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The Analysis of Periodic Signal Detection Method Based on Duffing System Chaotic Dynamics

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This article presents the analysis of periodic signal detection method based on Duffing system sensitivity to weak influences. The described signal detection method is developed with using of Duffing system that oscillates in chaotic state, without transitions to periodic state. The main advantage of such method is the absence of periodic oscillation modes with low sensitivity. The divergence of Duffing system phase trajectories is investigated with influences of different periodic signals under low signal-to-noise ratio values. The estimation of phase trajectories divergence is performed with using of numeric integration. The signal detection method is analyzed with different forms of input signal: sinusoidal, square, triangle. The analysis shows that a reliable detection of periodic signal can be performed for any of the three presented forms of signal with repeating frequency near the frequency of the driving signal. The obtained results show wide capabilities of Duffing system applications for detection of weak periodic signals.

Key words: weak signal detection; chaotic systems; signal-to-noise ratio; phase portrait

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Introduction

At the present time the number of radio electronic devices increases [1–3]. This leads to strict requirements to noise immunity and sensitivity of new communication devices [4–6].

Now, there is a wide variety of methods that are used for realization of communication devices with high noise immunity on the base of linear and nonlinear filtering methods [1, 2, 4, 5].

The efficiency of linear filtering [7, 8] is limited by superposition principle which leads to the constant proportion between signal and noise at each frequency considered separately The nonlinear signal processing methods have much greater capabilities [9–12] but they require more complicated mathematical models and algorithms for avoiding the nonlinear distortions [9, 13, 14].

One of the main parts of digital signal reception process is the determining whether high or low logical level is present at the input of signal processing device [1, 15, 16]. Thus the development of efficient signal detection methods is necessary for the design of new digital electronics and communication systems with high noise immunity.

During the last 20 years the novel methods of periodic signal detection are developed on the base of chaotic dynamics theory [17]. These methods are based on chaotic system sensitivity to initial conditions and low-energy influences [18–21].

There are known chaos-based methods of signal detection with using of different chaotic systems (Duffing Chua [22], Lorenz [23] and other systems [24–27]).

The most of known chaos based signal detection methods use the transition between chaotic state and periodic state for indication of presence or absence of signal with required parameters [25, 27–30]. The main disadvantage of such methods is the need to provide the state closed to critical.

The critical state of chaotic system is situated between chaotic and periodic states [30, 31]. It corresponds to very small ranges of driving signal parameters, such as amplitude, frequency and phase. Thus small signals can drive chaotic system oscillations out of critical state and decrease the sensitivity dramatically.

By the other side, if the chaotic system state changed to periodic, then its sensitivity to weak signals becomes lower.

The described problems do not allow wide practical applications of chaos-based signal detection methods.

Therefore, the development of signal detection methods on the base of chaotic oscillations analysis without state transition is an important problem of chaos-based signal processing [32]. For this purpose the Duffing system is selected in accordance with its relatively simple structure and double-well potential that allows to obtain a high sensitivity to weak signals.

1 The chaotic dynamics of Duffing system

The Duffing system is characterized by the presence of chaotic and periodic states that depend on the parameters of external influences [31].

A generalized form of Duffing system model is described by differential equation (1):

$$a \cdot x''(t) + b \cdot x'(t) + c \cdot F(d \cdot x(t)) = s(t), \quad (1)$$

where $s(t)$ is the input signal; $x(t)$ is the output signal; $F(x)$ is a nonlinear function that provides double-well potential. In this article we consider the function $F(x) = x^3 - x$.

The Duffing system is excited by signal $s(t)$ that consists of two components:

$$s(t) = s_0(t) + g(t), \quad (2)$$

where $s_0(t) = A_0 \sin(\omega t)$ is the driving signal that provides the required oscillation mode; $g(t)$ is the input signal that consists of useful information signal $s_{\inf}(t)$ and noise $n(t)$:

$$g(t) = s_{\inf}(t) + n(t). \quad (3)$$

In this work the information signal is considered as a periodic signal with amplitude A_s and repeating frequency ω. The noise is a random value with uniform distribution.

The coefficients a, b, c, d determine the oscillation damping and scaling by frequency and amplitude:

$$\begin{aligned} a &= 1/\omega, \quad b = k/\omega, \\ c &= B_{set}/B_0, \quad d = B_0/B_{set}, \end{aligned} \quad (4)$$

where ω is the cyclic frequency of driving signal; k is the damping coefficient; B_0 is the driving signal amplitude under $c = d = 1$; B_{set} is the established amplitude of driving signal. The values of B_0 and B_{set} determine the range of output signal amplitude.

Thus, we can obtain Duffing system models, which can provide the same form of phase portrait at different frequencies with different amplitudes of input and output signals.

In this article the Duffing system dynamics is analyzed with parameters: $\omega = 1$; $k = 0.5$; $c = 1$; $d = 1$; $A_0 = 0.41$.

But the model can be easily rescaled to any different frequency and amplitude by changing of the coefficients a, b, c, d. The differential equation (1) is solved numerically with using of trapezoidal integration method.

An example of Duffing system chaotic oscillations (1) is shown in Fig. 1.

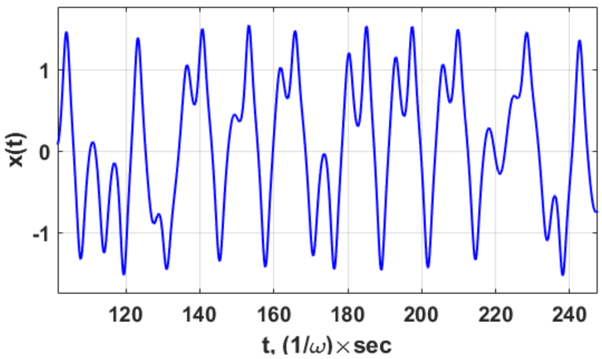

Fig. 1. Chaotic oscillations of Duffing system

The shown chaotic oscillations of Duffing system strongly depend on the parameters of input signal $s(t)$. A small changing of input signal parameters can cause significant changing of output signal after some time.

2 The divergence of Duffing system phase trajectories under the influence of periodic signals

The phase plane representation of chaotic oscillations is convenient for analysis of Duffing system response to weak signals [23, 31, 32].

The typical phase portrait of Duffing system chaotic oscillations is shown in Fig. 2.

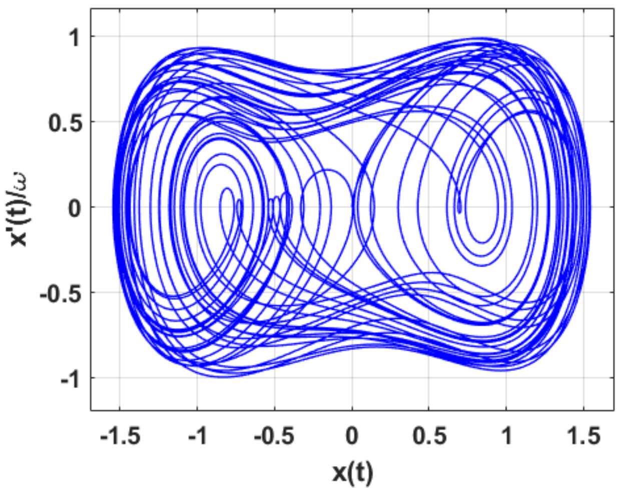

Fig. 2. A typical phase portrait of Duffing system chaotic oscillations

As it is shown in Fig. 2, the phase trajectories diverge with time. The difference between Duffing system phase trajectories can be caused by weak signals with periodic components at the frequency of driving signal [32].

For example, the process of phase trajectories divergence is shown in Fig. 3 for changing of driving signal amplitude $A_0 = 0.41$ by small value $\Delta A = 4 \cdot 10^{-4}$. The time of divergence is $15 \cdot T$ (noted by points in Fig. 3), where $T = 2 \cdot \pi/\omega$ is the period of driving signal $s_0(t)$.

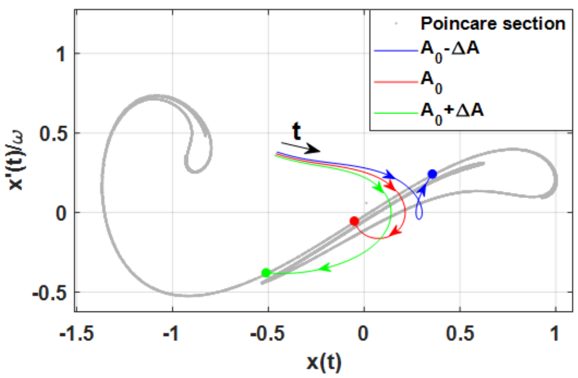

Fig. 3. Divergence of Duffing system phase trajectories without input noise

The phase trajectories diverge along the Poincare section of Duffing system phase portrait. The Poincare section is a set of points $(x(m \cdot T, \varphi_0),\ x'(m \cdot T, \varphi_0))$ selected with one driving signal period T and initial phase φ_0, where $m = 0, 1, 2, 3, ..., M$.

Fig. 4 shows the divergence of phase trajectories for the same amplitudes A_0, $A_0 - \Delta A$, $A_0 + \Delta A$ and the same time $15 \cdot T$ under the presence of strong noise.

The noise is an aperiodic random waveform with uniform distribution. The noise level is characterized by signal-to-noise ratio (5) in the frequency range $\omega_n \in [0.7\omega;\ 1.3\omega]$.

$$SNR = 20\log_{10}\left(\sqrt{\frac{\int\limits_0^t (s_{\inf}(\tau))^2 d\tau}{\int\limits_0^t (n(\tau))^2 d\tau}}\right) \tag{5}$$

In Fig. 4, the three phase trajectories are obtained under the same noise level with $SNR = -21\ dB$. The SNR value is estimated by expression (5) for harmonic input signal with ΔA amplitude ($s_{\inf}(t) = \Delta A \cdot \sin(\omega t)$).

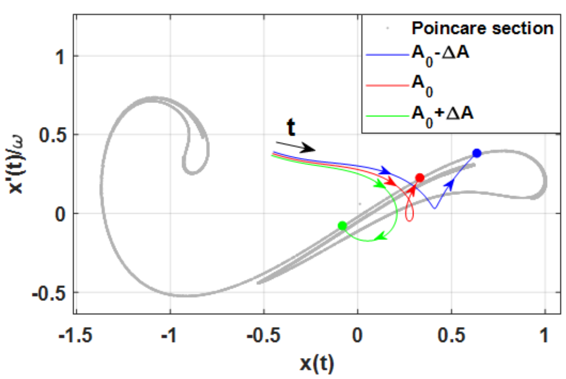

Fig. 4. Divergence of Duffing system phase trajectories with $SNR = -21\,dB$ at the input

Fig. 4 shows that noise does not change the direction of phase trajectory shifts along Poincare section caused by changing of periodic component amplitude.

The nature of Duffing system sensitivity to weak periodic signals is shown in Fig. 5.

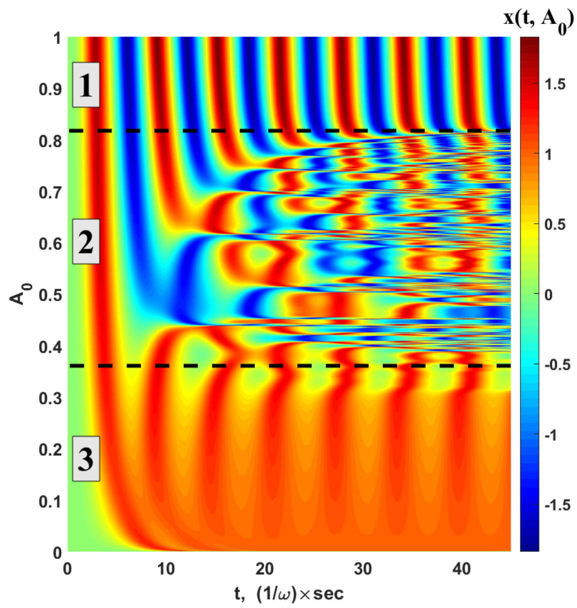

Fig. 5. The dependence between periodic signal amplitude oscillation mode: 1, 3 — periodic states; 2 — chaotic state; dashed lines — critical states

Fig. 5 shows that if amplitude of input signal periodic component is in the range $[0;\ 1]$, then Duffing system can be in chaotic or periodic state. The periodic states appear when $A_0 < 0.375$ or $A_0 > 0.754$. If $A_0 \in [0.375;\ 0.754]$, then Duffing system is in chaotic state.

As it is shown in Fig. 5, in periodic states the sensitivity of Duffing system to weak signals is low, such as at different A_0 values the output signal oscillation forms are almost the same. In chaotic state a small change of periodic component can lead to significant change of output signal form.

Thus the design of methods and algorithms for estimation of noisy signal parameters with analysis of Duffing system response divergences can allow to realize the signal reception under low SNR values.

3 Periodic signal detection based on the estimation of phase trajectories divergence

As it is shown in Fig. 3 and Fig. 4, the Duffing system phase trajectories diverge along the Poincare section under small changes of input signal periodic component.

For convenient expression of phase trajectories divergence estimation we perform the next replacements:

$$y(t) = x'(t), \tag{6}$$

$$g(t) = A \cdot g_n(t), \tag{7}$$

where $x'(t)$ is the derivative of output signal; $g_n(t)$ is the normalized input signal:

$$\sqrt{\frac{2}{t}\int_0^t (g_n(\tau))^2 d\tau} = 1. \tag{8}$$

Therefore, the divergence of Duffing system phase trajectories under influence of input signal $g(t)$ can be estimated by expression:

$$L(t,A) = \int_0^A \sqrt{\left(\frac{dx(t,u)}{du}\right)^2 + \left(\frac{dy(t,u)}{du}\right)^2} du. \tag{9}$$

The estimation of phase trajectories divergence is performed for three different periodic signals:

— sinusoidal signal:

$$s_{\inf}(t) = A_s \cdot \sin(\omega t); \tag{10}$$

— square signal:

$$s_{inf}(t) = \begin{cases} A_s, t \in \left[mT;\ \left(m+\frac{1}{2}\right)T\right], \\ -A_s, t \in \left[\left(m+\frac{1}{2}\right)T;\ (m+1)T\right]; \end{cases} \tag{11}$$

— triangle signal:

$$s_{inf}(t) = A_s \cdot \frac{2}{\pi} \arcsin\left(\sin(\omega t)\right). \tag{12}$$

The values of phase trajectories divergence estimation L are calculated with using of trapezoidal integration method.

The results of estimation of Duffing system phase trajectories divergence are shown in Fig. 6 under the influence of sinusoidal, square and triangle signals for the time $22 \cdot T$.

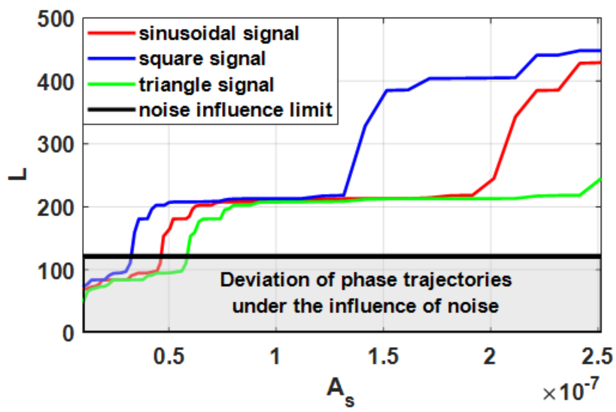

Fig. 6. The dependence between periodic signal amplitude and divergence of phase trajectories

Fig. 6 shows that if the amplitude of input signal periodic component is greater than $7 \cdot 10^{-8}$, then after 22 periods of driving signal the phase trajectory divergence estimation L is much greater than under aperiodic noise influence only. The region that corresponds to phase trajectories divergence caused by noise is shown in grey color.

The corresponding values of SNR are presented in Fig. 7.

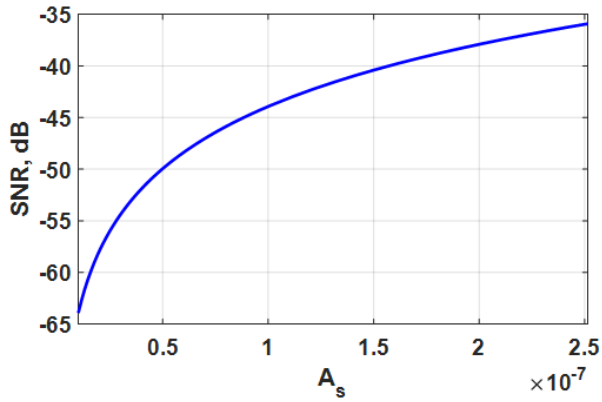

Fig. 7. SNR values for the presented amplitudes of periodic signals

So, we can detect periodic signals with repeating frequencies near the driving signal frequency ω with using of the expressions:

$$\begin{aligned} H_0: &\quad L < L_{thr1}, \\ H_1: &\quad L \geqslant L_{thr2}, \end{aligned} \tag{13}$$

where H_0 is the hypothesis of signal absence; H_1 is the hypothesis of signal presence; L_{thr1} and L_{thr2} the lower and upper threshold values of the divergence estimation L correspondingly. The threshold values can be defined in accordance with the statistical methods used for making the decisions between hypotheses H_0 and H_1.

4 Discussion

The presented analysis of dependence between amplitude of input signal periodic components and Duffing system phase trajectories divergence shows that the described chaotic system has great capabilities of application for signal detection purposes.

The presented results (Fig. 6, Fig. 7) demonstrate that Duffing system phase trajectories divergence increases significantly under weak periodic influences at the frequency of driving signal. The corresponding divergence of phase trajectories under noise influence is much less. For example, the noise with low level of periodic components ($SNR = -60\ dB$) causes the maximum estimated divergence only $L = 122$, while we obtain $L = 213$ under the influence of periodic signal with $SNR = -44\ dB$. Thus, we can establish a reliable threshold value of L for detection of periodic signals.

Also, the phase trajectories divergence depends on the form of periodic signal. So the square waveform causes the maximum divergence and triangle waveform causes the minimum divergence. The dependence between phase trajectories divergence and the form of periodic signal requires additional investigations. Such dependence is related to energy of periodic signals at the driving signal frequency.

For digital signal reception applications, the development of new methods is required for analysis of the phase trajectories divergence estimation L that changes with time, amplitude and form of periodic signals. The presented estimation approach can be used in program realization of signal detection methods based on discrete Duffing system modeling. For hardware realization the fast estimation methods are required.

Thus the future works of authors are concentrated on the development of discrete phase trajectories divergence estimation methods.

Conclusions

This article presents the analysis of periodic signal detection method based on Duffing system chaotic dynamics.

The obtained results show that accurate estimation of Duffing system phase trajectories divergence allows to detect periodic signals under low SNR values. The presented results show the capabilities of signal detection under $SNR = -44\ dB$ and higher in the frequency range $[0.7\omega;\ 1.3\omega]$, where ω is the driving signal frequency.

The advantage of the presented method is that the divergence of phase trajectories is not limited by Duffing system output signal dynamic range in accordance with the fractal geometry of its Poincare section that changes with time.

One of the most important directions for future development of chaos-based signal detection methods is the design of efficient digital estimators of phase trajectories divergence.

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Аналіз методу виявлення періодичних сигналів на основі хаотичної динаміки системи Дуффінга

Мартинюк В. В., Гаврилко Є. В., Бойко Ю. М., Федула М. В.

У статті запропоновано аналіз методу виявлення періодичних сигналів, який базується на чутливості системи Дуффінга до слабких впливів.

Процес виявлення періодичних сигналів є однією із найважливіших задач сучасної радіотехніки та зв'язку. Зокрема, чутливість і завадостійкість в процесі виявлення періодичних сигналів суттєво впливає на якість прийому цифорвих сигналів із амплітудною, частотною та фазовою маніпуляцією.

Наприкінці XX ст. було розроблено нову групу методів виявлення періодичних сигналів на основі властивостей чутливості хаотичних систем до слабких періодичних впливів за умови наявності вхідних шумів.

Більшість відомих методів виявлення періодичних сигналів із застосуванням хаотичних систем засновані на ідентифікації переходу коливань із хаотичного режиму в періодичний режим внаслідок збільшення амплітуд періодичних складових вхідного сигналу на частотах, близьких до частоти задаючого сигналу.

Перевагою приведеного методу виявлення періодичних сигналів є використання системи Дуффінга у хаотичному режимі, без переходів до періодичного режиму, в якому спостерігається значно нижча чутливість до слабких періодичних коливань вхідного сигналу.

Описаний у статті метод базується на тому, що при різних амплітудах періодичних складових вхідного сигналу системи Дуффінга її фазові траєкторії у хаотичному режимі розходяться з різною швидкістю. Приведено формулу для оцінки величини розходження фазових траєкторій.

Досліджено процеси розходження фазових траєкторій системи Дуффінга при різних формах періодичних сигналів на вході. Зокрема, наведено аналіз процесів розходження фазових траєкторій внаслідок впливу синусоїдального, прямокутного та трикутного сигналів. Вказано методику розрахунку значень коефіцієнтів рівняння Дуффінга, необхідних для реалізації хаотичних режимів коливань для різних амплітуд і частот задаючого сигналу.

Результати аналізу показують можливості виявлення періодичних сигналів різних форм із частотою повторення, близькою до частоти задаючого сигналу, за умов низьких значень відношення сигнал/шум на вході.

Отримані результати показують широкі можливості застосування системи Дуффінга для виявлення слабких періодичних сигналів.

Ключові слова: виявлення слабких сигналів; хаотичні системи; відношення сигнал-шум; фазовий портрет

Анализ метода обнаружения периодических сигналов на основе хаотической динамики системы Дуффинга

Мартынюк В. В., Гаврилко Е. В., Бойко Ю. Н., Федула М. В.

В статье предложен анализ метода обнаружения периодических сигналов, базирующийся на чувствительности системы Дуффинга к слабым воздействиям.

Метод обнаружения сигналов разработан с использованием системы Дуффинга в хаотическом режиме без переходов в периодический режим. Главное преимущество метода – отсутствие переходов в периодические режимы с низкой чувствительностью.

Исследованы процессы расхождения фазовых траекторий при разных периодических сигналах на входе (синусоидальный, прямоугольный, треугольный).

Результаты анализа показывают возможности обнаружения периодических сигналов с частотой повторения, близкой к частоте задающего сигнала, при низких значениях отношениях сигнал/шум.

Полученные результаты показывают широкие возможности применения системы Дуффигна для обнаружения слабых периодических сигналов.

Ключевые слова: обнаружение слабого сигнала; хаотические системы; отношение сигнал шум; фазовый портрет