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Investigation of the Field Scattered by Phased Equidistant Arrays Based on Asymptotic Methods of Electrodynamics

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It is suggested that the asymptotic method of saddle point be used for solution of integral electrodynamic equations of the electromagnetic field scattered by phased equidistant antenna arrays. This makes it possible to study the regularities of the re-radiated field in two arbitrary planes. The obtained expressions will contribute to design of new antenna systems, which will reduce electromagnetic field scattering. This will as well improve the electromagnetic compatibility of electronic equipment installed in the same or in adjacent objects equipped with antenna systems. The research results can be used to design algorithms for detection, recognition and identification of radar targets.

Key words: phased antenna arrays, study of electromagnetic field, asymptotic methods, effective scattering surface

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Introduction

The study of electromagnetic field scattering characteristics (re-radiation) caused by air and ground objects is important for radio detection and navigation. Sometimes it means "radar visibility"of the objects.

The antenna systems of air and ground objects are the largest contributors to their radar visibility.

They are the main sources of follow-up radiation or re-radiation of electromagnetic waves from the probing radar stations (RS). This phenomenon increases radar visibility of such objects in addition to negative impact on electromagnetic compatibility of radio electronic devices installed therein.

According to a number of sources [1–5], the contribution of antenna systems can be up to 98% of the total effective scattering surface (ESS) of the objects.

This is especially true of aperture antennas and phased antenna arrays (PAA) of RS [5–7].

1 Analysis of recent research and publications

Analysis of the causes and regularities of PAA scattering proves complexity of its elimination. Special design and application of special coatings that reduce the level of reflected signal from the objects are not always acceptable for their antenna systems. Typically, such improvements result in degradation of other main characteristics of antenna systems – gain, directional operation, etc. Therefore there is a need to optimize them according to the "efficiency–visibility"characteristic [4–7].

Unfortunately, it is not always appropriate.

The problem is that each radio system is a source of radiation and any antenna scatters more than half of the incident energy [5].

Moreover, if the antenna system does not scatter any energy, it and does not receive it. It means that it is impossible to avoid such scattering (re-radiation) completely, but the scattering can be significantly reduced [2–5].

So it is essential to study the electromagnetic field and scattered PAA in order to develop methods reducing it.

The scattering properties of any objects are usually described by effective surfaces (widths, areas) the scattering is to be described by integral (σ_Σ) differential (σ_d) surfaces and the scattering matrix (M) [8].

When studying scattering or wave re-radiation, PAA antenna system waves shall be considered a group of radiators, which represent an ensemble of shiny spots.

In this case, the problem of finding integral and differential scattering surfaces is reduced to the calculation of ESS group of radiators, with surface current brought upon each and electromagnetic field strength amplitudes excitation [5–7].

The complex strength amplitude E_r of the electromagnetic field scattered by the reflector, which is at a distance R from the observation point shall be calculated according to expression [7]:

$$\vec{E}_r = \frac{\vec{E}_n}{R\sqrt{4\pi}}\sqrt{\vec{\sigma}_1}e^{jkR}, \tag{1}$$

where $\vec{E}_n$ is a complex wave strength amplitude on the n reflector; $\vec{\sigma}_1$ is ESS for one radiator; $k = 2\pi/\lambda$ is wave number; λ is wave length.

On the basis of the superposition principle for a linear equidistant array from N radiators [7,9], the re-radiated signal at the receiving end shall be created by the interference of the signals reflected from all the radiators located along y axis (fig. 1).

The complex field strength amplitude $\vec{E}_{r\Sigma}$ for a linear equidistant PAA with N emitters in [7] shall be calculated according to the expression proposed in [7]:

$$\vec{E}_{r\Sigma} = \sum_{n=1}^{N}\frac{\vec{E}_{in}}{R_n\sqrt{4\pi}}\sqrt{\vec{\sigma}_n}e^{jkR}, \tag{2}$$

where $\vec{E}_{in}$ is a complex strength amplitude for the wave falling on n radiator; $\vec{\sigma}_n$ is ESS of n radiator; R_n is a distance from observation point to n radiator.

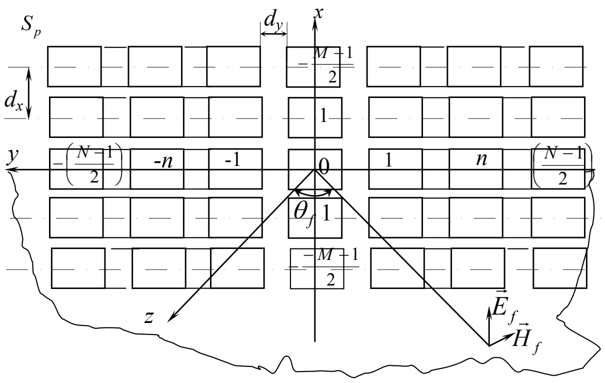

Fig. 1. Parameters of equidistant rectangular PAA for calculation of the scattered field at an incidence angle θ_f for a plane electromagnetic wave normally polarized to the incidence plane

The expression for back scattering $\sigma_\Sigma(\varphi,\theta_f)$ (of an integral single-position ESS) for a rectangular PAA shown in Fig. 1, shall be as follows [7]:

$$\begin{aligned}\sigma_\Sigma(\varphi,\theta_f) &= \sigma_n(\varphi,\theta)\times\\ &\times\left(\sum_{m=1}^{M}\sum_{n=1}^{N}|\rho_{m,n}|\cos\left(k\Delta R_{m,n}(\varphi,\theta)\right)+\right.\\ &\left.+F_{m,n}(t,\varphi,\theta)+\arg(\rho_{m,n})\right)^2, \end{aligned}\tag{3}$$

where $\sigma_n(\varphi,\theta)$ is a back scattering diagram for n radiator; $\rho_{m,n}$ is a reflection coefficient; $\Delta R_{m,n}(\varphi,\theta)$ is a travel path deviation for a plane equidistant antenna array; $F_{m,n}(t,\varphi,\theta)$ is a phase distribution of the incident signal as a function of the spatial and temporal aperture coordinates; M is a number of radiators along axis x; N is a number of radiators along axis y.

The study of the causes and regularities of secondary emission show that the main parameters of the system "PAA – probing RS"that affect the level of the reflected signal, are the angles θ_f,φ. They characterize the aspect of PAA re-radiation.

The re-radiated signal level is influenced by a complex of PAA characteristics: orientation diagram $f(\theta_f,\varphi)$, number of radiators along axis y M and along axis x N , the distance between them d_x and d_y and the electromagnetic waves travel path deviation.

A possibility to reduce ESS of the antenna system by means of effective phase distribution of the incident signal $F_{m,n}(t,\varphi,\theta)$ as a function of spatial and temporal PAA aperture coordinates was considered in [7].

$f(\theta,\varphi), d_x, d_y, M, N$ are a priori known design characteristics of PAA class. They are practically unchanged during the service life of RS [7].

Angles θ,φ vary depending on PAA spatial position relative to the probing RS and PAA operating mode [7].

Consequently, we can reduce ESS of an equidistant PAA by changing design properties of all radiators, or by changing distances between them proportionally d_x, d_y.

For this purpose, we must study the amplitudes in the waveguide aperture of n radiator (Fig. 2).

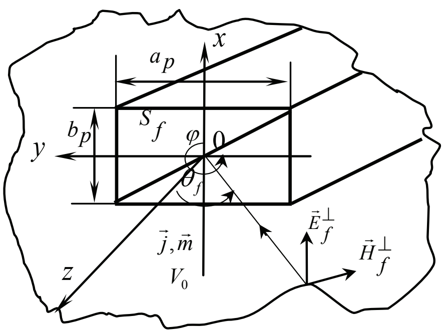

Fig. 2. Waveguide characteristics used for calculation of the scattered field with incident normally polarized wave

The amplitudes of the field, which is excited on a linear equidistant array (Fig. 3) were considered in [10].

However, the final expressions concerned determination of the amplitudes, excited during aperture of such array only if the incident wave is normally polarized to the incidence plane.

In other free sources [7,9,11], the excited amplitudes and re-radiation field expressions are simplified and rather approximate. They do not allow to study the total field scattered from PAA with necessary accuracy in the case it is exposed to the wave normally polarized to the incidence plane at an angle θ_f from arbitrarily selected φ in order to reduce it.

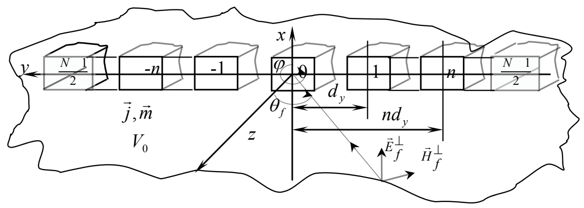

Fig. 3. Characteristics of horn-type radiator line used for calculation of the scattered field with incident plane electromagnetic wave normally polarized to the incidence plane

Against this background, the purpose of the article is to study the field re-radiated by a linear equidistant PAA, which is formed by an incident wave normally polarized to the incidence plane at an angle φ, and to determine causes and regularities of this phenomenon in order to reduce it.

2 Presentation of the basic material

Let the electromagnetic wave $\vec{E}_f, \vec{H}_f$ which is formed by external currents distributed with density $\vec{j}$ and $\vec{m}$ be incident to the PAA aperture consisting from n horn-type radiator linear arrays (see Fig. 1).

It is important to find a field scattered by such antenna.

Solution. We shall number the radiators from the center to the edges so that the central one is a zero one, while the extreme radiators are $(N-1)/2$ as shown in (see Fig. 1). The total number of the elements in the array shall be odd and equal to N.

The strength of the field E_{fn}, formed by a pair of radiators symmetrically located about the center shall be written as

$$E_{fn} = E_f^{+n} + E_f^{-n}, \tag{4}$$

where E_f^{+n} is the strength of radiator field $n=1$; E_f^{-n} is the strength of field $n=$-1.

In this case, the total field scattered by the linear radiator array shall be:

$$E_{fp} = E_{f0} + \sum_{n=1}^{\left(\frac{N-1}{2}\right)} E_{fn}, \tag{5}$$

where E_{f0} is the strength of the field, which is formed by the central radiator.

In order to determine E_{f0} it is necessary to investigate the wave amplitude in the aperture.

Let an independent source be placed inside the central radiator in the transmission mode (see Fig. 1) and form a field, which shall be marked $\vec{E}_{f0}, \vec{H}_{f0}$ inside and outside.

Inside the horn such a field shall be usually considered eigenfunction and marked -mn. However, due to the fact that index is already in use, we shall propose a field with a unit amplitude as $\vec{E}_{\pm m_x m_y}, \vec{H}_{\pm m_x m_y}$. It is reflected from the aperture with a reflection coefficient $\rho_{+m_x m_y}$:

$$\begin{cases} \vec{E}_{P0} = \left(\vec{E}_{-m_x m_y} + \rho_{+m_x m_y} \vec{E}_{+m_x m_y}\right), \\ \vec{H}_{P0} = \left(\vec{H}_{-m_x m_y} + \rho_{+m_x m_y} \vec{H}_{+m_x m_y}\right), \end{cases} \tag{6}$$

where $(-m_x, m_y)$ is the number of standing semi-waves, which fall on the sides of the cross-section and extend from the neck to aperture; $(+m_x, m_y)$ is the number of standing semi- waves propagating from the aperture to the neck.

Consequently, on the surface of the antenna aperture S_f from the inner side, a full field can be presented by an eigenfunction expansion [9,10]:

$$\begin{cases} \vec{E} = \sum\limits_{m_x m_y=1}^{\infty} C_{\pm m_x m_y} \left(\vec{E}_{+m_x m_y} + \rho_{-m_x m_y} \vec{E}_{-m_x m_y}\right), \\ \vec{H} = \sum\limits_{m_x m_y}^{\infty} C_{\pm m_x m_y} \left(\vec{H}_{+m_x m_y} + \rho_{-m_x m_y} \vec{H}_{+m_x m_y}\right), \end{cases} \tag{7}$$

where $A_{\pm m_x m_y}$ are eigenfunction amplitudes; $\vec{E}_{+m_x m_y}$, $\vec{H}_{+m_x m_y}$ are eigenfunctions, which spread from the aperture to the neck; $\vec{E}_{-m_x m_y}$, $\vec{H}_{-m_x m_y}$ are eigenfunctions spreading from the neck to the aperture; $\rho_{-m_x m_y}$ – coefficient of eigenfunction reflection from internal inhomogeneities in the horn.

In order to study the field $\vec{E}_{Pp}$ scattered by aperture of one or n-th radiator of the antenna, we shall use asymptotic methods with application of Lorentz lemma.

In order we could determine such a field, it is necessary to implement strict boundary conditions, i.e. continuity and tangential components of the total field $\vec{E}$ and $\vec{H}$ to aperture S_P.

Let's place coordinate origin in the center of the array $(z=0)$ (see Fig. 3).

For this case we have:

$$\begin{cases} \left(\vec{E}_f\,(z=0) + \vec{E}_{Pp}\,(z=0)\right)_\tau = \left(\vec{E}\,(z=0)\right)_\tau, \\ \left(\vec{H}_f\,(z=0) + \vec{H}_{Pp}\,(z=0)\right)_\tau = \left(\vec{H}\,(z=0)\right)_\tau, \end{cases} \tag{8}$$

where $\vec{E}_f, \vec{H}_f$ are the strengths of electric and magnetic fields incident on the aperture and excited by currents beyond the horn; $\vec{E}, \vec{H}$ are the strengths of the electric and magnetic fields from the aperture (4).

A component of the field $\vec{E}_{Pp}$ scattered throughout the space may be presented as a continuous spectrum of plane waves [9]:

$$\vec{E}_{Pp} = \frac{1}{4\pi^2} \iint\limits_{-\infty}^{\infty} \vec{A}\left(k_x, k_y\right) e^{-i(k_x x + k_y y + k_z z)} dk_x dk_y, \tag{9}$$

where $\vec{A}\left(k_x, k_y\right)$ is a spectral function of the complex amplitudes of plane waves; k_x, k_y, k_z are projections of the wave vector to the axes x, y, z, which are connected by formula $k^2 = k_x^2 + k_y^2 + k_z^2$.

After we plug (4) and (6) into (5), we get:

$$\vec{E}_{f\tau} + \frac{1}{4\pi^2} \iint\limits_{-\infty}^{\infty} A_\tau\left(k_x, k_y\right) e^{-i(k_x x + k_y y)} dk_x dk_y = \\ = \sum_{\nu=1}^{\infty} C_{+\nu} E_{+\nu\tau}\left(1 + \rho_{+\nu}\right), \tag{10}$$

where $C_{+\nu}$ is an amplitude of the excited wave.

A similar expression will be used for H field components.

Let's multiply both sides of the equation (7) by $e^{i(k_x x + k_y y)}$ and integrate them over x and y on the aperture surface S_p.

We shall write the Lorentz lemma [9] for the volume limited by an infinitely distant from the antenna surface and a bounded surface S_p (see Fig. 2) as:

$$\int\limits_V \left\{ \left(\vec{j}, \vec{E}_P\right) - \left(\vec{m}, \vec{H}_P\right) \right\} dV = \\ = \oint\limits_{S_p} \left(\left[\vec{E}, \vec{H}_P\right] - \left[\vec{E}_P, \vec{H}\right] \right), d\vec{S}, \tag{11}$$

and then complete the integration from $E_{P\tau}$ to infinite limits provided $E_{P\tau}\left(z = 0\right) = 0$ beyond the surface S_p. If we use the relation obtained in [9]:

$$A_\tau\left(k_x, k_y\right) = \frac{1}{4\pi^2} \iint\limits_{-\infty}^{\infty} \times \\ \times \left[\iint\limits_{-\infty}^{\infty} A_\tau\left(k_x, k_y\right) e^{-i(k_x x + k_y y)} dk_x dk_y \right] \times \\ \times e^{i(k_x x + k_y y)} dx dy, \tag{12}$$

we shall have:

$$A_\tau\left(k_x, k_y\right) = -\iint\limits_{(S_P)} E_{\Pi\tau}\left(z = 0\right) e^{i(k_x x + k_y y)} dx dy + \\ + \iint\limits_{(S_P)} \sum_{\nu=1}^{\infty} C_{+\nu} E_{+\nu\tau}\left(1 + \rho_{+\nu}\right) e^{i(k_x x + k_y y)} dx dy. \tag{13}$$

Let's plug (10) into (7)

$$\vec{E}_{Pp} \cong \frac{\vec{e}_{Pp}}{4\pi^2} \iint\limits_{-\infty}^{\infty} \left\{ \iint\limits_{(S_P)} \left[\sum_{\nu=1}^{\infty} C_{+\nu} E_{+\nu\tau}\left(1 + \rho_{-\nu}\right) \cdot \right.\right. \\ \left.\left. \cdot e^{i(k_x x + k_y y)} - E_{\Pi\tau}(z = 0) e^{i(k_x x + k_y y)} \right] dx dy \right\} \\ \cdot e^{-i(k_x x + k_y y + k_z z)} dk_x dk_y. \tag{14}$$

The magnetic components of the scattered field can be obtained from the Maxwell's equations:

$$\vec{H}_{Pp} = \frac{i}{\omega\mu} rot\vec{E}. \tag{15}$$

In order to calculate the scattered field according to [5] we shall apply the cross-section method. We must take into consideration that in addition to the waves excited in the aperture and reflected from the internal inhomogeneities, there are so-called "parasite"waves.

The field scattered by n-th radiator shall be written as:

$$\vec{E}_{Pp} = \frac{\vec{e}_p}{4\pi^2} \cdot \\ \iint\limits_{-\infty}^{\infty} \left\{ \iint\limits_{S_p} \left[\sum_{n=1}^{\infty} \left[\vec{A}_{+n} E_{+n}(1 + \rho_{-n}) \right] e^{i(k_x x + k_y y)} \right.\right. \\ \left.\left. \cdot E_{\nu\tau}(z = 0) e^{i(k_x x + k_y y)} \right] dx dy \right\} \\ e^{-i(k_x x + k_y y + k_z z)} dk_x dk_y, \tag{16}$$

where S_p is the aperture integration surface of n-th radiator (see Fig. 2) from the inner side; $\vec{e}_p$ is a unit electromagnetic field strength vector; $\vec{A}_{+n}$ is a spectral function of the complex amplitudes of the plane waves excited at the aperture; E_{+n} is the strength vector of the electromagnetic field incident on the n-th radiator; k_x, k_y, k_z are the projections of the wave vector on axes x, y, z; $E_{\nu\tau}\left(z = 0\right)$ is the strength vector of the electromagnetic field after taking into account the boundary conditions and integration ad infinitum beyond the surface S_p; ρ_{-n} the internal inhomogeneities reflection coefficient of the n-th radiator.

For the case of normal polarization of the wave in the plane of incidence, after plugging eigenfunctions [9] into (13) and their integration over x, y the tangential component of the field $\vec{E}^{\perp}_{Pp\tau}$ will be:

$$\vec{E}^{\perp}_{Pp\tau} = -\frac{1}{4\pi^2}\iint_{-\infty}^{\infty}\left\{\frac{b_p \sin\left(\frac{k_x b_p}{2}\right)}{\frac{k_x b_p}{2}} \cdot 2\sum_{n=1}^{\infty} C^{H\perp}_{+n}\left(\frac{n\pi}{a_p}\right)^2 \cdot \frac{\sin^2\left(\frac{n\pi}{2}\right)\cos\left(\frac{k_y a_p}{2}\right) - i\cos^2\left(\frac{n\pi}{2}\right)\sin\left(\frac{k_y a_p}{2}\right)}{\left(\frac{n\pi}{a_p}\right)^2 - (k_y)^2} + E_0 a_p \frac{\sin\left(\frac{a_p}{2}(k_y - k\sin\theta_f)\right)}{\frac{a_p}{2}(k_y - k\sin\theta_f)}\right\} \cdot e^{(-i(k_x x + k_y y + k_z z))} dk_x dk_y, \quad (17)$$

where a_p and b_p are the dimensions of the rectangular aperture of the n-th horn radiator (see Fig. 3).

We have to decide on the most effective or appropriate method studying the field $\vec{E}^{\perp}_{Pp\tau}$ in order to solve the problem.

Generally, the theory of wave propagation suggests a limited number of problems that allow exact solution. In those few cases where strict relations are known, they are quite complicated and do not allow to reveal physical nature or cause of the process regularities even with the help of advanced software packages.

We can understand the attention to the approximate methods of the wave theory, particularly asymptotic methods [10, 11] in recent years.

They are still relevant. In order we could apply the above methods, we shall rewrite (14) as:

$$\vec{E}^{\perp}_{Pp\tau} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} I_{x1}(k_x)\exp(-ik_x x)dk_x \times \int_{-\infty}^{\infty} I_{x2}(k_y)\exp(-i(k_y y + k_z z))dk_y, \quad (18)$$

where

$$I_{x1}(k_x) = b_p\left(\sin\frac{k_x b_p}{2}\right)\frac{2}{k_x b_p}; \quad (19)$$

$$I_{x2}(k_y) = 2\sum_{n=1}^{\infty} -C^{H\perp}_{+0n}\left(1+\rho^H_{-n}\right)\frac{f_{-n}(k_y)}{1-\left(\frac{a_p k_y}{n\pi}\right)^2} - E_0 a_p \frac{\sin\left(\frac{a_p}{2}(k_y - k\sin\theta_\Pi)\right)}{\frac{a_p}{2}(k_y - k\sin\theta_\Pi)}. \quad (20)$$

The functions $I_{x1}(k_x)$ and $I_{x2}(k_y)$ under the integrals in the expression (15) depend upon several parameters characterizing the system.

In this case, it will be the method of asymptotic evaluations helping us both take the integral and obtain the explicit dependence from the parameters specified in arbitrary planes of the incident wave.

The method of saddle point (steepest descent) is one of such asymptotic methods. It gives us an adequate accuracy and has a wide application for the study of different wave phenomena: acoustic, electromagnetic, etc [10].

In order to apply the above method, we shall present the expression (15) in a spherical coordinate system:

$$x = r\sin\theta\cos\varphi, \quad y = r\sin\theta\sin\varphi, \quad z = r\cos\varphi.$$

We get the following dependence:

$$\vec{E}^{\perp}_{Pp\tau} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} I_{x1}(k_x)\exp(-ik_x r\sin\theta\cos\varphi)\,dk_x \times \int_{-\infty}^{\infty} I_{x2}(k_y)\exp(-ir(k_y\sin\theta\sin\varphi + k_z\cos\theta))dk_y. \quad (21)$$

Using the method of saddle point, we solve the equation (21):

$$\vec{E}^{\perp}_{Pp\tau} \approx \frac{1}{4\pi^2}\cos W_0\sqrt{\frac{2\pi}{r\,|f''(W_0)|}}\exp(i\varphi_0)\,I_{x2} \times (k\sin W_0\cos z_0)\sqrt{\frac{2\pi\cos z}{r\,|f''(z_0)|}} \times I_{x1}(k\sin z_0)\,k\cos z_0\exp(i\varphi_m) \exp\left[rk\left(-i\sin\theta\cos\varphi\sin z_0 + \cos z_0 f(W_0)\right)\right]. \quad (22)$$

In order we could identify the physical nature of the phenomenon, let us consider a field scattered in some planes.

Usually we use a plane $\varphi = 3\pi/2$ and $\varphi = \pi$.

In the plane $\varphi = 3\pi/2$ we have:

$$\left.\begin{array}{l} \sin\varphi = -1;\ \cos\varphi = 0;\ W_0 = -\theta; \\ z_0 = 0;\ f(W_0) = -i;\ f''(W_0) = i; \\ \varphi_0 = \frac{\pi}{4};\ \varphi_m = \frac{\pi}{4};\ f''(z_0) = i \end{array}\right\}, \quad (23)$$

$$\vec{E}^{\perp}_{rp\tau|\varphi=\frac{3}{2}\pi} \approx \frac{k\cos\theta}{2\pi r}\exp\left(-i\left(kr - \frac{\pi}{2}\right)\right) \times I_{x1}(k_x = 0)\,I_{x2}(k_y = -k\sin\theta). \quad (24)$$

In the plane $\varphi = \pi$ we get:

$$\left.\begin{array}{l} \sin\varphi = 0;\ \ \cos\varphi = -1;\ \ W_0 = 0; \\ f(W_0) = -i\cos\theta_i;\ \ z_0 = -\theta;\ \ f''(W_0) = i\cos\theta; \\ \varphi_0 = \frac{\pi}{4};\ \ \varphi_m = \frac{\pi}{4};\ \ f''(z_0) = i \end{array}\right\}, \quad (25)$$

$$\vec{E}^{\perp}_{rp\tau|\varphi=\pi} \approx \frac{k\cos\theta}{2\pi r}\exp\left(-i\left(kr - \frac{\pi}{2}\right)\right) \times I_{x1}(k_x = -k\sin\theta)\,I_{x2}(k_y = 0). \quad (26)$$

The expression (22) shall be as follows for a linear equidistant antenna array:

$$\vec{E}^{\perp}_{Pp\tau} = \frac{1}{4\pi^2} \iint\limits_{-\infty}^{\infty} \left\{ \frac{b_p \sin(\frac{k_x b_p}{2})}{\frac{k_x b_p}{2}} \right.$$
$$\times \left[\sum_{m_y=1}^{\infty} (-)\, 2C^{\perp H}_{+0m_y} \times \left(\frac{m_y \pi}{a_p}\right)^2 (1+\rho^H_{0m_y}) \exp(-ink_y d_y) \right.$$
$$\times \frac{\sin^2\left(\frac{m_y\pi}{2}\right) s\left(\frac{k_y a_p}{2}\right) - i\,\cos^2\left(\frac{m_y\pi}{2}\right)\sin\left(\frac{k_y a_p}{2}\right)}{\left(\frac{m_y\pi}{a_p}\right)^2 - k_y^2}$$
$$- E_0 \exp\left(ind_y\left(k\ \sin\theta_\Pi - k_y\right)\right)$$
$$\left.\left. \frac{\sin\left(\frac{a_p}{2}\left(k_y - k\sin\theta_\Pi\right)\right)}{\frac{a_p}{2}\left(k_y - k\sin\theta_\Pi\right)} \right] \right\} \times$$
$$\times \exp\left(-i\left(k_x x + k_y y + k_z z\right)\right) dk_x dk_y, \quad (27)$$

where $C^{H_\perp}_{+0m_y}$ has been obtained in [9, 10].

In contrast to the existing approaches, the expression (27) allows to calculate a field both for linear and equidistant rectangular PAA.

The expression (27) differs from (21) because it takes into account the number of n radiator and the distance between them d_y.

The difference is also in use of (27) for more accurate expression for the amplitude $C^{H_\perp}_{+0m_y}$ and additional multipliers $\exp\left(-ink_y d_y\right)$ and $\exp\ \left(ind_y\left(k\sin\theta_\Pi - k_y\right)\right)$.

$$C^{H\perp}_{0n} = -\frac{4E_0 a_p\left(1+\cos\left(\theta_\Pi\right)\right)}{(n\pi)^2} \times$$
$$\frac{\left(\sin\frac{(n\pi)}{2}\right)^2 \cos\left(\frac{ka_p}{2}\sin\theta_\Pi\right)}{\left(1+\sqrt{1-\left(\frac{n\lambda}{2a_p}\right)^2}\right)\left(1-\rho^2_{mn}\right)\left(1-\left(\frac{2a_p}{n\lambda}\sin\theta_\Pi\right)^2\right)} +$$
$$\frac{j\left(\cos\left(\frac{(n\pi)}{2}\right)\right)^2 \sin\left(\frac{ka_p}{2}\sin\theta_\Pi\right)}{\left(1+\sqrt{1-\left(\frac{n\lambda}{2a_p}\right)^2}\right)\left(1-\rho^2_{mn}\right)\left(1-\left(\frac{2a_p}{n\lambda}\sin\theta_\Pi\right)^2\right)}. \quad (28)$$

Fig. 4 shows the maximum amplitude superposition's for the most common modes of electromagnetic field strength $\sum\limits_{m_y=1}^{\infty} -2C^{\perp H}_{+0m_y}$ for a linear equidistant PAA depending on the changing coefficient of reflection $\rho^H_{0m_y}$ (27) from the internal heterogeneities of each radiator.

The simulation was performed for $\rho^H_{0m_y} = 0,8$ – Curve 1 and for $\rho^H_{0m_y} = 0,6$ – Curve 2.

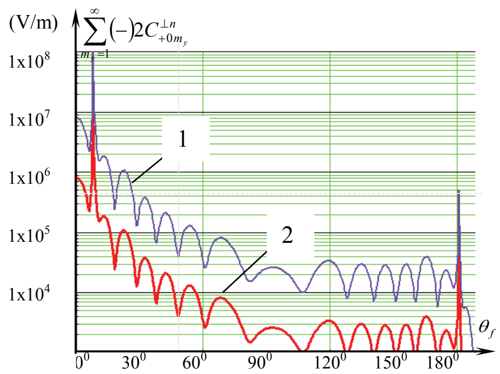

Fig. 4. Maximum amplitudes of the electromagnetic field

Fig. 4 demonstrates that an improvement of adjustment in the antenna path shall result in an increase in the maximum amplitude of the signal in the transmission mode.

Fig. 5 shows the normalized maximum amplitudes of the waves excited at the aperture of the linear equidistant PAA depending on the side-scan remote sensing angle θ_f with specified φ: for incident normally polarized wave $C^{\perp H}_{+0m_y}$ (Curves $1, 2, 3, 4, 5$) and if the incidence plane and the plane of polarization of the waves coincide $C^{||H}_{+m_x m_y}$ (lines $6, 7$).

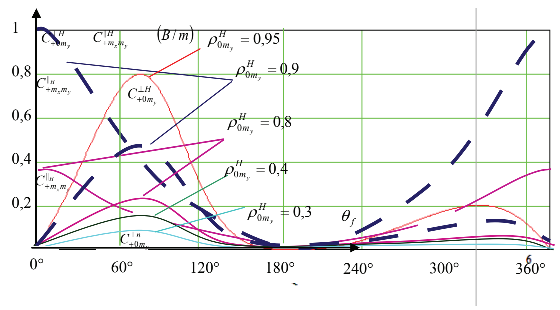

Fig. 5. Dependence of the maximum normalized wave amplitude from the side-scan remote sensing angle resulting from reflection coefficient changes

The studies were carried out for different values of the reflection coefficients.

The wave amplitudes (lines $1, 2, 3$) in Fig. 5 are almost identical with the sensing angle wave amplitudes in a polar coordinate system (lines $1, 2, 3$) in Fig. 6, which were obtained using a simplified expression.

We can check the reliability of the obtained mathematical expressions using asymptotic methods for solution of integral equations.

To reveal the causes and regularities of re-radiation from the aperture of the equidistant PAA for the specified arbitrarily selected φ provided a normal polarization of the incident wave in the incident plane, we shall use again the asymptotic method of saddle point in the plane $\varphi = 3/2\pi$ and $\varphi = \pi$.

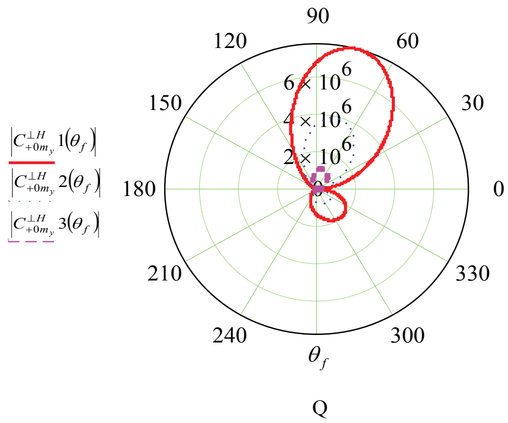

Fig. 6. How the wave amplitude depends from the side-scan remote sensing angle in a polar coordinate system

After we take an integral (16) the field scattered by the antenna array shall be as follows in the plane $\varphi = 3/2\pi$:$\varphi = \pi/2$

$$\vec{E}^{\perp H}_{Pp\ (\varphi=\frac{3\pi}{2}\ )} \cong \frac{k\cos\theta}{2\pi r}\exp\left(-i\left(kr-\frac{\pi}{2}\right)\right)\times$$
$$\times I_{x1}\left(k_x=0\right)I_{x2}\left(k_y=-k\sin\theta\right), \quad (29)$$

where

$$\begin{cases} I_{x1} = b_p\sin\left(\dfrac{k_x b_p}{2}\right)\dfrac{1}{\left(\frac{k_x b_p}{2}\right)}, \\ I_{x1}\left(0\right) = b_p,\ \text{if } k_x = 0. \end{cases} \quad (30)$$

$$I_{x2}\left(k_y=-k\sin\theta\right) = -2\sum_{m_y=1}^{\infty} C^{H\perp n}_{+0m_y}\left(1+\rho_{-0m_y}\right)$$
$$e^{-iknd_y\sin\theta}\left(\frac{\sin^2\left(\frac{m_y\pi}{2}\right)\cos\left(\frac{ka_p}{2}\sin\theta\right)}{1-\left(\frac{ka_p}{m_y\pi}\sin\theta\right)^2}+\right.$$
$$\left.+\frac{i\cos^2\left(\frac{m_y\pi}{2}\right)\sin\left(\frac{ka_p}{2}\sin\theta\right)}{1-\left(\frac{ka_p}{m_y\pi}\sin\theta\right)^2}\right)-$$
$$\frac{E_0\cdot e^{ind_y k\ (\sin\theta+\sin\theta_\Pi)}\cdot\sin\left(\frac{ka_p}{2}\left(\sin\theta+\sin\theta_\Pi\right)\right)}{\frac{ka_p}{2}\left(\sin\theta+\sin\theta_\Pi\right)}. \quad (31)$$

Thus, the field scattered in the plane $\varphi = 3/2\pi$ shall be:

$$\vec{E}^{\perp H}_{Pp\ (\varphi=\frac{3\pi}{2}\ )} \cong \frac{kb_p\sin\theta}{2\pi r}\exp\left(-i\left(kr-\frac{\pi}{2}\right)\right)\times$$
$$\Bigg\{-E_0 a_p\exp\left(ind_y k\ \left(\sin\theta+\sin\theta_f\right)\right)\times$$
$$\times\frac{\sin\left(\frac{ka_p}{2}\left(\sin\theta+\sin\theta_\Pi\right)\right)}{\frac{ka_p}{2}\left(\sin\theta+\sin\theta_\Pi\right)}$$
$$+2\sum_{m_y=1}^{\infty} -C^{H\perp n}_{+0m_y}\left(1+\rho_{-0m_y}\right)\exp\left(-iknd_y\sin\theta\right)\times$$
$$\left.\frac{\sin^2\left(\frac{m_y\pi}{2}\right)\cos\left(\frac{ka_p}{2}\sin\theta\right)+i\cos^2\left(\frac{m_y\pi}{2}\right)\sin\left(\frac{ka_p}{2}\sin\theta\right)}{1-\left(\frac{ka_p}{m_y\pi}\sin\theta\right)^2}\right\}. \quad (32)$$

In the plane $\varphi = \pi$ we get:

$$\vec{E}^{\perp H}_{Pp\ (\varphi=\pi)} \cong \frac{k\cos\theta}{2\pi r}\exp\left(-i\left(kr-\frac{\pi}{2}\right)\right)\times$$
$$\times I_{x1}\left(k_x=-k\sin\theta\right)I_{x2}\left(k_y=0\right), \quad (33)$$

where

$$I_{x1}\left(kx=-k\sin\theta\right) = \frac{b_p\sin\left(\frac{kb_p}{2}\sin\theta\right)}{\frac{kb_p}{2}\sin\theta}, \quad (34)$$

$$I_{x2}\left(k_y=0\right) = 2\sum_{m_y=1}^{\infty} -C^{H\perp n}_{+0m_y}\left(1+\rho^H_{-0m_y}\right)\times$$
$$\sin^2\frac{m_y\pi}{2} - E_0 a_p\frac{\sin\left(\frac{ka_p}{2}\sin\theta\right)}{\frac{ka_p}{2}\sin\theta}\exp\left(iknd_y\sin\theta_f\right). \quad (35)$$

Taking into account (33), (34) the expression for the scattered field in the plane $\varphi = \pi$ shall be:

$$\vec{E}^{\perp H}_{Pp(\varphi=\pi)} \cong \frac{kb_p\cos\theta}{2\pi r}$$
$$\exp\left(-i\left(kr-\frac{\pi}{2}\right)\right)\times\frac{\sin\left(\frac{kb_p}{2}\sin\theta\right)}{\frac{kb_p}{2}\sin\theta}$$
$$2\left[\sum_{m_y=1}^{\infty} -C^{H\perp n}_{+0m_y}\left(1+\rho^H_{-0m_y}\right)\sin^2\frac{m_y\pi}{2}\right]-$$
$$-E_0 a_p\frac{\sin\left(\frac{ka_p}{2}\sin\theta\right)}{\frac{ka_p}{2}\sin\theta}\exp\left(iknd_y\sin\theta_f\right). \quad (36)$$

Conclusion

The application of asymptotic methods of electrodynamics allows us to determine the field scattered from the horn aperture in the case of normal polarization of the incident wave to the plane of incidence, and in the case of coincidence of the plane of incidence and wave polarization.

The electromagnetic field scattered by phased equidistant RS antenna arrays can be reduced by application of asymptotic methods for solution of integral equations as evidenced from the simulation according to the obtained expressions.

We have proved that in order to determine the scattered field it is advisable to apply the method of saddle point.

The wave amplitude diagrams as functions of the side-scan remote sensing angle shown in Figure 4, 5, 6 demonstrate that an improvement of adjustment in the antenna path shall result in an increase in the maximum amplitude of the signal in the transmission mode. According to the antenna reciprocity principle, such adjustment will improve absorption of the top type waves at the aperture of an individual radiator or an equidistant antenna array.

This will reduce the voltage standing wave ratio and the side lobe level.

Consequently, the probing RS will receive a reduced re-radiated signal, which will improve reconnaissance protection of PAA [1, 9].

The above expressions (6) for an individual radiator and (16) for an equidistant antenna array have computational, practical, and methodological value. Their consistent development and physical interpretation will allow us to estimate their use in the study of the scattered (re-radiated field) for PAA and other antenna systems with pyramidal horns used as exciters.

The results obtained apply both to the development of electrodynamics theory and to the improvement of calculation methods [12–16]. They can be applied in the development of algorithms for detection and recognition of radar targets.

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Застосування асимптотичних методів для дослідження електромагнітного поля, розсіяного еквідистантними антенними решітками

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Будь-яка антенна система є джерелом вторинного випромінювання, оскільки розсіює не менше половини падаючої на неї енергії, що впливає на радіолокаційну помітність всього об'єкта, на якому вони встановлені.

Це означає, що такого розсіювання (перевипромінювання) антеною, у тому числі фазированою антенною решіткою, усунути неможливо, проте його можна суттєво зменшити.

Запропоновано використання асимптотичного методу перевалу для розв'язання інтегральних електродинамічних рівнянь електромагнітного поля, розсіяного фазованими еквідистантними антенними решітками. Це дозволяє дослідити закономірності перевипроміненного поля в двох довільних площинах.

Результати моделювання амплітуд хвилі, як функцій кута зондування свідчать, що покращення узгодження в антенному тракті призведе до збільшення максимальної амплітуди сигналу в режимі передачі. Відповідно до принципу оберненості антен таке узгодження покращить поглинання вищих типів хвиль, що наводяться на розкриві одиночного випромінювача або еквідистантної антенної решітки. Це дозволить зменшити коефіцієнт стоячої хвилі за напругою та рівень бічних пелюсток.

Отримані вирази стануть у нагоді під час проектування нових антенних систем, конструктивні особливості яких сприятимуть зменшенню розсіювання від них електромагнітного поля. Такі дії також дозволять покращити електромагнітну сумісність радіоелектронних засобів, що встановлені на одному, або на сусідніх об'єктах, до складу яких входять антенні системи.

Результати досліджень може бути використано у процесі розробки алгоритмів виявлення, розпізнавання та ідентифікації радіолокаційних об'єктів.

Ключові слова: фазовані антенні решітки, дослідження електромагнітного поля, асимптотичні методи, ефективна поверхня розсіювання

Применение асимптотических методов для исследования электромагнитного поля, рассеянного эквидистантными антенными решетками

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Апертурные антенны и фазированные антенные решетки вносят наибольший вклад в радиолокационную заметность воздушных и наземных объектов и могут составлять до 98% их общей эффективной рассеивающей поверхности. Известно, что любая антенная система является источником вторичного излучения, поскольку она рассеивает по меньшей мере половину падающей на нее энергии. Это означает, что такого рассеяния (переизлучения) антенной или фазированной антенной решеткой полностью избежать невозможно, но его можно существенно уменьшить.

Реализация известных способов устранения переизлучения (рассеяния) приводит к ухудшению других основных характеристик антенных систем – коэффициентов усиления, направленного действия и так далее. На уровень переизлучаемого сигнала также влияет множество характеристик самой антенной решетки: диаграмма направленности, количество излучателей, расстояние между ними и разница в распространении электромагнитных волн. Таким образом, исследование электромагнитного поля, рассеянного фазированной антенной решеткой, и выяснение причин и закономерностей такого явления, с целью его снижения, является весьма актуальной задачей.

Для исследования поля, рассеянного одиночным или n-м антенным излучателем, был применен асимптотический метод с использованием леммы Лоренца и точных граничных условий – непрерывности тангенциальных составляющих полного поля $\vec{E}$ и $\vec{H}$ к раскрыву рупора S_P. Используемый для этого математический аппарат позволяет рассчитывать поле не только для линейной, но и для прямоугольной эквидистантной фазированной антенной решетки. Его особенностью является учет количества облучателей и расстояния между ними.

Для решения интегральных электродинамических уравнений электромагнитного поля, рассеянного фазированными антенными решетками, используется асимптотический метод седловой точки (перевала). Это позволяет исследовать закономерности переизлученного поля в двух произвольных плоскостях.

Выражения, приведенные для одного излучателя и эквидистантной антенной решетки, имеют не только расчетно-практичное, но и методическое значение. Их последовательный вывод и физическая интерпретация позволят оценить пределы их использования при изучении рассеянного (переизлученного поля) не только фазированных антенных решеток, но и других антенных систем, содержащих рупора пирамидальной формы.

Диаграммы амплитуд волн, построенные как функции угла зондирования, показывают, что улучшение согласования в антенном тракте приведет к увеличению максимальной амплитуды сигнала в режиме передачи. В соответствии с принципом взаимности антенн такое согласование улучшит поглощение волн высших типов в апертуре отдельного излучателя или эквидистантной антенной решетки. Что, в свою очередь, уменьшит коэффициент стоячей волны по напряжению и уровень боковых лепестков.

Выведенные математические выражения будут полезны при проектировании новых антенных систем, особенности конструкции которых помогут уменьшить рассеивание электромагнитного поля от них. Такие действия также улучшат электромагнитную совместимость радиоэлектронных устройств, установленных на одном или соседних объектах, содержащих антенные системы.

Полученные результаты могут быть применены при разработке алгоритмов обнаружения и идентификации радиолокационных целей.

Ключевые слова: фазированные антенные решетки, исследование электромагнитного поля, асимптотические методы, эффективная поверхность рассеяния