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Mutual Coupling Coefficients of Rotated Rectangular Dielectric Resonators in Open Space

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The coefficients of mutual coupling of rectangular dielectric resonators in open space are calculated under the condition of their rotation relative to one of the axes of a given rectangular coordinate system. Analytical formulas for complex coupling coefficients are obtained. The expressions found give complete information about the frequencies and Q-factor of coupled oscillations of dielectric resonators. The dependences of the coupling coefficients on the angles of rotation and spatial coordinates of resonators in the case of excitation of the main magnetic types of natural oscillations in them are considered. The concept of pseudo-rotation of resonators is introduced. Cases are noted when the pseudo-rotation of the resonators does not lead to a change in the coupling coefficients. The dependences of the coupling coefficients for different types of resonator pseudo-rotations are investigated. New integral representations are derived for the mutual coupling coefficients of rectangular dielectric resonators provided that their axes rotate in open space. In particular cases of parallelism of the resonator axes of one of the coordinate axes, the analytical expressions found in the work coincide with those obtained earlier. For each case of rotation, approximate analytical formulas are found for the integral representations obtained in this work, expressed in terms of the spherical Hankel functions of the second kind. Comparison of calculations of coupling coefficients by integral formulas and approximate expressions is carried out. It is shown that the approximate expressions have acceptable accuracy for all the considered cases of rotations. The dependences of the coupling coefficients on the coordinates of the resonators are investigated. The regions are marked in which the found integral representations make it possible to correctly describe the coupling coefficients of rectangular resonators. In contrast to integral representations, approximate formulas are correct in the entire spatial region of resonator interaction. The results obtained make it possible to construct analytical models of antennas, multi-element arrays and devices of infrared and optical wavelength ranges, made with the use of rectangular dielectric resonators; significantly reduce computation timecompared to numerical methods and optimize complex multi-cavity structures of microwave and optical communication systems.

Keywords: coupling coefficient; mutual coupling coefficient; rotation; rectangular dielectric resonator; open space

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Introduction

Currently, rotatable dielectric resonators (DRs) have found application in various lattices for the implementation of optical metasurfaces [2, 4, 6–9], arrays [1, 5, 13], metamaterials [11, 15] and another frequency-selective structures, for the formation and detection of twisted waves, Airy rays [9], as well as antennas [3, 10, 12, 14] and filters [16, 18]. The main advantage of DR lattices in comparison with metal metasurfaces is the low level of dissipative losses. For calculation and optimization the scattering parameters, it is necessary to build adequate electromagnetic models describing the properties of such lattices. The most convenient way to calculate the physical properties of the lattices is the development of the scattering theory based on the use of the coupling coefficients of resonators in various structures [17,18]. Knowledge of the coupling coefficients makes it possible to calculate and optimize the basic scattering parameters of the lattices with the least amount of computing power.

1 Statement of the problem

The purpose of this article is calculation and analysis of coupling coefficients of the rectangular dielectric resonators in open space in the rotation of the resonator axes.

Attempts to describe the rotation of dielectric resonators relative to each other lead, usually, to complex and cumbersome analytical expressions. To minimize the size of the resulting formulas, in this work the simplest rotations about only one axis, the allocated rectangular coordinate system, are considered. For simplicity, we considered only the basic dipole oscillations of rectangular magnetic resonators H_{111}.

2 Coupling coefficients of X-rotation Rectangular DRs in the Open Space

The main magnetic type of natural oscillations H_{111}, the field of a rectangular dielectric resonator in the local coordinate system (x', y', z') (Fig. 1) is represented as:

$$
\begin{aligned}
e_{x'} &= \frac{h_1 i\omega\mu_0}{k_1^2-\beta_z^2}\beta_y\cos(\beta_x x')\sin(\beta_y y')\cos(\beta_z z');\\
e_{y'} &= \frac{-h_1 i\omega\mu_0}{k_1^2-\beta_z^2}\beta_x\sin(\beta_x x')\cos(\beta_y y')\cos(\beta_z z');\\
e_{z'} &= 0;\\
h_{x'} &= \frac{h_1}{k_1^2-\beta_z^2}\beta_x\beta_z\sin(\beta_x x')\cos(\beta_y y')\sin(\beta_z z');\\
h_{y'} &= \frac{h_1}{k_1^2-\beta_z^2}\beta_y\beta_z\cos(\beta_x x')\sin(\beta_y y')\sin(\beta_z z');\\
h_{z'} &= h_1\cos(\beta_x x')\cos(\beta_y y')\cos(\beta_z z').
\end{aligned}
\tag{1}
$$

Here h_1 is the amplitude; ω is the circular frequency of natural oscillations; μ_0 is the magnetic permeability; $(\beta_x, \beta_y, \beta_z)$ are the wave numbers: $\beta_x^2+\beta_y^2+\beta_z^2 = k_1^2$; $k_1 = \omega\sqrt{\mu_0\varepsilon_1}$; $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$; ε_0, ε_1 is the dielectric permittivity of the external space and resonator, respectively.

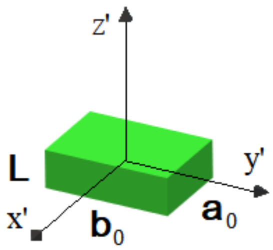

Fig. 1. Rectangular dielectric resonator in the local coordinate system (x', y', z')

In the general case, the complex coefficient of mutual coupling of two resonators in a waveguide with a cross section $a\times b$, can be represented as an expansion:

$$
k_{12} = \frac{i}{\omega w_2}\sum_t (c_t^{1\pm})_0 (c_t^{2\pm})_0^* e^{-i\Gamma|z_2-z_1|}, \tag{2}
$$

where $c_t^{s\pm}$ is the expansion coefficients of the DR field in terms of the waveguide field:

$$
c_t^{s\pm} = i/2\omega(\varepsilon_1-\varepsilon_0)\int\limits_V (\vec{e}^s, (\vec{E}_t^{\pm})^*)dv,
$$

where $\vec{e}^s$ – electric component field (1) of the s-th resonator; t – multi-index determined by the type of waveguide mode $(\vec{E}_t^{\pm}, \vec{H}_t^{\pm})$, $c_t^{s\pm} = (c_t^{s\pm})_0 e^{i\Gamma z_s}$; $(c_t^{s\pm})_0$ – expansion coefficient without taking into account the dependence on the longitudinal coordinate z_s; w_2 – energy stored in the resonator material.

To calculate the coupling coefficients in open space, we assumed that the walls of the "virtual" waveguide tend to infinity:

$$
\kappa_{12} = \lim_{a,b\to\infty} k_{12} \tag{3}
$$

as a result, sum (2) was transformed into the integral.

In the case of rotation of the resonator axes relative to the x-axis of the selected rectangular coordinate system, we will assume that the initial orientation of the axes of both resonators z' is parallel to the z-axis (Fig. 2, a). Let us also assume that the coordinates of the center of the first resonator are equal (x_1, y_1, z_1), and the coordinates of the second resonator are equal (x_2, y_2, z_2) and the main type of magnetic oscillations is excited in the resonators H_{111}, the field of which is described by expression (1).

In the case of rotation of 1 resonator relative to the x-axis (Fig. 2, a) by an angle β_1 and the second resonator relative to the x-axis by an angle β_2, the mutual coupling coefficient takes the form:

$$
\begin{aligned}
\kappa_{12} = i\kappa_0 \int_0^\infty\int_0^\infty \cos[\xi k_0\Delta x]\cdot\frac{e^{-i\gamma k_0\Delta z}}{\gamma(\xi^2+\eta^2)}\times\\
\times\Bigg\{\prod_{s=1}^{2}[\eta\beta_y\omega_x(\xi)\varpi_y(\eta\cos\beta_s+\gamma\sin\beta_s)+\xi\beta_x\cos\beta_s\varpi_x(\xi)\omega_y(\eta\cos\beta_s+\gamma\sin\beta_s)]\omega_z(\eta\sin\beta_s-\gamma\cos\beta_s)e^{-i\eta k_0\Delta y}+\\
+\prod_{s=1}^{2}[\eta\beta_y\omega_x(\xi)\varpi_y(\eta\cos\beta_s-\gamma\sin\beta_s)+\xi\beta_x\cos\beta_s\varpi_x(\xi)\omega_y(\eta\cos\beta_s-\gamma\sin\beta_s)]\omega_z(\eta\sin\beta_s+\gamma\cos\beta_s)e^{i\eta k_0\Delta y}+\\
+\prod_{s=1}^{2}[\xi\gamma\beta_y\omega_x(\xi)\varpi_y(\eta\cos\beta_s+\gamma\sin\beta_s)-\eta\gamma\beta_x\cos\beta_s\varpi_x(\xi)\omega_y(\eta\cos\beta_s+\gamma\sin\beta_s)+
\end{aligned}
\tag{4}
$$

$$+\ (\xi^2+\eta^2)\beta_x \sin\beta_s \varpi_x(\xi)\omega_y(\eta\cos\beta_s+\gamma\sin\beta_s)\big]\,\omega_z(\eta\sin\beta_s-\gamma\cos\beta_s)e^{-i\eta k_0\Delta y}+$$
$$+\prod_{s=1}^{2}\big[\xi\gamma\beta_y\omega_x(\xi)\varpi_y(\eta\cos\beta_s-\gamma\sin\beta_s)-\eta\gamma\beta_x\cos\beta_s\varpi_x(\xi)\omega_y(\eta\cos\beta_s-\gamma\sin\beta_s)+$$
$$+\ (\xi^2+\eta^2)\beta_x\sin\beta_s\varpi_x(\xi)\omega_y(\eta\cos\beta_s-\gamma\sin\beta_s)\big]\,\omega_z(\eta\sin\beta_s+\gamma\cos\beta_s)e^{-i\eta k_0\Delta y}\Big\}\,d\xi d\eta;$$

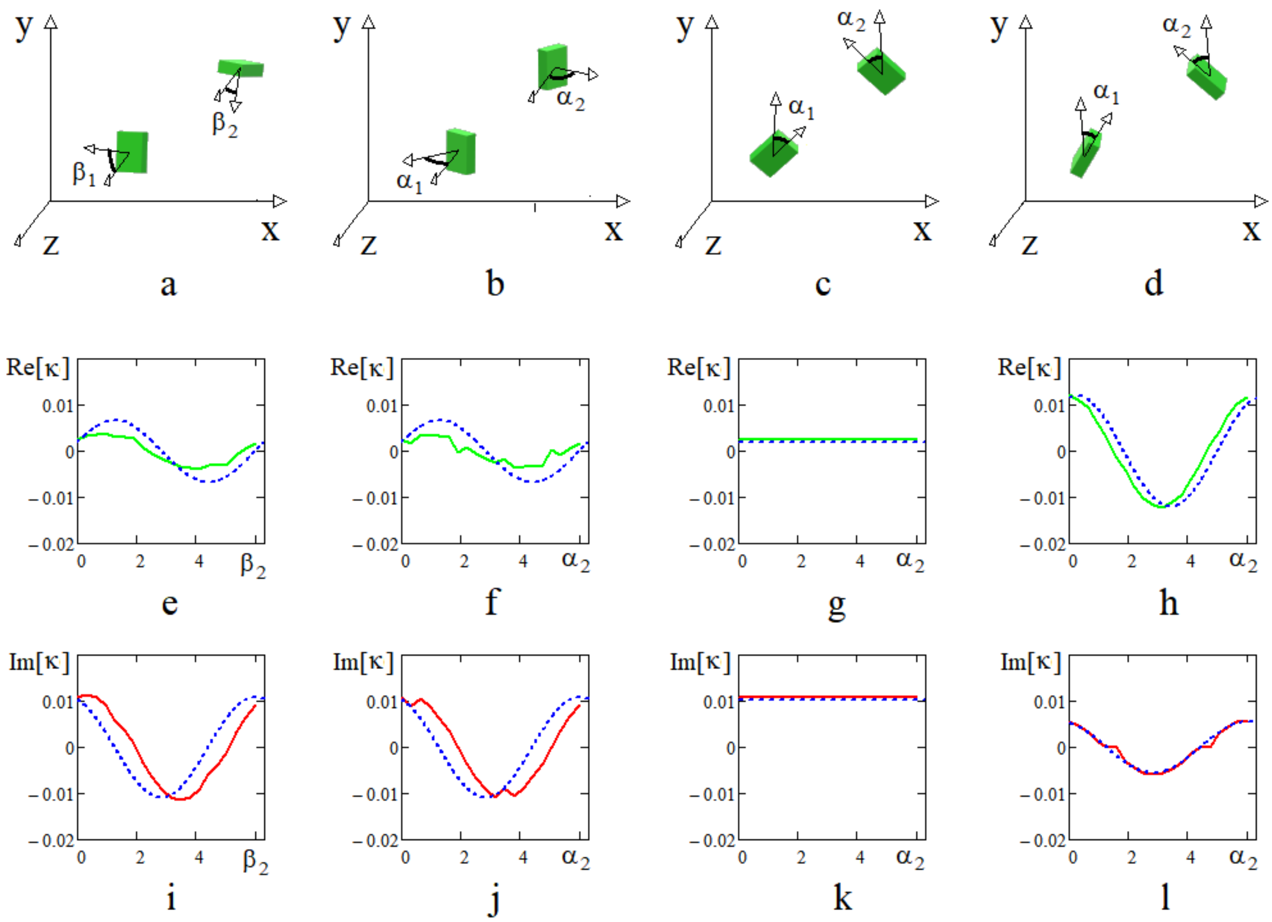

Fig. 2. Rotation of Rectangular DR relatively the x-axis (a); y-axis (b); z-axis (c, d) of the coordinate system in the open space. Dependence of the coupling coefficients on the rotation angle β_2 relatively the x-axis (e, i). Dependence of the DR coupling coefficients on the rotation angle α_2 relatively the y-axis (f, j). Dependence of the coupling coefficients on the rotation angle α_2 relatively the z-axis (g, k) for the initial DR position $z' \parallel z$, $x' \parallel x$. Dependence of the coupling coefficients on the angle of rotation α_1 for the initial DR position $z' \parallel x$ (h, l). ($k_0\Delta x = 1$; $k_0\Delta y = 1$; $k_0\Delta z = 2$; $\alpha_1, \beta_1 = 0$).

here $\xi^2+\eta^2+\gamma^2=1$;

$$\kappa_0 = \frac{32k_0^2}{\pi^2}(\varepsilon_{1r}-1)^2\,\frac{p_xp_yp_zq_xq_yq_z}{\upsilon H};$$

$$\upsilon H = \varepsilon_{1r}k_0^2[\beta_y^2\pi_x^+\pi_y^-\pi_z^+ + \beta_x^2\pi_x^-\pi_y^+\pi_z^+]+$$
$$+\beta_x^2\beta_z^2\cdot\pi_x^-\pi_y^+\pi_z^- + \beta_y^2\beta_z^2\cdot\pi_x^+\pi_y^-\pi_z^- + (\beta_x^2+\beta_y^2)^2\cdot\pi_x^+\pi_y^+\pi_z^+;$$

$\pi_{\rm v}^{\pm} = p_{\rm v} \pm \sin p_{\rm v}\cos p_{\rm v}$; ${\rm v} = (x,y,z)$; $\varepsilon_{1r} = \varepsilon_1/\varepsilon_0$; $p_x = \beta_x a_0/2$; $p_y = \beta_y b_0/2$; $p_z = \beta_z L/2$ as well as $q_x = k_0a_0/2$; $q_y = k_0b_0/2$; $q_z = k_0L/2$ are the characteristic parameters of the resonators. For H_{111} oscillations (1)

$$\omega_{\rm v}(\xi) = \frac{1}{p_{\rm v}^2-(q_{\rm v}\xi)^2}\times$$
$$\times\,[p_{\rm v}\sin p_{\rm v}\cos(q_{\rm v}\xi) - q_{\rm v}\xi\cos p_{\rm v}\sin(q_{\rm v}\xi)];$$

$$\varpi_{\rm v}(\xi) = \frac{i}{p_{\rm v}^2-(q_{\rm v}\xi)^2}\times$$
$$\times\,[p_{\rm v}\cos p_{\rm v}\sin(q_{\rm v}\xi) - q_{\rm v}\xi\sin p_{\rm v}\cos(q_{\rm v}\xi)].$$

Integral (4) is not very convenient for calculations, it is characterized by poor convergence, therefore, to calculate it, we will use the approximation.

The essential region of integration (4) is in the region of small values of the arguments of the functions $\omega_{\rm v}(z)$, $\varpi_{\rm v}(z)$. Therefore we use here an approximate representation of the functions $\omega_{\rm v}(z)$, $\varpi_{\rm v}(z)$, $\rm v = (x, y, z)$, typical for small values of variables (ξ, η):

$$\omega_{\rm v}(z) \approx A_{\rm v}, \ \varpi_{\rm v}(z) \approx C_{\rm v} z, \tag{5}$$

where $A_{\rm v}$, $C_{\rm v}$ are constants. Applying (5) and Sommer feld's integrals [18], we obtain a simpler formula, expressed in terms of the angles of rotation of the resonators:

$$\kappa_{12} \approx \kappa_{ZZ} \cos\beta_1 \cos\beta_2 + \kappa_{YY} \sin\beta_1 \sin\beta_2 + \kappa_{YZ} \sin(\beta_1+\beta_2), \tag{6}$$

where

$$\kappa_{ZZ} = \kappa_1 \times \left\{ \frac{\Delta x^2 + \Delta y^2}{\Delta r^2} h_0^{(2)}(k_0 \Delta r) - \left[1 - 3\frac{\Delta z^2}{\Delta r^2}\right] \frac{h_1^{(2)}(k_0\Delta r)}{k_0 \Delta r} \right\}; \tag{7}$$

$$\kappa_{YZ} = -\kappa_1 \frac{\Delta y \Delta z}{\Delta r^2} h_2^{(2)}(k_0 \Delta r); \tag{8}$$

(x_s, y_s, z_s) is the coordinates of the center of the s-th resonator in the rectangular coordinate system ($s = 1, 2$); $\Delta x = x_1 - x_2$; $\Delta y = y_1 - y_2$; $\Delta z = |z_1 - z_2|$; $\Delta r = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$; $h_n^{(2)}(z)$ is the Hankel spherical functions of the second kind.

Under the conditions: $A_x = A_y$ and $\beta_x C_x = \beta_y C_y$, which occur, for example, for resonators with a square cross-section ($a_0 = b_0$), the coefficient $\kappa_1 = i\frac{\pi}{2}\kappa_0 \times |A_x \beta_x C_x A_z|^2$. The easiest way to choose a numerical value of κ_1 is based on the mean theorem, directly for the given parameters of the resonator from (4) for a particular orientation, for example, for $\beta_1 = \beta_2 = 0$ [17].

3 Coupling coefficients of Y-rotation Rectangular DRs

In the case of rotation of each of the resonators, respectively, at an angle α_1, α_2 (Fig. 2, b) relative to the y-axis, we obtain similarly to (4):

$$\begin{aligned}
\kappa_{12} = i\kappa_0 \int_0^\infty \int_0^\infty \cos[\eta k_0 \Delta y] \cdot \frac{e^{-i\gamma k_0 \Delta z}}{\gamma(\xi^2+\eta^2)} \times \\
\times \Bigg\{ \prod_{s=1}^{2} [\eta \beta_y \cos\alpha_s \varpi_y(\eta) \omega_x(\xi\cos\alpha_s + \gamma \sin\alpha_s) + \xi \beta_x \omega_y(\eta) \varpi_x(\xi \cos\alpha_s + \gamma\sin\alpha_s)] \omega_z(\xi \sin\alpha_s - \gamma\cos\alpha_s) e^{-i\xi k_0 \Delta x} + \\
+ \prod_{s=1}^{2} [\eta \beta_y \cos\alpha_s \varpi_y(\eta) \omega_x(\xi\cos\alpha_s - \gamma \sin\alpha_s) + \xi \beta_x \omega_y(\eta) \varpi_x(\xi \cos\alpha_s - \gamma\sin\alpha_s)] \omega_z(\xi \sin\alpha_s + \gamma\cos\alpha_s) e^{-i\xi k_0 \Delta x} + \\
+ \prod_{s=1}^{2} [\xi\gamma \beta_y \cos\alpha_s \varpi_y(\eta) \omega_x(\xi\cos\alpha_s + \gamma \sin\alpha_s) - \eta\gamma \beta_x \omega_y(\eta) \varpi_x(\xi \cos\alpha_s + \gamma\sin\alpha_s) - \\
- (\xi^2+\eta^2)\beta_y \sin\alpha_s \varpi_y(\eta)\omega_x(\xi\cos\alpha_s + \gamma\sin\alpha_s)\big] \, \omega_z(\xi \sin\alpha_s - \gamma\cos\alpha_s) e^{-i\xi k_0 \Delta x} + \\
+ \prod_{s=1}^{2} [\xi\gamma \beta_y \cos\alpha_s \varpi_y(\eta) \omega_x(\xi\cos\alpha_s - \gamma \sin\alpha_s) - \eta\gamma \beta_x \omega_y(\eta) \varpi_x(\xi \cos\alpha_s - \gamma\sin\alpha_s) + \\
+ (\xi^2+\eta^2)\beta_y \sin\alpha_s \varpi_y(\eta)\omega_x(\xi\cos\alpha_s - \gamma\sin\alpha_s)\big] \, \omega_z(\xi \sin\alpha_s + \gamma\cos\alpha_s) e^{i\xi k_0 \Delta x} \Bigg\} \, d\xi d\eta.
\end{aligned} \tag{9}$$

Using again approximation (5), we find:

$$\kappa_{12} \approx \kappa_{ZZ} \cos\alpha_1 \cos\alpha_2 + \kappa_{XX} \sin\alpha_1 \sin\alpha_2 + \kappa_{XZ} \sin(\alpha_1 + \alpha_2). \tag{10}$$

Here

$$\kappa_{XX} = \kappa_1 \cdot \left\{ \frac{2}{3} h_0^{(2)}(k_0 \Delta r) + \frac{1}{2}\left[\frac{1}{3} - \frac{\Delta y^2 - \Delta x^2 + \Delta z^2}{\Delta r^2}\right] h_2^{(2)}(k_0\Delta r) \right\}; \tag{11}$$

$$\kappa_{XZ} = -\kappa_1 \frac{\Delta x \Delta z}{\Delta r^2} h_2^{(2)}(k_0 \Delta r). \tag{12}$$

4 Coupling coefficients of the Rectangular DRs under Z1-rotation

In the case of rotation of 1 and 2 resonators relative to z-axis (Fig. 2, c), respectively, at an angle α_1, α_2, relative to z-axis, the mutual coupling coefficient takes the form:

$$\begin{aligned}
\kappa_{12} = i\kappa_0 \int_0^\infty \int_0^\infty \frac{e^{-i\gamma k_0 \Delta z}}{\gamma(\xi^2+\eta^2)} \cdot |\omega_z(\gamma)|^2 \times \\
\times \Bigg\{ \prod_{s=1}^{2} [\beta_y(\eta\cos\alpha_s + \xi\sin\alpha_s)\omega_x(\xi\cos\alpha_s - \eta\sin\alpha_s)\varpi_y(\xi\sin\alpha_s + \eta\cos\alpha_s) - \\
- \beta_x(\eta\sin\alpha_s - \xi\cos\alpha_s)\varpi_x(\xi\cos\alpha_s - \eta\sin\alpha_s)\omega_y(\xi\sin\alpha_s + \eta\cos\alpha_s)] \cos(\xi k_0\Delta x + \eta k_0 \Delta y) + \\
+ \prod_{s=1}^{2} [\beta_y(\eta\cos\alpha_s - \xi\sin\alpha_s)\omega_x(\xi\cos\alpha_s + \eta\sin\alpha_s)\varpi_y(\xi\sin\alpha_s - \eta\cos\alpha_s) - \\
- \beta_x(\eta\sin\alpha_s + \xi\cos\alpha_s)\varpi_x(\xi\cos\alpha_s + \eta\sin\alpha_s)\omega_y(\xi\sin\alpha_s - \eta\cos\alpha_s)] \cos(\xi k_0\Delta x - \eta k_0 \Delta y) + \\
+ \prod_{s=1}^{2} [\beta_y\gamma(\xi\cos\alpha_s - \eta\sin\alpha_s)\omega_x(\xi\cos\alpha_s - \eta\sin\alpha_s)\varpi_y(\xi\sin\alpha_s + \eta\cos\alpha_s) - \\
- \beta_x\gamma(\xi\sin\alpha_s + \eta\cos\alpha_s)\varpi_x(\xi\cos\alpha_s - \eta\sin\alpha_s)\varpi_y(\xi\sin\alpha_s + \eta\cos\alpha_s)] \cos(\xi k_0\Delta x + \eta k_0 \Delta y) + \\
+ \prod_{s=1}^{2} [\beta_y\gamma(\xi\cos\alpha_s + \eta\sin\alpha_s)\omega_x(\xi\cos\alpha_s + \eta\sin\alpha_s)\varpi_y(\xi\sin\alpha_s - \eta\cos\alpha_s) - \\
- \beta_x\gamma(\xi\sin\alpha_s - \eta\cos\alpha_s)\varpi_x(\xi\cos\alpha_s + \eta\sin\alpha_s)\varpi_y(\xi\sin\alpha_s - \eta\cos\alpha_s)] \cos(\xi k_0\Delta x - \eta k_0 \Delta y) \Bigg\} d\xi d\eta. \quad (13)
\end{aligned}$$

Using again approximation (5), we obtain from (13):

$$\kappa_{12} \approx \kappa_{ZZ}. \quad (14)$$

5 Coupling coefficients of the Rectangular DRs under Z2-rotation

Let us consider another important type of resonator rotation about the z-axis, in which the initial position of the axes of both resonators is oriented parallel to the x-axis (Fig. 2, d). In this case

$$\begin{aligned}
\kappa_{12} = i\kappa_0 \int_0^\infty \int_0^\infty \frac{e^{-i\gamma k_0 \Delta z}}{\gamma(\xi^2+\eta^2)} \times \\
\times \Bigg\{ \prod_{s=1}^{2} [\beta_x(\eta\sin\alpha_s - \xi\cos\alpha_s)\varpi_x(\gamma)\omega_y(\xi\sin\alpha_s + \eta\cos\alpha_s)\omega_z(\xi\cos\alpha_s - \eta\sin\alpha_s)] \cos(\xi k_0\Delta x + \eta k_0 \Delta y) + \\
+ \prod_{s=1}^{2} [\beta_x(\eta\sin\alpha_s + \xi\cos\alpha_s)\varpi_x(\gamma)\omega_y(\xi\sin\alpha_s - \eta\cos\alpha_s)\omega_z(\xi\cos\alpha_s + \eta\sin\alpha_s)] \cos(\xi k_0\Delta x - \eta k_0 \Delta y) + \\
+ \prod_{s=1}^{2} [\beta_x\gamma(\xi\sin\alpha_s + \eta\cos\alpha_s)\varpi_x(\gamma)\omega_y(\xi\sin\alpha_s + \eta\cos\alpha_s) + \beta_y(\xi^2+\eta^2)\omega_x(\gamma)\varpi_y(\xi\sin\alpha_s + \eta\cos\alpha_s)] \times \\
\times \omega_z(\xi\cos\alpha_s - \eta\sin\alpha_s) \cos(\xi k_0\Delta x + \eta k_0 \Delta y) + \\
+ \prod_{s=1}^{2} [\beta_x\gamma(\xi\sin\alpha_s - \eta\cos\alpha_s)\varpi_x(\gamma)\omega_y(\xi\sin\alpha_s - \eta\cos\alpha_s) + \beta_y(\xi^2+\eta^2)\omega_x(\gamma)\varpi_y(\xi\sin\alpha_s - \eta\cos\alpha_s)] \times \\
\times \omega_z(\xi\cos\alpha_s + \eta\sin\alpha_s) \cos(\xi k_0\Delta x - \eta k_0 \Delta y) \Bigg\} d\xi d\eta. \quad (15)
\end{aligned}$$

Taking into account (5), expression (15) takes the form:

$$\kappa_{12} \approx \kappa_{XX} \cos\alpha_1 \cos\alpha_2 + \kappa_{YY} \sin\alpha_1 \sin\alpha_2 - \kappa_{XY} \sin(\alpha_1 + \alpha_2), \quad (16)$$

where

$$\kappa_{YY} = \kappa_1 \left\{ \frac{2}{3} h_0^{(2)}(k_0\Delta r) + \frac{1}{2}\left[\frac{1}{3} - \frac{\Delta x^2 - \Delta y^2 + \Delta z^2}{\Delta r^2}\right] h_2^{(2)}(k_0\Delta r) \right\}, \quad (17)$$

$$\kappa_{XY} = \kappa_1 \frac{\Delta x \Delta y}{\Delta r^2} h_2^{(2)}(k_0 \Delta r). \tag{18}$$

Relations (4, 9, 13, 15) as well as (6-8, 10-12, 16-18) make it possible to calculate the mutual coupling coefficients of the rectangular DRs in the cases of the considered rotations of their axes. Relations (4, 9, 13, 15) are valid under the condition $\Delta z > d_{\max}$, where $d_{\max}$ is the maximum size of the DR is in the direction of the straight line connecting their centers. In contrast to integral representations, the found approximate formulas (6-8, 10-12, 16-18) are finite in the entire spatial region of resonator interaction.

6 Analysis of mutual coupling coefficients

In particular cases of parallelism of the resonator axes of one coordinate axes, the found analytical expressions (4-18) coincide with those obtained earlier [17].

The results of calculating the dependences of the mutual coupling coefficients on the coordinates and the angle of rotation of the resonator are shown in Fig. 2-5 for $\varepsilon_{1r} = 36; a_0 = b_0;\ L/2a_0 = 0,4; f_0 = 8\,\mathrm{GHz}$.

Fig. 2 compares the results of the numerical calculation of integral representations and the calculation of the mutual coupling coefficients of the resonators using approximate formulas. Solid curves show the dependences calculated by the formulas (4, 9, 13, 15); the dotted curves show the dependences obtained by approximate formulas (6, 10, 14, 16). As can be seen from the data presented, the approximate relations are in satisfactory agreement with the integral representation of the coupling coefficients in the region of resonator coordinates of interest to us.

Fig. 3 shows the dependences of the coupling coefficients in the case of pseudo-rotation of resonators: $\alpha_2, \beta_2 = \alpha_1, \beta_1 + a$, where $a = const$, according to formulas (6, 10, 14, 16). By pseudo-rotation of resonators we mean rotation in which the relative direction of the axes of each resonator remains fixed. A change in the mutual coupling coefficients during pseudo-rotation is associated with a change in the relative coordinates of the DR. In the latter case (Fig. 3, d, h), if the centers of the resonators are located on the axis of rotation, then the coefficients of mutual coupling remain constant.

Figures 4, 5 show the dependences of the coupling coefficients upon variation of the coordinates of the second resonator, for different orientations of its axis. The reason for the appearance of asymmetry in the dependence in the case of y-rotation (Fig. 5, b, f) is due to the different spatial distribution of the vector fields of natural oscillations relative to the plane of symmetry $x = 0$ of the first resonator.

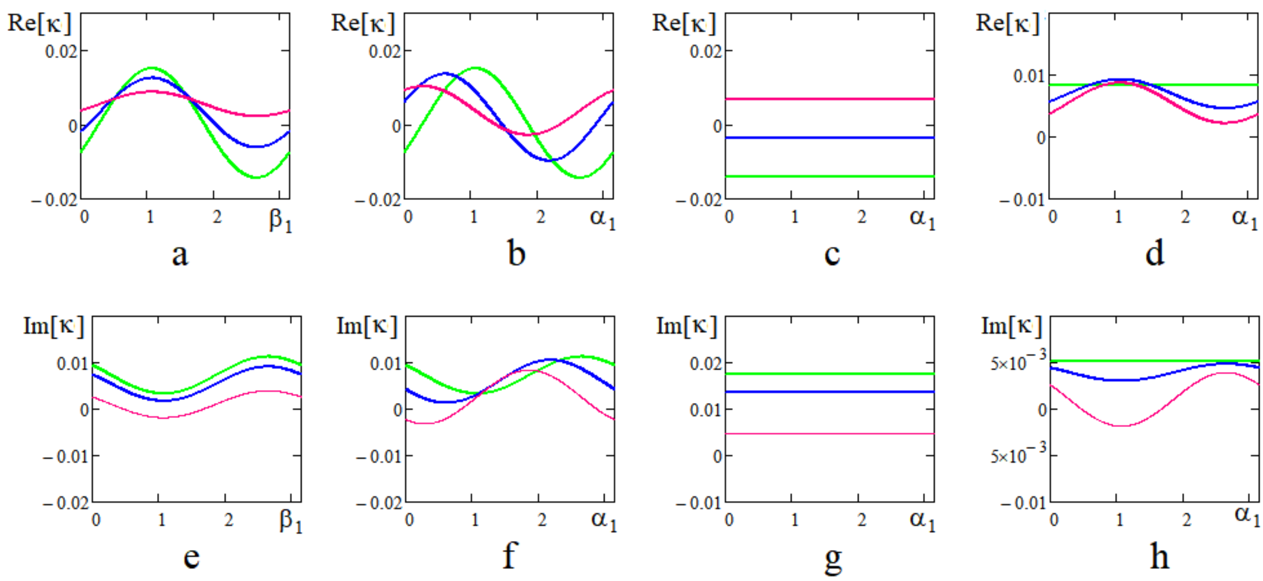

Fig. 3. Pseudo-rotations ($\alpha_2, \beta_2 = \alpha_1, \beta_1 + 1$) of rectangular DRs relatively the x-axis (a, e); y (b, f) and z (c, g); (d, h); ($k_0\Delta x = 0$; $k_0\Delta y = 0$; $k_0\Delta z = 2$) – green curves; ($k_0\Delta x = 1$; $k_0\Delta y = 0$; $k_0\Delta z = 2$) – blue curves; ($k_0\Delta x = 2$; $k_0\Delta y = 0$; $k_0\Delta z = 2$) – red curves.

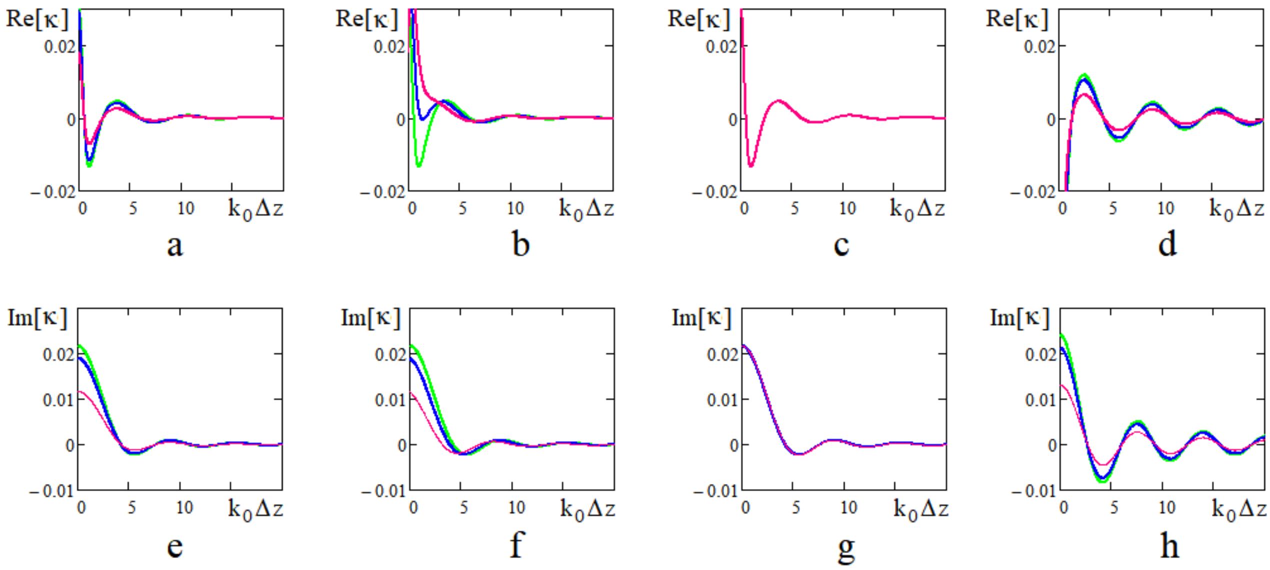

Fig. 4. Dependence of the DRs rotated about the x-axis (Fig. 2, a) on the distance $k_0\Delta z$ (a, e); rotated relatively the y-axis (Fig. 2, b) (b, f); rotated relatively z-axis (Fig. 2, c) (c, g); rotated relatively z-axis (Fig. 2, d) (d, h): $k_0\Delta x = 1$; $k_0\Delta y = 0$; for $\alpha_1, \beta_1 = 0$; $\alpha_2, \beta_2 = 0$ – green curves; $\alpha_2, \beta_2 = 0,5$ – blue curves; $\alpha_2, \beta_2 = 1$ – red curves

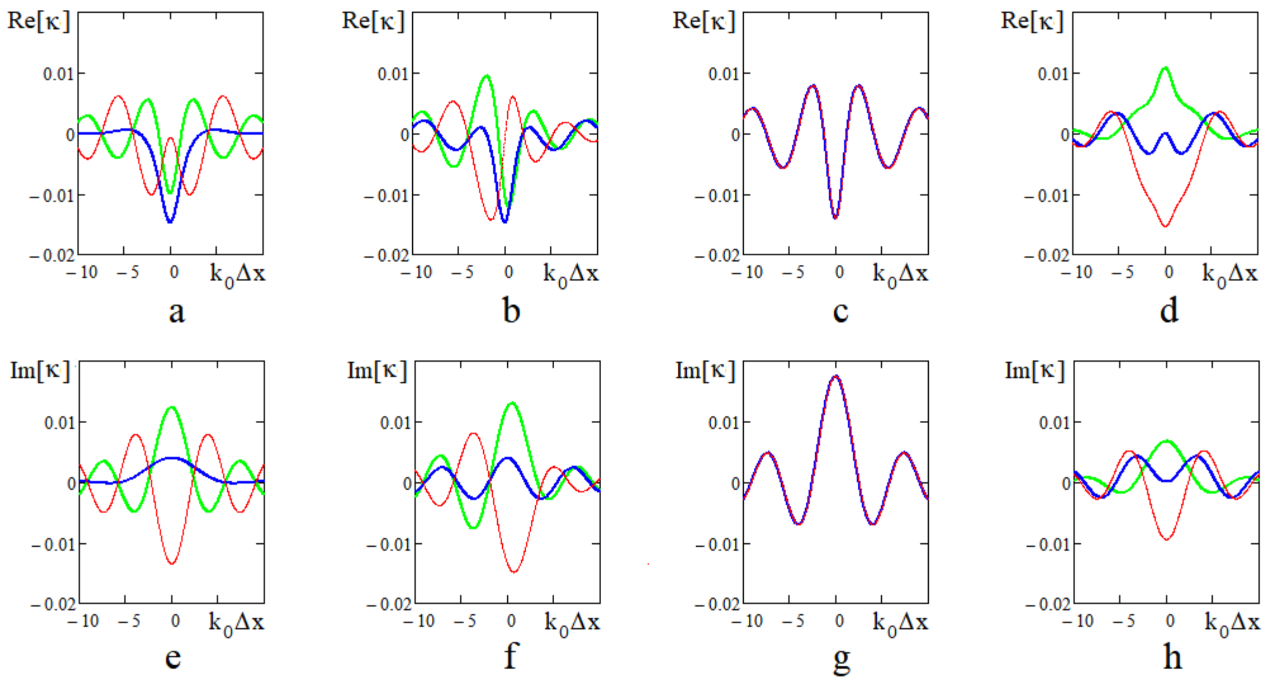

Fig. 5. Dependence of the coupling coefficients on the distance $k_0\Delta x$ (a, d) for rotated DRs about the x-axis (Fig. 2, a) (a, e); rotated DRs relatively the y-axis (Fig. 2, b) (b, f); rotated DRs relatively z-axis (Fig. 2, c) (c, g); rotated DRs relatively z-axis (Fig. 2, d) (d, h). ($k_0(y_1 - y_2) = 0$; $k_0\Delta z = 2$; $\alpha_1, \beta_1 = -\pi/4$; $\alpha_2, \beta_2 = 0$ – green curves; $\alpha_2, \beta_2 = \pi/4$ – blue curves; $\alpha_2, \beta_2 = 3\pi/4$ – red curves)

Discussion and Conclusion

We obtained simple analytical expressions for the mutual coupling coefficients of rectangular DRs in the open space in the case of their rotations about one of the axes of given coordinate system.

The dependences of the coupling coefficients on the angles of rotation and spatial coordinates of resonators are considered.

The concept of pseudo-rotation of resonators is introduced. The cases are investigated when the pseudo-rotation of the resonators does not lead to a change in the values of the mutual coupling coefficients.

The formulas found have good accuracy and make it possible to calculate and optimize the electromagnetic parameters of complex multi-element filters, antennas and different metasurfaces containing a large number of dielectric resonators much faster compared to numerical simulation methods.

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Коефіцієнти взаємного зв'язку обертових прямокутних діелектричних резонаторів у відкритому просторі

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Розраховані коефіцієнти взаємного зв'язку діелектричних резонаторів прямокутної форми у відкритому просторі при їх обертанні щодо однієї з осей заданої прямокутної системи координат. Знайдені вирази дають повну інформацію про частоти та добротності зв'язаних коливань діелектричних резонаторів. Отримані аналітичні формули для комплексних коефіцієнтів взаємного зв'язку. Розглядаються залежності коефіцієнтів зв'язку від кутів обертання і просторових координат резонаторів в разі збудження в них основних магнітних типів власних коливань. Введено поняття псевдообертання резонаторів. Досліджено залежності коефіцієнтів зв'язку при різних видах псевдообертання резонаторів. Відзначено випадки, коли псевдообертання резонаторів не призводить до зміни коефіцієнтів зв'язку. Виведено нові інтегральні співвідношення для коефіцієнтів взаємного зв'язку діелектричних резонаторів прямокутної форми при обертанні їх осей у відкритому просторі. В окремих випадках паралельності осей резонаторів одній з координатних осей, знайдені в роботі аналітичні вирази збігаються з отриманими раніше. Для кожного випадку обертання знайдені наближені аналітичні формули для отриманих в роботі інтегральних співвідношень, що

виражаються через сферичні функції Ханкеля другого роду. Проведено порівняння обчислень коефіцієнтів зв'язку за інтегральними формулами і наближеними виразами. Показано, що наближені вирази мають прийнятну точність для всіх розглянутих випадків обертань. Розглянуто залежності коефіцієнтів зв'язку від координат резонаторів. Відзначено області, в яких знайдені інтегральні вирази дозволяють коректно описувати коефіцієнти зв'язку прямокутних резонаторів. На відміну від інтегральних співвідношень, наближені формули коректні у всій просторовій області взаємодії резонаторів. Отримані результати дозволяють будувати аналітичні моделі антен, багатоелементних решіток і пристроїв інфрачервоного і оптичного діапазонів довжин хвиль, виконаних з застосуванням діелектричних резонаторів прямокутної форми; значно скорочувати час обчислень в порівнянні з чисельними методами і оптимізувати складні багаторезонаторні структури мікрохвильових і оптичних систем зв'язку.

Ключові слова: коефіцієнт зв'язку; коефіцієнт взаємного зв'язку; обертання; прямокутний діелектричний резонатор; відкритий простір

Коэффициенты взаимной связи вращаемых прямоугольных диэлектрических резонаторов в открытом пространстве

Трубин А. А.

Рассчитаны коэффициенты взаимной связи диэлектрических резонаторов прямоугольной формы в открытом пространстве при условии их вращения относительно одной из осей заданной прямоугольной системы координат. Найденные выражения дают полную информацию о частотах и добротности связанных колебаний диэлектрических резонаторов. Выводятся аналитические формулы для комплексных коэффициентов взаимной связи. Рассматриваются зависимости коэффициентов связи от углов вращения и пространственных координат резонаторов в случае возбуждения в них основных магнитных типов собственных колебаний. Введено понятие псевдовращения резонаторов. Отмечены случаи, когда псевдовращение резонаторов не приводит к изменению значений коэффициентов связи. Исследованы зависимости коэффициентов связи при разных видах псевдовращений резонаторов. Выведены новые интегральные представления для коэффициентов взаимной связи диэлектрических резонаторов прямоугольной формы при условии вращения их осей в открытом пространстве. В частных случаях параллельности осей резонаторов одной из координатных осей, найденные в работе аналитические выражения совпадают с полученными ранее. Для каждого случая вращения получены приближенные аналитические формулы для найденных в работе интегральных представлений, выражаемые через сферические функции Ханкеля второго рода. Проведено сравнение расчетов коэффициентов связи по интегральным формулам и приближенными выражениями. Показано, что приближенные выражения обладают приемлемой точностью для всех рассмотренных случаев вращений. Рассмотрены зависимости коэффициентов связи от координат резонаторов. Отмечены области, в которых найденные интегральные представления позволяют корректно описывать коэффициенты связи прямоугольных резонаторов. В отличие от интегральных представлений, найденные приближенные формулы корректны во всей пространственной области взаимодействия резонаторов. Полученные результаты позволяют строить аналитические модели антенн, многоэлементных решеток и устройств инфракрасного и оптического диапазонов длин волн, выполненных с применением диэлектрических резонаторов прямоугольной формы; значительно сокращать время вычислений по сравнению с численными методами и оптимизировать сложные многорезонаторные структуры микроволновых и оптических систем связи.

Ключевые слова : коэффициент связи; коэффициент взаимной связи; вращение; прямоугольный диэлектрический резонатор; открытое пространство