the radial direction also leads to a complicated correlation of the radial coupling coefficients (fig. 2, e, g, h).

In the majority cases the imaginary part values of the coupling coefficients at least one tenth as many as it real parts. Degree of the imaginary part of the coupling approximately is equal to Q^{-1}.

Filter parameters calculation

Obtained results allow to design electrodynamic models of various filters in the millimeter and infrared wavelength ranges. It's considerable interest to the determining of the feasible scattering parameters of the bandstop and bandpass filters, building up on basis of the Spherical microresonators [16].

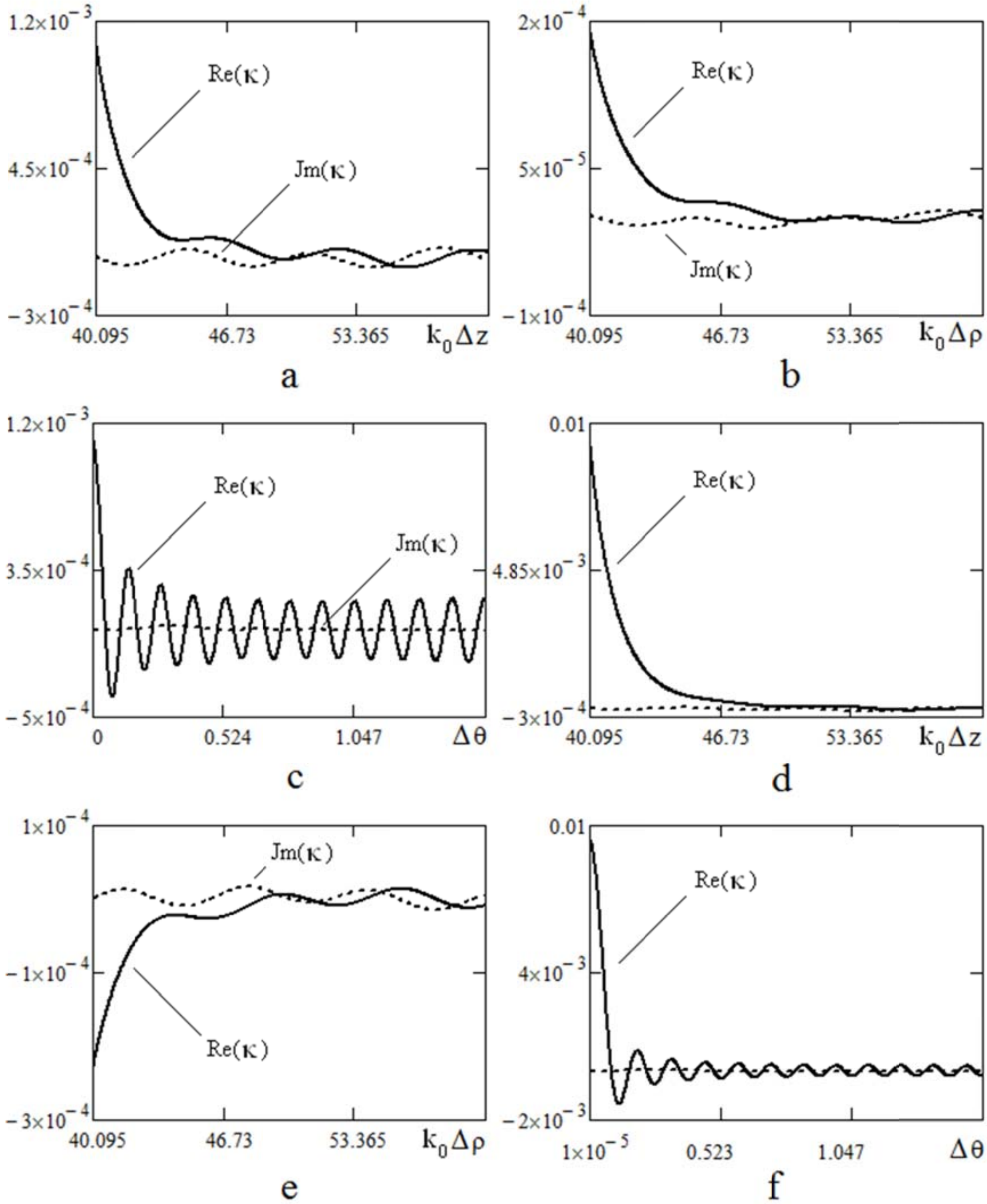

Fig. 3. Complex coupling coefficients as functions of the microresonator coordinates for the WG modes H_{n01} (a - c); H_{n11} (d - f) ($n = 25$; $\varepsilon_{1r} = 2,2$); $\Delta\rho = 0$ (a, d); $\Delta z = 0$ (b, e); $\Delta r = 2r_0$, $\Delta\varphi = 0$ (c, f).

Basic difficulty of the filter design is necessity of supply power with excitation only one kind of confluent modes, inherent to this type of the microresonators. One further difficulty is necessity of support accurate dimensions of the filter elements, such as equality of the microresonators as well as its precise arrangement relatively one another.

Example of calculation on the S-matrix parameters of the bandstop filter on

WG modes H_{mm1} $(m = 30)$ is shown in the fig. 4. The Spherical microresonator parameters are: $f_0 = 200\,\text{THz}$; $\varepsilon_{1r} = 2{,}2$; $Q^D = 10^{10}$. The coupling coefficients between microresonators were calculated on (5); the coupling coefficients between microresonators and dielectric waveguide are equal: $k_L = 5 \cdot 10^{-3}$. Here and below $S_{21} = S_{21}(\lambda) = 20\lg|T(\lambda)|$; $S_{11} = S_{11}(\lambda) = 20\lg|R(\lambda)|$, are the scattering matrix elements, where $T(\lambda)$ – is the transmission coefficient and R(λ) – is the reflection coefficient response of the microresonator system in the waveguide.

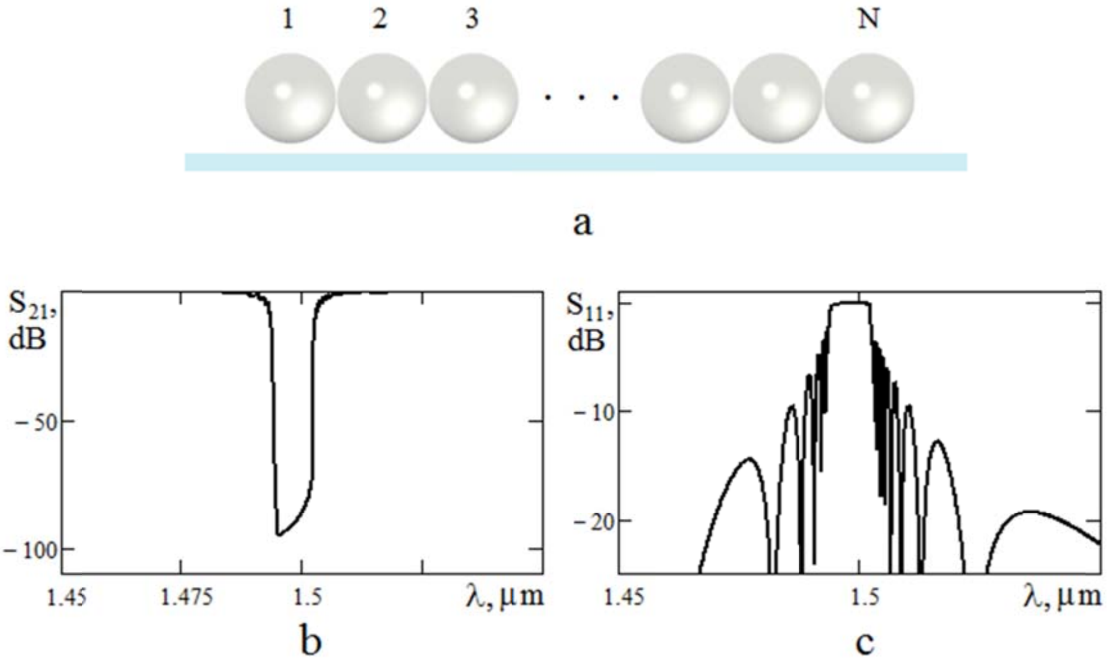

Fig. 4 Sketch of bandstop filter on the laterally coupled Spherical microresonators (a) and dielectric waveguide. S-matrix responses of one band of the 25-section Spherical microresonators (b - c).

The fig. 5 shows theoretical scattering parameters of several bandpass filters on the Spherical microresonators with WG modes. Here by the continuous curves denoted $S_{21}(\lambda)$ and by the dotted curves are denoted $S_{11}(\lambda)$ responses on the wavelength. Practically zero coupling between not adjacent microresonators supports a well symmetry of the S-matrix parameters relatively central wavelength.

Calculated a group delay (fig. 5, c, f, i) also are showing possibility of such filters using for the signal latency.

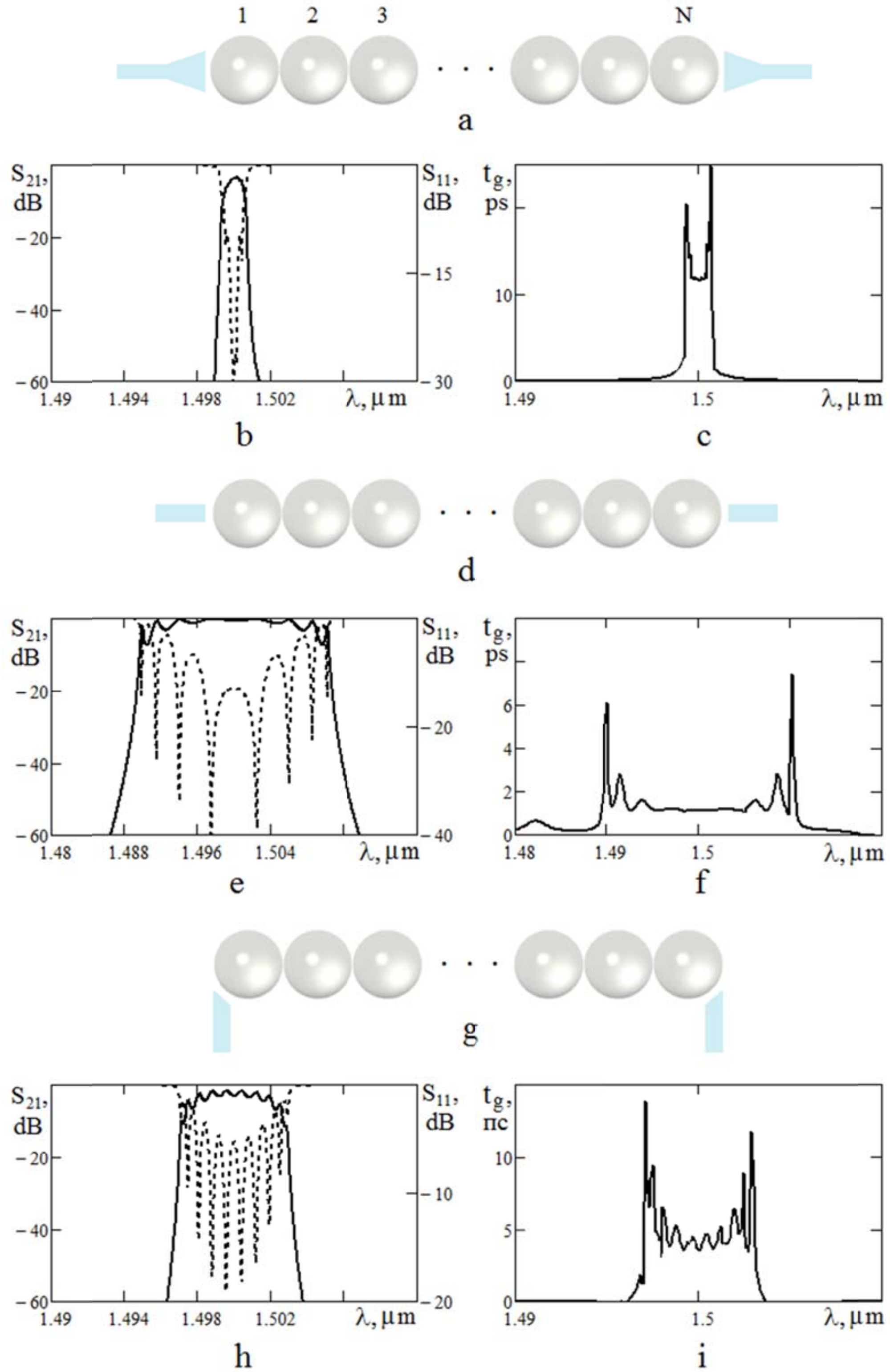

Fig. 5 Bandpass filters on the Spherical microresonators with WG modes H_{n01} ($n = 30$), (a); H_{n11} ($n = 31$), (d); H_{mm1} ($m = 30$), (g). S-matrix of the 7-section (b); 10-section (e, h) filters as a function of the wave-length. Group delay (c, f, i) as a function of the wave-length. $f_0 = 200$ THz; $\varepsilon_{1r} = 2,2$; $Q^D = 10^{10}$. The coupling coefficient with dielectric waveguide: $k_L = 0,0005$ (b); $k_L = 0,009$ (c); $k_L = 0,001$ (h).

Conclusions

An analytical relationships for the coupling coefficients of the Spherical microresonator in the Open space has been obtained and investigated.

It's showed that the WG mode coupling coefficient describes by more complicated dependencies on the structure parameters.

The real and imaginary parts of the coupling coefficients of the WG modes can be differed more than one degree.

Obtained results showed a possibility of building different types of varied bands filters with symmetrical characteristics.

References

1. Braginskiy V.B. and Ilchenko V.S. (1987) Properties of Optical Dielectric Microresonators, *Doklady AS USSR*, Vol. 293, No 6, pp.1358-1361.

2. Mackowsky D. W. and Mishenko M. I. (1996) Calculation of the T matrix and the scattering matrix for ensembles of spheres. *J. Opt. Soc. Am.*, Vol. 13, No. 11, pp. 2266 - 2278.

3. Gorodetsky M. L. and Ilchenko V. S. (1999) Optical microsphere resonators: Optimal coupling to high-Q whispering-gallery modes. *J. Opt. Soc. Amer. B*, Vol. 16, pp. 147–154.

4. Kippenberg T. J., Spillane S. M. and Vahala K. J. (2002) Modal coupling in traveling-wave resonators. *Optics Letters*, Vol. 27, No. 19, pp. 1669 - 1671.

5. Astratov V. N. (2010) Fundamentals and Applications of Microsphere Resonator Circuits. *Photonic Microresonator Research and Applications*. Springer, pp. 423-457.

6. Chen Z., Taflove A. and Backman V. (2006) Highly efficient optical coupling and transport phenomena in chains of dielectric microspheres. *Optics Letters*, Vol. 31, No. 3. pp. 389-391.

7. Kieu K. and Mansuripur M. (2007) Fiber laser using a microsphere resonator as a feedback element. *Optics Letters*, Vol. 32, No 3, pp. 244 - 246.

8. Thompson D.B., Keating D.A., Guler E., Ichimura K., Williams M.E. and Fuller K.A. (2010) Separation-sensitive measurements of morphology dependent resonances in coupled fluorescent microspheres. *Optics Express*, Vol. 18, No. 18, pp. 19209 -19218.

9. Mitsui T., Onodera T., Wakayama Y., Hayashi T., Ikeda N., Sugimoto Y., Takamasu T. and Oikawa H. (2011) Influence of micro-joints formed between spheres in coupled-resonator optical waveguide. *Optics Express*, Vol. 19, No. 22, pp. 22258-22267.

10. Xifre-Perez E., Domenech J.D., Fenollosa R., Munoz P., Capmany J. and Mesenguer F. (2011) All silicon waveguide spherical microcavity coupler device. *Optics Express*, Vol. 19, No. 4. pp. 3185-3192.

11. Ilchenko M.E. and Trubin A.A. (2004) *Electrodynamics' of Dielectric Resonators,* Kiev, Naukova Dumka, 265 p.

12. Abramowitz M. eds. and Stegun I. eds. (1964) *Handbook of mathematical functions*. National bureau of standards., 830 p.

13. Trubin A. A. (1991) Mutual coupling coefficients of the Spherical Dielectric Resonators in Rectangular Waveguide, *Vestnik kievskogo politekhnicheskogo instituta. Radiotekhnika*, No. 28, pp. 27-32.

14. Trubin A.A. (2007) Natural oscillations of High-Q Resonance Macrostructures in Infrared and Optical ranges on the Systems with Coupled Dielectric Resonators. *Microwave & Telecommunication Technology, CriMiCo*, pp. 473-476.

15. Trubin A. A. Several specified integrals of special functions, Available at: http://eqworld.ipmnet.ru/ru/auxiliary/aux-specfunc.htm

16. Trubin A. A. (1997) Scattering of electromagnetic waves on the Systems of Coupling Dielectric Resonators. *Radio electronics*, No 2, pp. 35-42.

Трубін О. О. ***Коефіцієнти зв'язку сферичних діелектричних мікрорезонаторів з коливаннями шепочучеї галереї.*** *Приведено результати теоретичних розрахунків коефіцієнтів зв'язку сферичних діелектричних мікрорезонаторів з коливаннями шепочучеї галереї. Розглянуті основні закономірності зміни коефіцієнтів зв'язку при варіації параметрів мікрорезонаторів. Розраховані частотні залежності матриці розсіювання режекторного та смугових фільтрів які побудовані на сферичних микрорезонаторах інфрачервоного діапазону.*

Трубин А. А. ***Коэффициенты связи сферических диэлектрических микрорезонаторов с колебаниями шепчущей галереи.*** *Рассчитаны распределения полей различных вырожденных собственных колебаний сферических микрорезонаторов вида шепчущей галереи. Приведены результаты теоретических расчетов комплексных коэффициентов взаимной связи сферических диэлектрических микрорезонаторов в открытом пространстве. Получены общие аналитические соотношения для коэффициентов связи. Найденные выражения совпадают с полученными ранее для низших типов колебаний сферических диэлектрических резонаторов. Рассмотрены основные закономерности изменения коэффициентов связи при вариации параметров микрорезонаторов, а также их относительного расположения при возбуждении в них разных видов колебаний типа шепчущей галереи. Показано, что в случае колебаний шепчущей галереи, коэффициенты связи изменяются более сложным образом при вариации параметров структуры. Установлено, что коэффициенты взаимной связи принимают видимые значения только в ближней области вблизи "касания" поверхностей микрорезонаторов, определяемой распределением поля их собственных колебаний, при этом быстро убывают при увеличении расстояния между центрами. Выведено несколько новых не табличных интегралов, содержащих функции Бесселя, и отсутствующих в научных публикациях и справочниках. На основании полученных формул, рассчитаны частотные зависимости матрицы рассеяния 25-звенного режекторного фильтра, построенного с применением сферических микрорезонаторов инфракрасного диапазона длин волн, выполненных из кварцевого стекла. Рассчитаны и исследованы характеристики рассеяния ряда многозвенных полосовых фильтров. Исследованы параметры полос пропускания, а также вносимой групповой задержки фильтров в зависимости от возбуждаемых в микрорезонаторах видов колебаний. Показано, что на одних и тех же структурах микрорезонаторов можно реализовывать как широкополосные, так и узкополосные фильтры в зависимости от способа возбуждения их вырожденных колебаний. Полученные результаты существенно расширяют возможности разработчиков, т.к. позволяют создавать электродинамические модели полосовых и режекторных фильтров, а также других устройств миллиметрового и инфракрасного диапазонов, построенных на основе применения микрорезонаторов сферической формы с колебаниями шепчущей галереи. Такие фильтры могут быть использованы в мультиплексерах, линиях задержки, а также других устройствах современных волоконно-оптических систем связи.*

Trubin A. A. ***Coupling coefficients of the Spherical dielectric microresonators with whispering gallery modes.*** *The calculation results of the Spherical dielectric microresonators coupling coefficients with whispering gallery modes are presented. Microresonator coupling coefficients as a functions of main parameters is considered. S-matrix frequency dependences of both bandstop and bandpass filters on the Spherical microresonators of infrared wavelength range are calculated.*

Keywords: *microresonator, coupling coefficient, whispering gallery mode, S-matrix, filter.*