УДК 621.372

COUPLING COEFFICIENTS OF DIFFERENT CYLINDRICAL DIELECTRIC RESONATORS IN THE OPEN SPACE[1]

Trubin A. A., Doctor of Engineering, Professor
National Technical University of Ukraine
"Kyiv Polytechnic Institute", Kyiv, Ukraine
atrubin@ukrpost.net

КОЕФІЦІЄНТИ ЗВ'ЯЗКУ РІЗНИХ ЦИЛІНДРИЧНИХ ДІЕЛЕКТРИЧНИХ РЕЗОНАТОРІВ У ВІДКРИТОМУ ПРОСТОРІ

Трубін О. О., д.т.н., професор
Національний технічний університет України "Київський політехнічний інститут",
м. Київ, Україна,

Introduction

Cylindrical DRs are applying today in various microwave devices [1 - 6]. For calculation and optimization of such devices is more convenient to use electrodynamic modeling with sufficient accuracy [7, 8]. Calculation of the systems, containing multiple DRs in the open space, is based on computation of mutual coupling coefficients. The coupling coefficients of the Cylindrical DRs did not studied in full measure even in case of the lowest modes. The goal of the present work is the calculation and analysis of the Cylindrical DR coupling coefficients with main modes, located in the open space, in the general case of its arbitrary spatial orientation as well as for cases of various shapes and materials.

Coupling coefficient calculating

Allocation microresonators side by side with each other leads to the coupling oscillations appearance. The fields and frequencies of the DR oscillations are defined by values of the mutual coupling coefficients. In the common case, the coupling coefficient can be determined as a surface integral [9]:

$$\kappa_{sn} = \frac{i}{2\omega_0 w_n (1 + \delta_{sn})} \oint_{s_n} \left\{ \left[\vec{e}_s, \vec{h}_n^* \right] + \left[\vec{e}_n^*, \vec{h}_s \right] \right\} \vec{n} ds, \qquad (1)$$

expressed via the eigenmode field $(\vec{e}_s, \vec{h}_s)$ of one (s-th) DR on the surface of another (n-th) DR. Here $s, n = 1, 2$; and $\vec{n}$ - is the normal to the surface s_n of n-th DR, ω_0 - is the resonance frequency of the resonators; w_n - is the energy, stored in the dielectric.

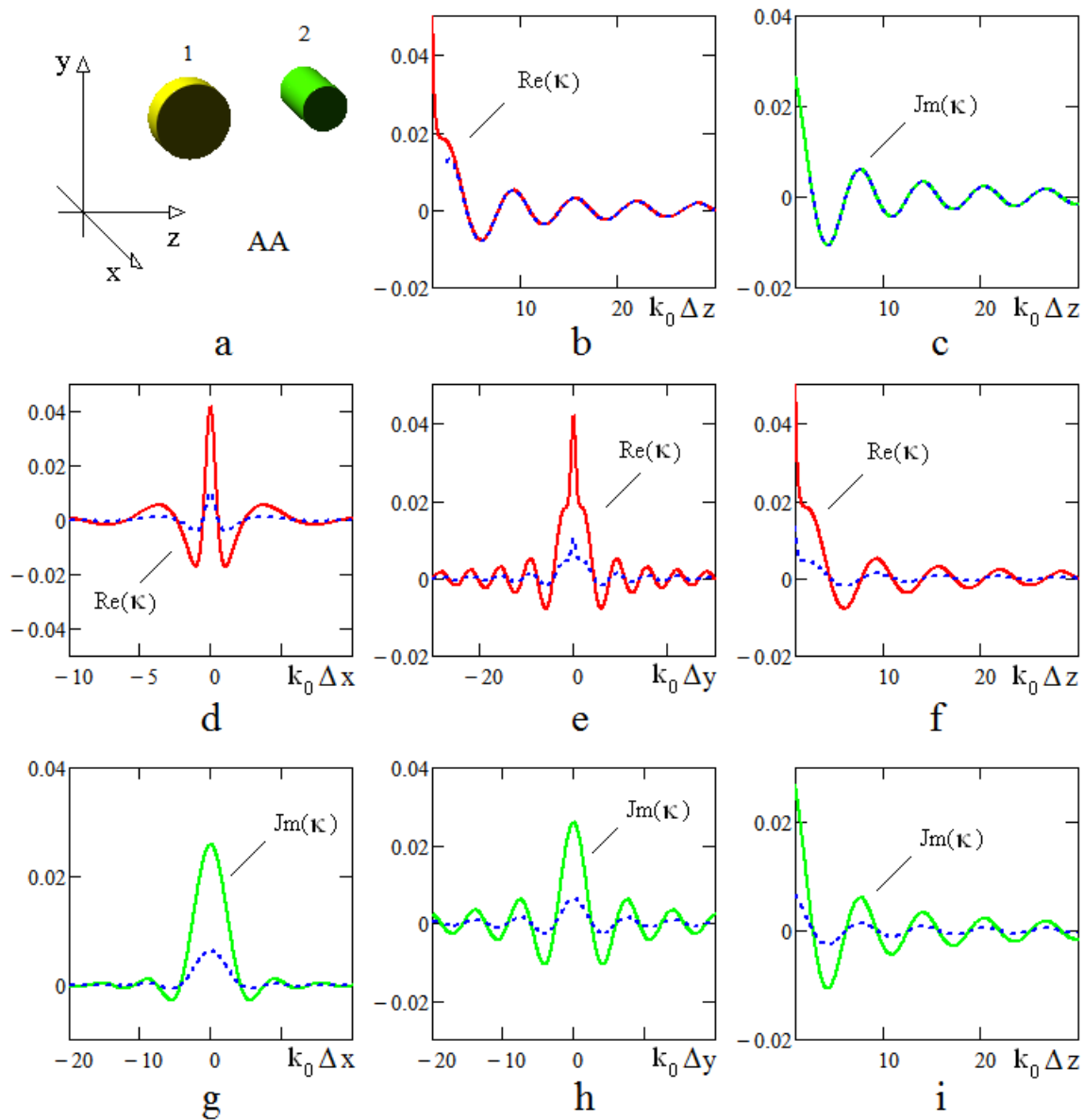[1] http://radap.kpi.ua/radiotechnique/article/view/1223

Fig. 1. A - sketch of two different Cylindrical DRs in the AA position in Open space. The real part of the coupling coefficients κ_{12} (solid lines); κ_{21} (dotted lines) (b, d - f) of two Cylindrical DRs with $H_{1,0,1}^{+}$ modes; ($m = 0$): $\varepsilon_{1r} = 36$; $\varepsilon_{2r} = 81$; $\Delta_1 = L_1 / 2r_1 = 0,4$; $\Delta_2 = L_2 / 2r_2 = 0,8$. Imaginary part of the coupling coefficients versus coordinates of the DR centers (c, g - i). B, c, f, i: $k_0 \Delta x = k_0 \Delta y = 0$; d, g: $k_0 \Delta y = 0$; $k_0 \Delta z = 1,1 k_0 (r_1 + r_2)$; e, h: $k_0 \Delta x = 0$; $k_0 \Delta z = 1,1 k_0 (r_1 + r_2)$.

As follow from (1), in the general case of unequal DRs: $\kappa_{11} \neq \kappa_{22}$; $\kappa_{12} \neq \kappa_{21}$. A direct calculation of the integral (1) in most cases is not possible, so we will use the well-known expressions for the mutual coupling coefficients found for the DR in a rectangular waveguide [10]. Then the same coupling coefficients for the open space can be simply obtained using the integral transformation of the known analytical expression based on assumption that the transmission line metal walls

have been "removed" to the infinity.

Using necessary expressions, for example, 5.2 of the [10], as well as integrals [9, 11], after simplifications obtain:

For the AA position (see fig. 1, a) in the case of two different Cylindrical DRs, with mode $H^+_{1,0,1}$, the mutual coupling coefficients can be obtained in the form:

in the area: $\Delta z \geq r_1 + r_2$:

$$\kappa_{1,2} = \kappa_0 \cdot \int_0^\infty \varphi_1(\sqrt{1-\xi^2}) \cdot \varphi_2(\sqrt{1-\xi^2})^* H_0^{(2)}(k_0 \Delta\rho\sqrt{1-\xi^2})\cos(k_0\Delta x\xi)d\xi \; ; \quad (2)$$

$$(\Delta\rho = \sqrt{\Delta y^2 + \Delta z^2})$$

For the AB position (fig. 2, a) in the area: $\Delta z \geq r_1 + r_2$:

$$\kappa_{1,2} = \kappa_0 \cdot \qquad\qquad\qquad\qquad (3)$$

$$\int_0^\infty \int_0^\infty \frac{e^{-i\gamma k_0 \Delta z}}{\gamma} \frac{\varphi_1(\sqrt{\eta^2+\gamma^2})}{\sqrt{\eta^2+\gamma^2}} \cdot \frac{\varphi_2(\sqrt{\xi^2+\gamma^2})^*}{\sqrt{\xi^2+\gamma^2}}\sin(k_0\Delta x\xi)\sin(k_0\Delta y\eta)\cdot\xi\eta\cdot d\xi d\eta \; ;$$

For the AC position (see fig. 3, a) in the area: $\Delta z \geq r_1 + L_2/2$:

$$\kappa_{1,2} = \kappa_0 \cdot \qquad\qquad\qquad\qquad (4)$$

$$\cdot\int_0^\infty \int_0^\infty e^{-i\gamma k_0 \Delta z}\frac{\varphi_1(\sqrt{\eta^2+\gamma^2})}{\sqrt{\eta^2+\gamma^2}} \cdot \frac{\varphi_2(\sqrt{\xi^2+\eta^2})^*}{\sqrt{\xi^2+\eta^2}}\sin(k_0\Delta x\xi)\cos(k_0\Delta y\eta)\cdot\xi\cdot d\xi d\eta \; ;$$

$$(\xi^2 + \eta^2 + \gamma^2 = 1);$$

For the CC position (see fig. 4, a) in the area: $\Delta z \geq L_1/2 + L_2/2$:

$$\kappa_{1,2} = \kappa_0 \cdot \int_0^\infty \frac{e^{-i\gamma k_0 \Delta z}}{\gamma}\varphi_1(\kappa)\cdot\varphi_2(\kappa)^* J_0(k_0\Delta\rho\kappa)\kappa d\kappa \; , \qquad (5)$$

$$(\kappa^2 + \gamma^2 = 1 \; ; \Delta\rho = \sqrt{\Delta x^2 + \Delta y^2}\;);$$

where; $\Delta x = x_1 - x_2$; $\Delta y = y_1 - y_2$; $\Delta z = |z_1 - z_2|$; (x_s, y_s, z_s) – are the rectangular coordinates of the DR' centers;

$$\kappa_0 = \frac{8i}{v_2}\frac{r_1}{r_2}\frac{\beta_2}{\beta_1}\frac{\beta_{2z}}{k_0}\frac{(\varepsilon_{1r}-1)(\varepsilon_{2r}-1)}{\varepsilon_{2r}}\; ; \qquad (6)$$

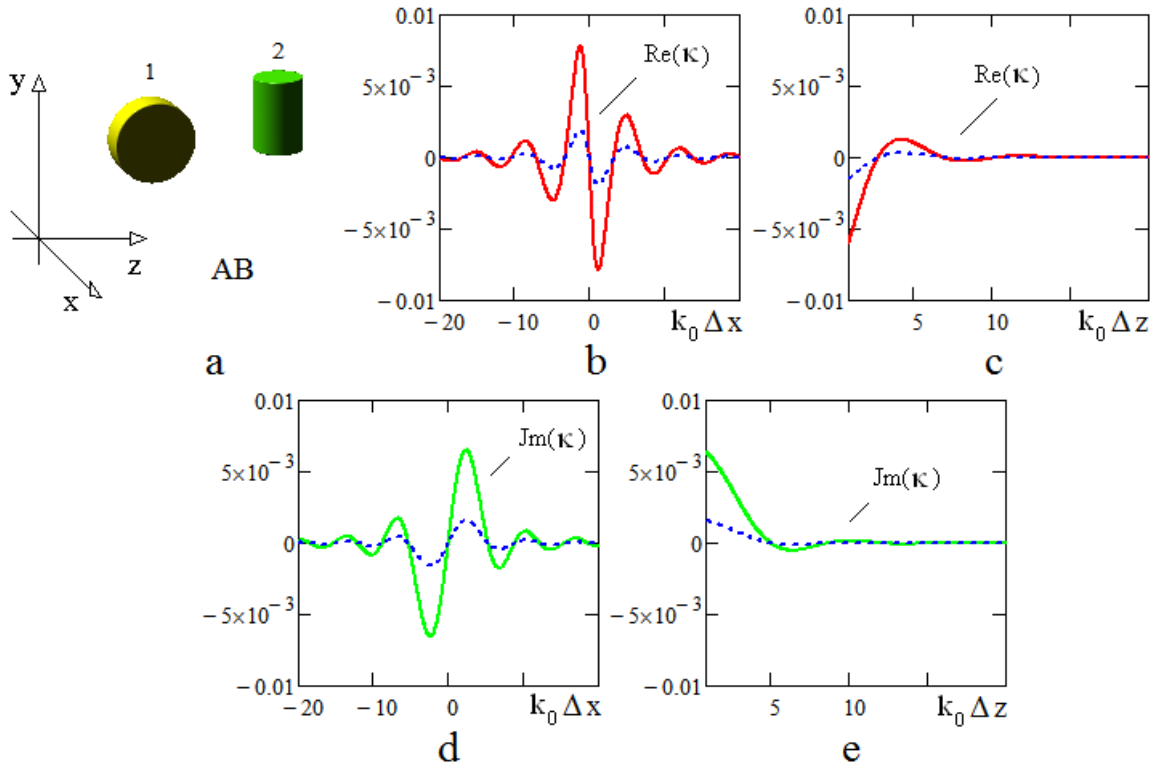$$v_2 = [J_1^2(p_{2\perp}) - J_0(p_{2\perp})J_2(p_{2\perp})](2p_{2z} + \sin 2p_{2z}).$$

Fig. 2. AB position of two Cylindrical DRs in Open space. Mutual coupling coefficients κ_{12} (solid lines); κ_{21} (dotted lines) (b - e) as a function coordinates of the DR centers with $\varepsilon_{1r} = 36$; $\varepsilon_{2r} = 81$; $\Delta_1 = L_1 / 2r_1 = 0,4$; $\Delta_2 = L_2 / 2r_2 = 0,8$. B, d: $k_0\Delta y = 2$; $k_0\Delta z = 1,1 k_0 (r_1 + r_2)$; c, e: $k_0\Delta x = k_0\Delta y = 2$.

Here

$$\varphi_s(\eta) = \frac{[\frac{p_{s\perp}}{q_{s\perp}} J_0(p_{s\perp})J_1(q_{s\perp}\eta) - \eta J_1(p_{s\perp})J_0(q_{s\perp}\eta)]}{[(\frac{p_{s\perp}}{q_{s\perp}})^2 - \eta^2]} \cdot \quad (7)$$

$$\cdot \frac{[\frac{p_{sz}}{q_{sz}} \sin p_{sz} \cos q_{sz}\sqrt{1-\eta^2} - \sqrt{1-\eta^2} \cos p_{sz} \sin q_{sz}\sqrt{1-\eta^2}]}{(\frac{p_{sz}}{q_{sz}})^2 - (\sqrt{1-\eta^2})^2}$$

and $p_{s\perp} = \beta_s r_s$; $p_{sz} = \beta_{sz} L_s / 2$; $q_{s\perp} = k_0 r_s$; $q_{sz} = k_0 L_s / 2$; β_s, β_{sz} - are the wave numbers of the eigenoscillation field of the s-th Cylindrical DR [10]; r_s - is the radius, L_s - is the height of the s-th DR ($s = 1, 2$); $H_n^{(2)}(\beta_0\rho)$ are the Hankel functions of the second kind; $J_n(x)$ - are the Bessel functions of the first kind of the n-th order; $k_1 = \sqrt{\varepsilon_{1r}} k_0$; $k_0 = \omega_0 / c$; ω_0 - is the circular resonance frequency; c -

is the light velocity; ε_{sr} - is the relative dielectric permittivity of the s-th DR' material.
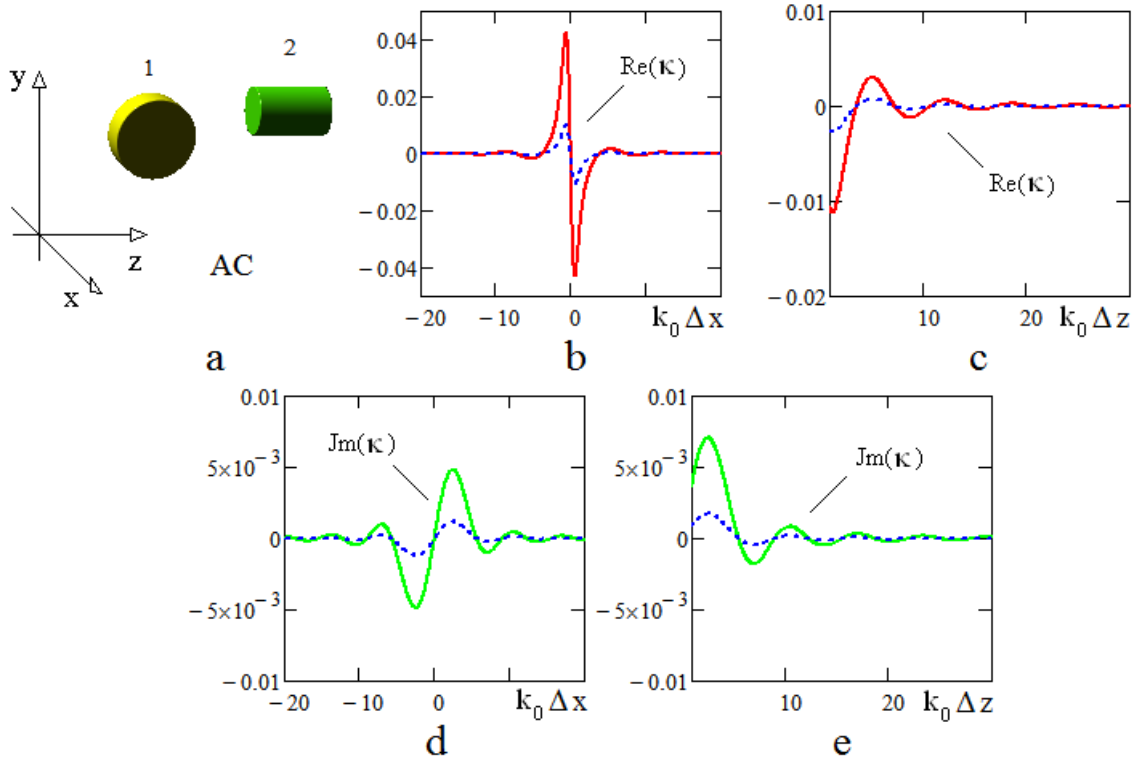

Fig. 3. Position AC of the Cylindrical DRs in Open space. Mutual coupling coefficients κ_{12} (solid lines); κ_{21} (dotted lines) (b - e) as a function coordinates of the DR centers with $\varepsilon_{1r} = 36$; $\varepsilon_{2r} = 81$; $\Delta_1 = L_1 / 2r_1 = 0,4$; $\Delta_2 = L_2 / 2r_2 = 0,8$. B, d: $k_0\Delta y = 0$; $k_0\Delta z = 1,3k_0(r_1 + L_2 / 2)$; c, e: $k_0\Delta x = 2$; $k_0\Delta y = 0$.

The integral convergence provides by choice of the radical signs for $\xi > 1$: $\sqrt{1-\xi^2} = -i\sqrt{\xi^2 - 1}$ in the (2 - 4), as well as for $\kappa > 1$: $\sqrt{1-\kappa^2} = -i\sqrt{\kappa^2 - 1}$ in the (5).

Note also that the function $\varphi_s(\eta)$ has no singularities in the region $0 \le \eta < \infty$. It makes a major contribution to the integrals (2-6) only for small values η. Expand the function $f_s(\eta) = \varphi_s(\eta) / \eta$ in a series of η at 1 and taking into account that $f_s(1) = \varphi_s(1)$ obtain:

$$\varphi_s(\eta) = \varphi_s(1) \cdot \eta + \frac{d}{d\eta}\left[\frac{\varphi_s(\eta)}{\eta}\right]_{\eta=1} \eta(\eta - 1) + ...$$

Next, using the approximation:

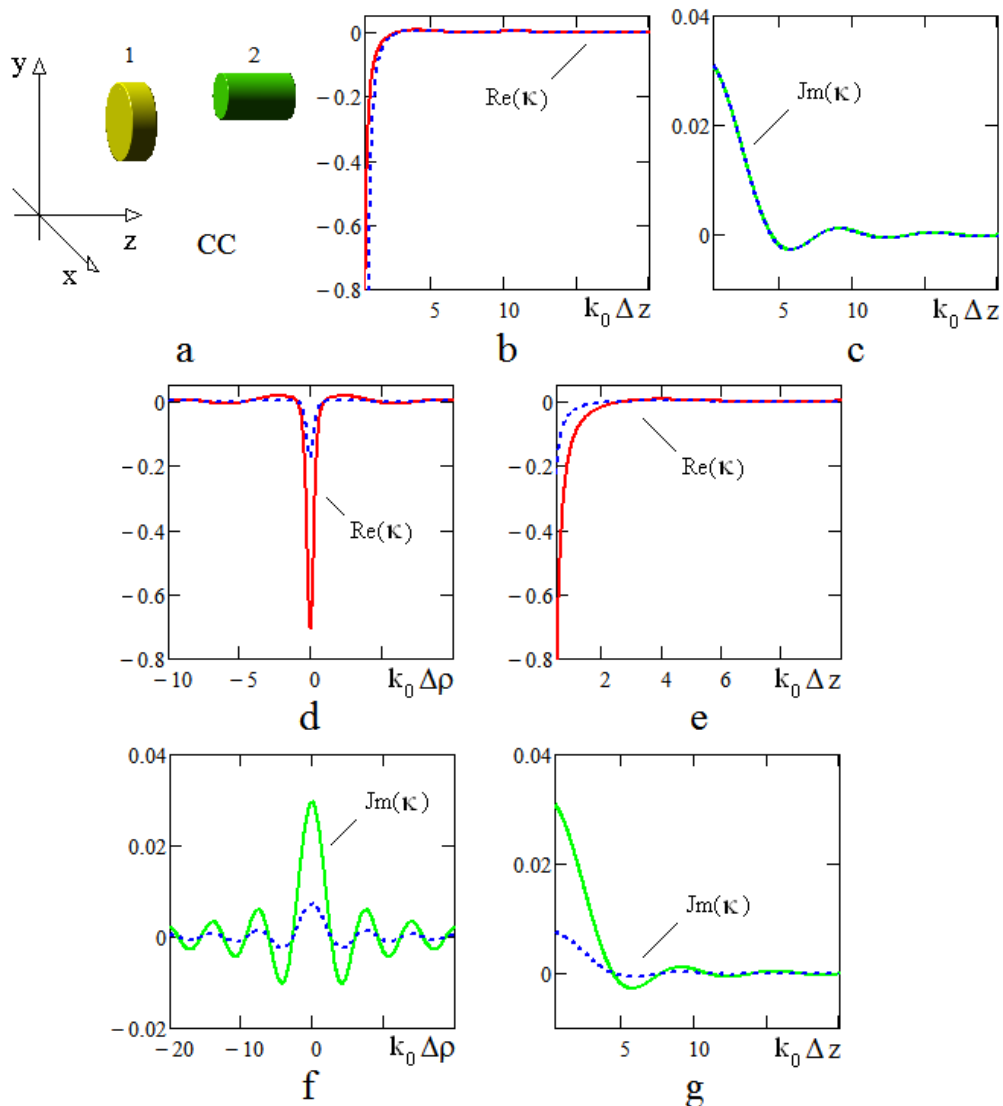$$\varphi_s(\eta) \approx \varphi_s(1) \cdot \eta. \tag{8}$$

we obtain

Fig. 4. CC position of two Cylindrical DRs in Open space. Mutual coupling coefficients (b -
g) as a function coordinates of the DR centers with $\varepsilon_{1r} = 36$; $\varepsilon_{2r} = 81$; $\Delta_1 = L_1 / 2r_1 = 0,4$;
$\Delta_2 = L_2 / 2r_2 = 0,8$. B, c, e, g: $k_0 \Delta x = k_0 \Delta y = 0$; d, f: $k_0 \Delta z = 1,7 k_0 (L_1 + L_2) / 2$.

For the AA position:

$$\kappa_{1,2} \simeq \kappa_0 \cdot \varphi_1(1) \varphi_2(1)^* \{ \left(\frac{\Delta\rho}{\Delta r} \right)^2 h_0^{(2)}(k_0 \Delta r) - [1 - 3 \left(\frac{\Delta x}{\Delta r} \right)^2] \frac{h_1^{(2)}(k_0 \Delta r)}{k_0 \Delta r} \}; \quad (9)$$

$$(\Delta\rho = \sqrt{\Delta y^2 + \Delta z^2})$$

For the AB position:

$$\kappa_{1,2} \simeq -\kappa_0 \cdot \varphi_1(1) \varphi_2(1)^* \frac{\Delta x \Delta y}{\Delta r^2} h_2^{(2)}(k_0 \Delta r); \quad (10)$$

For the AC position:

$$\kappa_{1,2} \simeq \kappa_0 \cdot \varphi_1(1)\varphi_2(1)^* \frac{\Delta x \Delta z}{\Delta r^2} h_2^{(2)}(k_0 \Delta r) ; \tag{11}$$

For the CC position:

$$\kappa_{1,2} \simeq \kappa_0 \cdot \varphi_1(1)\varphi_2(1)^* \left\{ \left(\frac{\Delta \rho}{\Delta r} \right)^2 h_0^{(2)}(k_0 \Delta r) - [1 - 3\left(\frac{\Delta z}{\Delta r} \right)^2] \frac{h_1^{(2)}(k_0 \Delta r)}{k_0 \Delta r} \right\} ; \tag{12}$$

$$(\Delta \rho = \sqrt{\Delta x^2 + \Delta y^2})$$

In the (9 - 12) $\Delta r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$; $h_n^{(2)}(z)$ - are the spherical Hankel functions of the second kind [12]. Here we have also used the generalized Sommerfeld's integral [11].

Found relations allow us to calculate the coupling coefficients of different Cylindrical DRs in the open space. It is interesting that at the same time restrictions on the range DR coordinates (2 - 5) are removed. In the particular case of identical DRs the relations (2, 5) coincide with [13].

Coupling coefficient analysis

Fig. 1, 4, b - c shows the dependence of the coupling coefficients of the DR center coordinates, calculated according to the formulas (9, 12) (solid curves) as well as the numerical formulas (2, 5) (dashed curves). As can be seen from these curves, the use of approximation (8) gives a very good accuracy.

It is easy to verify that the coupling coefficients found (9 - 12) are proportional to the respective magnetic field components of the first resonator in the axis of symmetry direction of the second resonator.

Given this observation, we can assume that, in general, mutual coupling coefficient of two different cylindrical DR with the mode $H_{1,0,1}^+$ will be represented as:

$$\kappa_{1,2} \simeq \kappa_0 \cdot \varphi_1(1)\varphi_2(1)^* \cdot (\vec{h}_1(\Delta r, \theta_1, \varphi_1), \vec{n}_2) \tag{13}$$

where $\vec{h}_1(\Delta r, \theta_1, \varphi_1)$ - is the magnetic field of the first DR in the center of the second one; $\vec{n}_2 = \vec{n}_2(\theta_2, \varphi_2)$ is the unit vector directed from the DR' center along the axis of second DR (fig. 5, a).

The relation (13) is exactly the same (9-12) in the case of AA; AB; AC and CC DR position. Fig. 5, b - e shows mutual coupling coefficients as a function of arbitrary relative DR orientation.
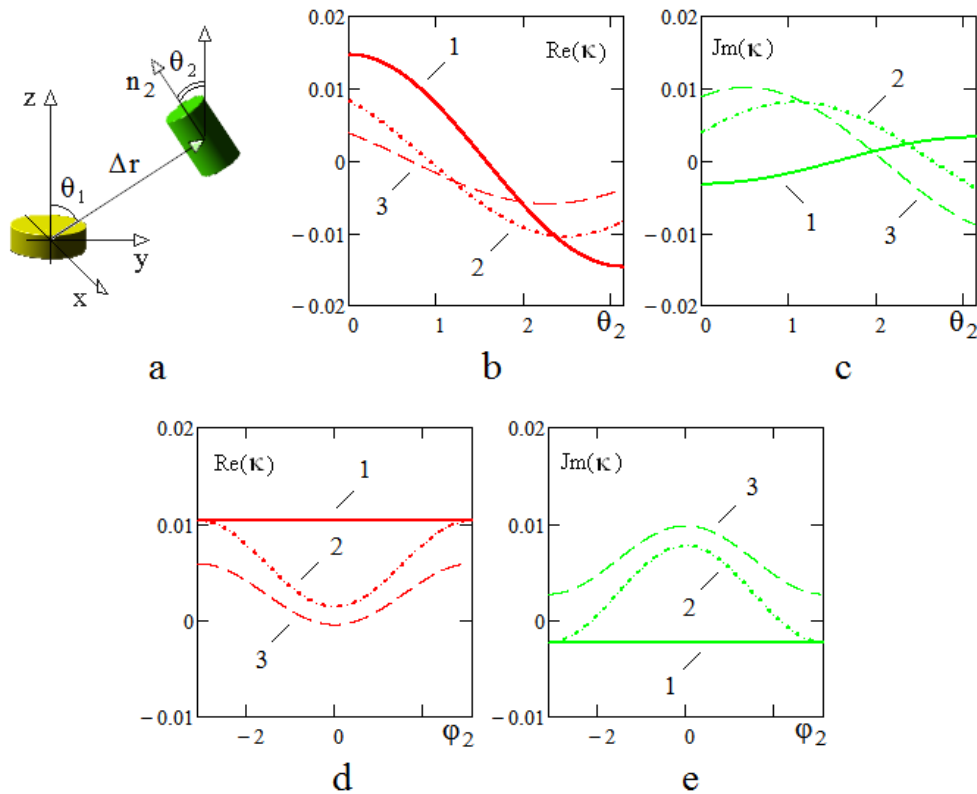
Fig. 5. General position of the Cylindrical DRs in the Open space. Mutual coupling coefficients (b - e) as a function of the relative DR orientation: $k_0 \Delta r = 3$; b, c: $\varphi_1 = \varphi_2 = 0$; curve 1: $\theta_1 = \pi/2$; curve 2: $\theta_1 = \pi/4$; curve 3: $\theta_1 = \pi/8$; d, e: $\varphi_1 = 0$; $\theta_2 = \pi/4$; curve 1: $\theta_1 = \pi/2$; curve 2: $\theta_1 = \pi/4$; curve 3: $\theta_1 = \pi/8$.

Conclusions

Analytical relationships for mutual coupling coefficients for the $H_{1,0,1}^+$ modes of different Cylindrical DR in the Open space has been obtained and investigated.

It stated, that mutual coupling coefficients are determined by the dependencies on the DR magnetic field and relative orientation of the DR axes.

The resulting ratio can be used for calculations of the DR natural oscillations, as well as the scattering parameters of the various element gratings in the communication devices with dielectric resonators.

References

1. Kajfez D. and Guillon P. (1998) *Dielectric Resonators*. Noble Publishing Corporation, Atlanta, 561 p.

2. Ueda T., Lai A. and Itoh T. (2007) Demonstration of Negative Refraction in a Cutoff Parallel-Plate Waveguide Loaded With 2-D Square Lattice of Dielectric Resonators. *IEEE Trans. on MTT*, Vol. 55, No. 6, pp. 1280-1287. DOI : 10.1109/tmtt.2007.897753

3. Kishk A. A. (2005) Directive Yagi-Uda dielectric resonator antennas. *Microwave and optical technology letters*, Vol. 44, No. 5, pp. 451-453. DOI: 10.1002/mop.20664

4. Benson T. M., Boriskina S. V., Sewell P., Vukovic A., Greedy S. C. and Nosich A. I. (2006) *Micro-Optical Resonators for Microlasers and Integrated Optoelectronics*. Frontiers in Planar Lightwave Circuit Technology, No 216, pp. 39. DOI : 10.1007/1-4020-4167-5_02

5. Preu S., Schwefel H. G. L., Malzer S., D¨ohler G. H., Wang L. J., Hanson M., Zimmer-

man J. D. and Gossard A. C. (2008) Coupled whispering gallery mode resonators in the Terahertz frequency range. *Optics Express*, Vol. 16, No. 10, pp. 7336-7343. DOI: 10.1364/oe.16.007336

6. Semouchkina E. A.*,* Semouchkin G. B., Lanagan M. and Randall C. A. (2005) FDTD Study of Resonance Processes in Metamaterials. *IEEE Trans. on MTT*, Vol. 53, No 4, pp. 1477-1487. DOI: 10.1109/tmtt.2005.845203

7. Trubin A. A. (1987) Perturbation theory analysis of the System Coupling Dielectric Resonators. *Vestnik Kievskogo politekhnicheskogo instituta. Radiotekhnika*, No 24, pp. 42-45. (in Russian)

8. Trubin A.A. (1997) Scattering of electromagnetic waves on the Systems of Coupling Dielectric Resonators. *Radioelectronics*. No 2, pp. 35-42.

9. Trubin A. A. (2016) *Lattices of Dielectric Resonators*, Series in Advanced Microelectronics Vol 53, Springer International Publishing, 159 p. DOI : 10.1007/978-3-319-25148-6

10. Ilchenko M. E. and Trubin A. A. (2004) *Electrodynamics' of Dielectric Resonators*, Kiev, Naukova Dumka, 265 p. (in Russian)

11. Trubin A. A. Several specified integrals of special functions, Available at : http://eqworld.ipmnet.ru/ru/auxiliary/Trubin-EqWorld-2010_02_06.pdf

12. Abramowitz M. ed. and Stegun I. (1964) *Handbook of mathematical functions*. National bureau of standards.

13. Trubin, A. A., Shmyglyuk, G. S. (2006) The modeling of the antenna lattce parameters on cylindrical dielectric resonators. *Visn. NTUU KPI, Ser. Radioteh. radioaparatobuduv.*, no. 33, pp. 101-108. (in Ukrainian)

Трубін О. О. **Коефіцієнти зв'язку різних циліндричних діелектричних резонаторів у відкритому просторі.** *Приведено результати розрахунків коефіцієнтів взаємного зв'язку циліндричних діелектричних резонаторів різних відносних розмірів з магнітними типами коливань. Знайдено загальна формула для коефіцієнтів взаємного зв'язку для довільної орієнтації циліндричних ДР у відкритому просторі. Розглянуті основні закономірності зміни зв'язку при варіації відносного положення резонаторів.*

Ключові слова: циліндричні діелектричні резонатори різних розмірів; коефіцієнти зв'язку; магнітні типи коливань.

Трубин А. А. **Коэффициенты связи различных цилиндрических диэлектрических резонаторов в открытом пространсте.** *Приведены результаты расчетов коэффициентов взаимной связи цилиндрических диэлектрических резонаторов различных относительных размеров с магнитными типами колебаний. Получена общая формула для коэффициентов взаимной связи для произвольной ориентации цилиндрических ДР в открытом пространстве. Рассмотрены основные закономерности изменения связи при вариации относительного положения резонаторов.*

Ключевые слова: цилиндрические диэлектрические резонаторы различных размеров, коэффициенты связи, магнитные типы колебаний

Trubin A. A. **Coupling coefficients of different cylindrical dielectric resonators in the open space.** *The calculation results of the mutual coupling coefficients of different relative sizes cylindrical dielectric resonators with magnetic modes are presented. A general formula for the mutual coupling coefficients for an arbitrary orientation of the cylindrical DRs in the open space are obtained. The basic patterns of coupling coefficient changing with the variation of the relative position of the resonators are examined.*

Keywords: different cylindrical dielectric resonator, coupling coefficient, magnetic mode.