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Scattering of Electromagnetic Pulses on Different Dielectric Resonator Systems

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The theory of scattering of electromagnetic pulses in a transmission line on systems of coupled dielectric resonators of different shapes and dielectric permittivity is developed. General analytic expressions for envelope pulses scattered on band-pass and rejection dielectric filters are obtained. It is shown that for identical resonators the expressions found in special cases become known. The results of calculating the rectangular and Gaussian pulses scattered by bandpass and bandstop filters of the terahertz and infrared wavelengths, constructed on various dielectric resonators in various transmission lines, are presented.

Key words: different dielectric resonator; pulses; scattering; bandstop filter; bandpass filter

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Introduction

As is known, the power loss of electromagnetic energy in conductors increases with growing frequency, which makes it impossible to build high-quality resonators on metal structures already in the millimeter wavelength range. The existence of dielectrics with anomalously low losses in the Millimeter, Terahertz and up to the visible wavelength range makes it possible to design high-Q resonators based on them. At present, there are no elements alternative to Dielectric Resonators (DRs) in these wavelength ranges. DRs are the main elements of filters, antennas, metamaterials, etc., consisting, generally, of a large number of unit. The construction of frequency-selective devices of communication systems based on a large number of resonators requires the study of the propagation of pulses in complex DR structures. Thus, the need to study the propagation of electromagnetic pulses in various Microwave, Terahertz and Optical devices is an actual task of telecommunication theory and practice [1–6]. However, the solution of this class of problems in scattering theory, by virtue of its complexity, is carried out most often by numerical methods. In some cases, the solution of similar problems can be obtained approximately in an analytical form, by means of electrodynamic modeling. Finding such analytical solutions also makes it possible to more clearly understand the physical mechanisms of scattering, greatly accelerates the computation process, besides it allows calculating and optimizing more complex resonator structures.

In the present paper we consider the most general solution of the problem of scattering, obtained with the help of perturbation theory, of electromagnetic pulses, propagating in the transmission lines, by a system, consisting of a number of different coupled DRs of various shapes and possibly consisting of different materials [7].

1 Statement of the problem

The goal of the current article is the development of the theory of scattering of electromagnetic pulses on the microwave, infrared and optical filters, consisting of different dielectric resonators in the time domain.

2 Pulses scattering theory on different DRs

Consider the system of N different DRs, possibly consisting of different materials. We assume that the eigenoscillation field of each isolated DR is known: $(\vec{e}_s, \vec{h}_s)$, $(s = 1, 2, ..., N)$. Here $\vec{e}_s$-is the electric field and $\vec{h}_s$ — is the magnetic field intensity of the s-th isolated DR. We assume also that the fields of different DRs satisfy approximately the orthogonality conditions:

$$\begin{aligned} \int\limits_{v_n} \vec{e}_n \vec{e}_m^{*} dv \approx \delta_{nm} \int\limits_{v_n} \left|\vec{e}_n\right|^2 dv; \\ \int\limits_{v_n} \vec{h}_n \vec{h}_m^{*} dv \approx \delta_{nm} \int\limits_{v_n} \left|\vec{h}_n\right|^2 dv. \end{aligned} \qquad (1)$$

Here $*$ — is the complex conjugation symbol; δ_{nm} is the Kronecker symbol; v_n- is the volume of n-th DR.

We consider the problem of scattering of an electromagnetic wave $\left(\vec{E}^{+}(t), \vec{H}^{+}(t)\right)$ on a system of coupled DRs. It is assume that the dependence on time $\left(\vec{E}^{+}(t), \vec{H}^{+}(t)\right)$ is known to us. We are looking for the solution of the scattering problem in the form:

$$\begin{aligned} \vec{E} &\approx \vec{E}^{+}(t) + \sum_{s=1}^{N} a^s(t)\vec{e}^s; \\ \vec{H} &\approx \vec{H}^{+}(t) + \sum_{s=1}^{N} a^s(t)\vec{h}^s, \end{aligned} \tag{2}$$

where $a^s(t)$ are the unknown time functions ($s = 1, 2, ..., N$) and $(\vec{e}^s, \vec{h}^s)$ are the s-th eigenoscillation field of the N-DR system [8]. The eigenoscillation field of the N-DR system we represented as a superposition of fields of the isolated resonators:

$$\begin{aligned} \vec{e}^s &= \sum_{u=1}^{N} b_u^s \vec{e}_u; \\ \vec{h}^s &= \sum_{u=1}^{N} b_u^s \vec{h}_u. \end{aligned} \tag{3}$$

The DR amplitudes of coupled oscillations b_u^s should satisfy the equation system [8] and do not depend on time.

Let us assume that the frequency of the carrier pulse (Ω) differs slightly from the frequencies of the coupled oscillations of the resonator system (ω^s): $|\Omega - \omega^s|/\omega_0 \ll 1$; $\omega_0 = \mathrm{Re}[\omega_s]$ ($s = 1, 2, ..., N$) is the real part of frequencies of isolated partial resonators. Using Maxwell's equations for the eigenoscillations of isolated and coupled DRs and the scattering field, as well as the orthogonality conditions (1), after integrating over the volume of each DR, we find a system of differential equations for the amplitudes $a^s(t)$:

$$\sum_{s=1}^{N}\left[(2Q_u^D - i)\frac{d}{dt}a^s(t) - 2i\omega^s Q_u^D a^s(t)\right]b_u^s = -\frac{\omega_0}{P_u^D}\left(c_u^{+}(t)\right)^{*}, \quad (u = 1, 2, ..., N), \tag{4}$$

where $Q_u^D = \omega_0 w_u / P_u^D$ — is the Q-factor of loss in dielectric of u-th DR; $w_u = 1/4 \int\limits_{v_u} \left(\varepsilon_u |\vec{e}_u|^2 + \mu_0 \left|\vec{h}_u\right|^2\right) dv$ — is the energy, stored in the dielectric of the u-th DR; $\varepsilon_u = \mathrm{Re}\,\tilde{\varepsilon}_u$; $\tilde{\varepsilon}_u = \varepsilon_u - i\varepsilon''_u$ — is the complex dielectric permittivity and the μ_0 is the permeability of the u-th DR. $P_u^D = \omega_0 \frac{\varepsilon''_u}{2} \int\limits_{v_u} |\vec{e}_u|^2 dv$ — is the loss power in the dielectric; and

$$c_u^{+}(t) = -\frac{1}{2}\oint\limits_{s_u}\left\{[\vec{e}_u, \vec{n}]\vec{H}^{+}(\vec{r}, t)^{*} + [\vec{n}, \vec{h}_u]\vec{E}^{+}(\vec{r}, t)^{*}\right\}ds; \tag{5}$$

is the integral over the surface s_u of u-th DR; $\vec{n}$ — is the normal to the surface.

The solution of the system of equations (4) together with (2) in the most general form determines the distribution of the field of the DR system when the time pulses of the field fall on them.

If the resonators are excited by a harmonic signal with a frequency ω: $d/dt[a^s(t)] = i\omega a^s(t)$, the system of equations (4) becomes known [7].

The solution of the equations system (4) can be found in an asymptotic form, if we take into account the presence of small parameters $\delta_v = (2Q_v^D)^{-1} \ll 1$ ($v = 1, 2, ..., N$):

$$a^s(t) \approx a_0^s(t) - e^{i\omega^s t}\sum_{v=1}^{N}\delta_v \frac{\det B_v^s}{\det B} \cdot \sum_{u=1}^{N}\omega^u b_v^u \int\limits_{-\infty}^{t} e^{-i\omega^s \tau} a_0^s(\tau)d\tau + O(\delta_v^2), \tag{6}$$

where

$$a_0^s(t) = -\frac{1}{2}e^{i\omega^s t}\int\limits_{-\infty}^{t} e^{-i\omega^s \tau}\frac{\det C_s(\tau)}{\det B}d\tau \tag{7}$$

is the principal term of the expansion and

$$B_v^s = \begin{bmatrix} b_1^1 & ... & 0 & ... & b_1^N \\ . & . & 0 & . & . \\ 0 & 0 & 1 & 0 & 0 \\ . & . & 0 & . & . \\ b_N^1 & ... & 0 & ... & b_N^N \end{bmatrix}$$

is the minor of matrix B in which s is the column and v is the row number;

$$C_s(t) = \begin{bmatrix} b_1^1 & ... & c_1^{+}(t)^{*}/w_1 & ... & b_1^N \\ b_2^1 & ... & c_2^{+}(t)^{*}/w_2 & ... & b_2^N \\ . & ... & . & ... & . \\ b_N^1 & ... & c_N^{+}(t)^{*}/w_N & ... & b_N^N \end{bmatrix};$$

$$B = \begin{bmatrix} b_1^1 & b_1^2 & ... & b_1^N \\ b_2^1 & b_2^2 & ... & b_2^N \\ . & . & . & . \\ b_N^1 & b_N^2 & ... & b_N^N \end{bmatrix}.$$

The delay of the incident pulse on DR is determined by the corresponding phase factor in the expansion coefficients $c_u^{+}(t)^{*}$. The choice of arbitrary constants in the solution of the equation (7) is determined by the physical conditions: $a^s(t) = 0$ if $\left(\vec{E}^{+}(t), \vec{H}^{+}(t)\right) = 0$.

In some cases it is convenient to use the time Green's functions. Green's functions for different DR can be calculated if we use the approximate expansion of the scattered field (7) with a harmonic effect on the resonator system. In this approximation, the Green's functions obtained also coincide with those obtained earlier [8].

In the particular case of identical DR and types of oscillations: $Q_u^D = Q^D$; $P_u^D = P^D$: the system (4) may be solved with respect to differential equations:

$$(2Q^D - i)\frac{d}{dt}a^s(t) - 2i\omega^s Q^D a^s(t) = -Q^D \frac{\det C_s(t)}{\det B}. \quad (8)$$

The solution of the equation (8), taking into account the linearity of the determinant $C_s(t)$, is [9]:

$$a_s(t) = -\frac{Q^D}{2Q^D - i} e^{\frac{2Q^D}{2Q^D - i} i\omega^s t} \cdot \int_{-\infty}^{t} e^{-\frac{2Q^D}{2Q^D - i} i\omega^s \tau} \frac{\det C_s(\tau)}{\det B} d\tau \approx a_0^s(t). \quad (9)$$

Solutions (6) and (2) allow us to directly calculate the envelopes of scattered pulses. Let us assume, that the incident pulse may be represented in the form:

$$\begin{pmatrix} \vec{E}^+(\vec{r},t) \\ \vec{H}^+(\vec{r},t) \end{pmatrix} \approx A_{in}(t) e^{i\Omega t} \begin{pmatrix} \vec{E}^+(\vec{r}) \\ \vec{H}^+(\vec{r}) \end{pmatrix}, \quad (10)$$

where $A_{in}(t)$ — is slowly varying amplitude:

$$|dA_{in}/dt| \,/\, |A_{in}| \ll \Omega.$$

$(\vec{E}^+(\vec{r}), \vec{H}^+(\vec{r}))$ satisfies the Maxwell equations:

$$rot\vec{H}^+(\vec{r}) \approx i\Omega\varepsilon_0 \vec{E}^+(\vec{r});$$

$$rot\vec{E}^+(\vec{r}) \approx -i\Omega\mu_0 \vec{H}^+(\vec{r})$$

and also the boundary conditions. Then, from (5): $c_s^+(t) = A_{in}(t)^* e^{-i\Omega t} c_s^+$, where

$$c_s^+ = -\frac{1}{2}\oint_{S_s} \{[\vec{e}_s, \vec{n}]\vec{H}^+(\vec{r})^* + [\vec{n}, \vec{h}_s]\vec{E}^+(\vec{r})^*\} ds.$$

Scattered pulses at a sufficiently large distance from the resonators in the transmission line can also be represented in the form:

$$\begin{pmatrix} \vec{E}^\pm_{out}(\vec{r},t) \\ \vec{H}^\pm_{out}(\vec{r},t) \end{pmatrix} \approx A^\pm_{out}(t) e^{i\Omega t} \begin{pmatrix} \vec{E}^\pm(\vec{r}) \\ \vec{H}^\pm(\vec{r}) \end{pmatrix}.$$

Then from (7), (2) the envelopes can be represented as:

$$A^+_{out}(t) \approx A_{in}(t) - \frac{\omega_0}{2}\sum_{s=1}^{N} \frac{\det B_s^+}{\det B} \left\{ e^{i(\omega^s - \Omega)t} \cdot \int_{-\infty}^{t} e^{i(\Omega - \omega^s)\tau} A_{in}(\tau) d\tau \right\};$$

$$A^-_{out}(t) \approx -\frac{\omega_0}{2}\sum_{s=1}^{N} \frac{\det B_s^-}{\det B} \left\{ e^{i(\omega^s - \Omega)t} \cdot \int_{-\infty}^{t} e^{i(\Omega - \omega^s)\tau} A_{in}(\tau) d\tau \right\}. \quad (11)$$

Here for the bandstop filters:

$$B_s^\pm = \begin{bmatrix} b_1^1 & \dots & \sum_{u=1}^{N} b_u^b \tilde{k}_{u1}^{\pm+} & \dots & b_1^N \\ b_2^1 & \dots & \sum_{u=1}^{N} b_u^b \tilde{k}_{u2}^{\pm+} & \dots & b_2^N \\ . & \dots & . & \dots & . \\ b_N^1 & \dots & \sum_{u=1}^{N} b_u^b \tilde{k}_{uN}^{\pm+} & \dots & b_N^N \end{bmatrix} \quad (12)$$

and for the bandpass filters [8]:

$$B_s^+ = \begin{bmatrix} b_1^1 & \dots & b_N^b \tilde{k}_{N1}^{++} & \dots & b_1^N \\ b_2^1 & \dots & 0 & \dots & b_2^N \\ . & \dots & . & \dots & . \\ b_N^1 & \dots & 0 & \dots & b_N^N \end{bmatrix}; \quad B_s^- = \begin{bmatrix} b_1^1 & \dots & b_1^b \tilde{k}_{11}^{-+} & \dots & b_1^N \\ b_2^1 & \dots & 0 & \dots & b_2^N \\ . & \dots & . & \dots & . \\ b_N^1 & \dots & 0 & \dots & b_N^N \end{bmatrix}. \quad (13)$$

(s is the column number of the matrix $B_s^\pm$ with the sum $\sum_{u=1}^{N} b_u^b \tilde{k}_{us}^{\pm+}$);

$$\tilde{k}_{sn}^{++} = (c_s^+ c_n^{+*})/(\omega_0 w_n) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s - z_n)};$$
$$\tilde{k}_{sn}^{-+} = (c_s^- c_n^{+*})/(\omega_0 w_n) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s + z_n)};$$

$(\tilde{k}_{sn})_0$ — is the coupling coefficient for the propagating wave, expressed without phase difference accounting in the transmission line ($s, n = 1, 2, ..., N$).

As might be expected from (11), the envelopes of scattered pulses are linear functions of the incident pulse. The dependence of the envelopes is determined by the carrier frequency difference from the frequencies of the coupled oscillations of the resonator system.

3 Pulses scattering on different DR filters

The obtained relations allow us to directly calculate the shape of the envelopes of scattered pulses on the systems of different DRs in the transmission line. Let us consider the scattering of pulses by dielectric filters whose amplitude-frequency characteristics were examined earlier in [7, 10, 11].

Fig. 1 shows example of scattered pulses on the bandstop filter, consisting of 10 Different Spherical microresonators. Each microcavity was excited by a magnetic type of oscillation H_{nm1} with $m = n$. The distance between adjacent DR centers was equal to $115\lambda_w/4$, where λ_w — is the guided wavelength. The rectangular and Gaussian scattered pulse envelops were calculated by the formula (11), (12).

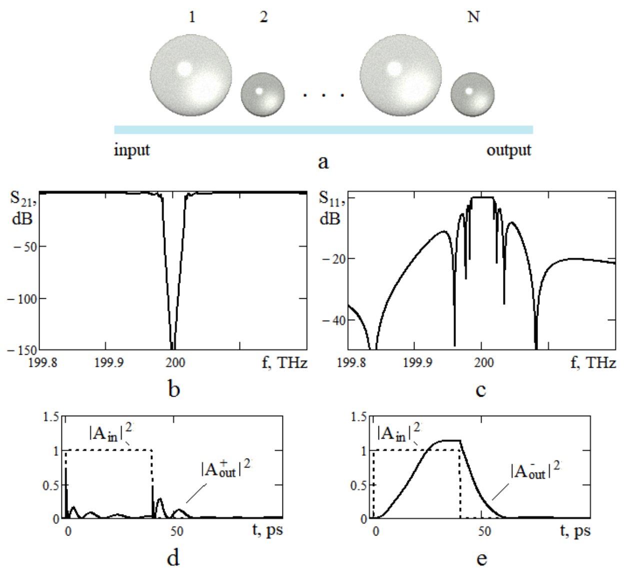

Fig. 1. Bandstop filter on coupled different spherical microresonators and dielectric waveguide (a). S-matrix responses of one band of the 10-section (a) filter on spherical microresonator H_{mm1} modes ($\varepsilon_{1r} = 9$; $\varepsilon_{2r} = 16$; $Q_1^D = 1/tg\delta_1 = 10^6$; $Q_2^D = 1/tg\delta_2 = 2 \cdot 10^6$; $m_1 = m_2 = n_1 = n_2 = 26$; $l_1 = l_2 = 1$). Envelops of the scattered rectangular pulse ($t_2 - t_1 = 40$ps) (d, e) on the bandstop filter (a - c).

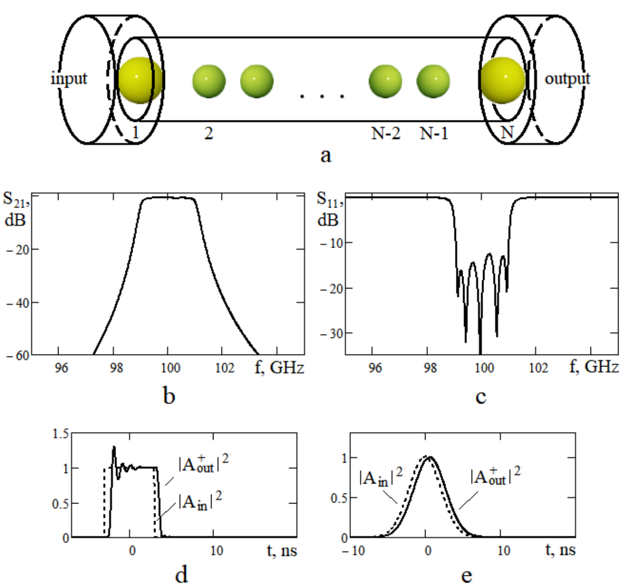

Fig. 2. Different spherical (a) DRs in the evanescent circular waveguide ($\varepsilon_{1r} = 16$;$\varepsilon_{2r} = 25$;$Q_1^D = 3 \cdot 10^3$; $Q_2^D = 2 \cdot 10^3$; $R = 0,9mm$; $R_z = 0,5mm$; $f_0 = 100$GHz). S-matrix module as function of frequency of 5-DR bandpass filter (b, c) with H_{111} modes. Envelops of the scattered rectangular ($t_2 - t_1 = 6$ns) (d), Gaussian ($\sigma = 3$ns) (e) pulses.

Fig. 2 shows rectangular and Gaussian pulses, scattered by bandpass filter on 5 different Spherical DRs ($N = 5$). The centers of all the resonators were located on the axis of symmetry of the waveguides. Each resonator was excited on the main magnetic mode H_{111}. The radius of the input and output waveguides determined the possibility of propagation of only one type of wave H_{11}. The presence of input and output resonators with a lowered dielectric permittivity made it possible to increase their coupling with the waveguides and obtain a smooth characteristic of the transmission coefficient (Fig. 2b).

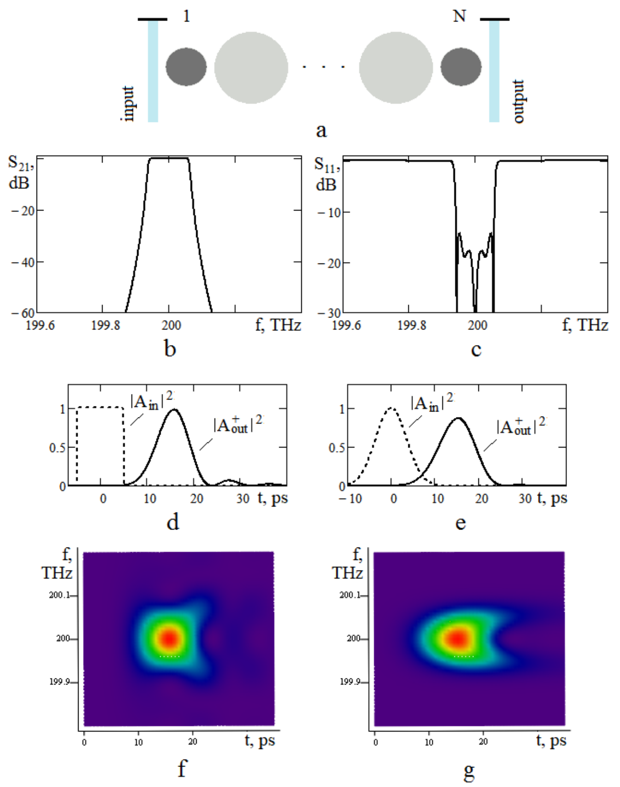

Fig. 3. Sketch of bandpass filter on laterally coupled different Disk microresonators (a). S-matrix responses of the 7-section bandpass filter on $HE^+_{1,m,1}$ modes as functions of the frequency (b - c). The coupling coefficients of the terminal resonators with transmission lines: $k_L = 7 \cdot 10^{-4}$; the 1-th and the N-th microresonator parameters are $\varepsilon_{1r} = 16$; $Q_1^D = 10^6$; $m_1 = 20$; another resonator parameters are $\varepsilon_{2r} = 9,6$; $Q_2^D = 2 \cdot 10^6$; $m_2 = 24$. Envelops of the scattered rectangular ($t_2 - t_1 = 10$ps) (d), Gaussian ($\sigma = 5$ps) (e) pulses at $\Omega = 2\pi f_0$ ($f_0 = 200$THz). Frequency versus time of rectangular (f) and Gaussian (g) transmitted pulses.

In Fig. 3 shows the result of scattering of a rectangular and Gaussian pulse on a bandpass filter with various disc microcavities [11]. The scattered pulse envelops were calculated by the formula (11), (13).

As can be seen, the developed model correctly describes the pulses scattering on the system of different DRs. The envelopes of rectangular pulses are most noticeable, while the envelopes of Gaussian pulses vary considerably less, and their scattering by filters is determined only by visible delay effects.

Discussion and Conclusion

A scattering theory of the pulses on different DR systems, based on perturbation theory, has been expanded.

Obtained new analytical relationships, which allow us to directly calculate the envelopes of pulses without using time Green functions.

At the same time, it should be noted that the above scattering theory has a number of limitations, related both to the assumption about the type of scattered pulses (10) and the less obvious assumption of neglecting the transient processes of wave propagation in the cavity volume. Obviously, similar processes of wave propagation will play an important role in resonators with higher types of oscillations. Another less obvious limitation is the assumption that the spectrum of the incident pulses is limited. However, this condition is necessary for effective frequency filtering in telecommunication systems. In the future, in our opinion, it is of interest to more subtly compare the experimental, numerical, and analytical results with the aim of elucidating the influence of various conditions for the scattering of electromagnetic pulses of different shapes on multiresonator structures.

A developed analytical model can be used to directly optimize telecommunication and other devices that built on different types of Dielectric Resonators.

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Розсіювання електромагнітних імпульсів на системах різноманітних діелектричних резонаторів

Трубін О. О.

Розвинута теорія розсіювання електромагнітних імпульсів в лінії передачи на системах зв'язаних діелектричних резонаторів різної форми та діелектричної проникності. Знайдено загальні аналітичні вирази для огинаючих імпульсів, які розсіюються на смугових та режекторних діелектричних фільтрах. Показано, що отримані вирази переходять в відомі в окремих випадках однакових резонаторів. Наведено результати розрахунків обвідної прямокутного та гауссовского імпульсів при їх розсіюванні на смугових та режекторних фільтрах терагерцового та інфрачервоного діапазонів довжин хвиль, побудованих на різноманітних діелектричних резонаторах в різноманітних лініях передачи.

Ключові слова: діелектричний резонатор; імпульси; розсіювання; режекторний фільтр; смуговий фільтр

Рассеяние электромагнитных импульсов на системах различных диэлектрических резонаторов

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Развита теория рассеяния электромагнитных импульсов в линии передачи на системах связанных диэлектрических резонаторов разной формы и диэлектрической проницаемости. Получены общие аналитические выражения для огибающих импульсов, рассеиваемых на полосовых и режекторных диэлектрических фильтрах. Показано, что найденные выражения переходят в известные в частных случаях одинаковых резонаторов. Приведены результаты расчета прямоугольного и гауссовского импульсов при их рассеянии на полосовых и режекторных фильтрах терагерцового и инфракрасного диапазона длин волн, построенных на различных диэлектрических резонаторах в различных линиях передачи.

Ключевые слова: диэлектрический резонатор; импульсы; рассеяние; режекторный фильтр; полосовой фильтр