534.232.082.73

# Mathematical Modelling of Cylindrical Piezoelectric Transducers for Electroacoustic Devices

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This paper will review the procedure and the results of the research conducted on developing mathematical models of cylindrical piezoelectric transducers that are extensively applied in electrical acoustics and hydro acoustics (for example, in devices designed for radiating and receiving acoustic oscillations in air or water medium). The distinctive feature of the developed models lies in the fact that the dependences established are a mathematical description of the electroacoustic connection between the wave fields located in different parts of a hollow piezoceramic cylindrical transducer. The analytical dependences obtained in the result of a simulation allow us to establish the electrical impedance and amplitude values of the electric current and electric charge on the electroded surface of a piezoelectric transducer (cylindrical piezoelectric shell of finite height) under the inverse piezoelectric effect, thus obtaining a complete solution for the problem of harmonic axisymmetric oscillations of a transducer of this type. In order to assess the results, the developed mathematical model was used in cylindrical shell transducers made of PZT-type (plumbum zirconate titanate) piezoelectric ceramics. Strong evidence of a frequency-dependent change of electric impedance and components of the displacement vector for material particles in the oscillating piezoelectric transducer was found with frequencies of electromechanical resonances within the range of 33-35 kHz and 82 kHz, when a sharp impedance decrease was observed (2.6-5 times). A comparative analysis of mathematically calculated and experimentally obtained values of the electrical impedance of the oscillating cylindrical piezoceramic shell revealed high convergence between them (the discrepancy between the simulation results and experimentally obtained data at the same values of operating frequency within the range up to 100 kHz did not exceed 17%).

*Keywords:* piezoelectric transducer; acoustoelectronics; mathematical mode; impedance; cylindrical shell

DOI: 10.20535/RADAP.2022.88.24-34

## Introduction

The widespread use of piezoelectric transducers in recent years [1] is explicated by their relative inexpensiveness, high reliability, progressing miniaturization, versatility and integration into microelectronics and mechatronics technologies. Another advantage of manufacturing and implementing piezoelectric devices is explained by the demand to actively introduce modern renewable, energy and resource saving technologies within the concept of sustainable development and the initiative "Industry 4.0" [2].

Piezoelectric transducers have won a special importance in electro and hydro acoustics, where they are designed to emit and receive acoustic vibrations in air or water medium (components of target detection, echolocation, ultrasonic non-destructive testing, health diagnostics, etc.) [3, 4].

According to the latest marketing research, the piezoelectric transducers market has grown significantly over the past five years. Thus, the revenue of the piezoelectric transducers market amounted to 725 million US dollars in 2016, increased to 1.1 billion US dollars in 2021 and, according to Precision Market Reports, a leading company in the field of business strategy, is anticipated to reach 2.1 billion US dollars in 2026 with an average annual growth rate of 17-19% in 2021-2026 [5].

At the same time, for new models of piezoelectric transducers to be developed, it is necessary to reinforce the scientific base in the areas of improving methodological, technological and, most importantly, mathematical support. Improvements in the essential mathematical software will involve development and practical implementation of new calculation methods into production and operation of piezoelectric transducers for electroacoustic devices, as well as designing and modelling piezoelectric transducers with various technical and design characteristics [6, 7].

# 1 Relevance of the study based on recent publications

Circular cylindrical transducers have been amply used in practice, particularly, in designing hydroacoustic antennas [8, 9], wherein either solid or sectional shells of finite height often serve as the active element. In this case, as discussed in [10,11], a standard design scheme of a cylindrical transducer includes a vertical set of a finite number of shells within a single structure, which, by electrical switching according to a certain scheme, allows designing transducers with directed properties, both across the width of the main petal and across the level of lateral petals. At the same time, as noted in [9], choosing the correct relation between the shell diameter and its height is immensely important, since it enables us to determine the ratio of resonance mode frequencies of different types and the associated energy of elastic oscillations of the shells on the mode selected as the working one for the whole transducer.

On the other hand, a number of works [12–15] are devoted to studying processes that occur in shell piezoelectric elements of various shapes with a fully electroded surface (solid electrodes). For example, source [12] reveals the research results for forced oscillations of a piezoceramic plate, which are induced by acting external mechanical harmonic load at the fundamental resonance frequency. Besides, the presented results consider the viscoelastic properties of the material and the plate's fastening scheme. In publications [13, 14] the research on the effects that occur in piezoceramic disks in microcircuitry piezoelectric motors are presented, and an attempt is made to calculate and simulate the vibration characteristics of variously sized thin piezoceramic disks. In [15], a mathematical model of a crystalline piezoelectric element is presented, which regards the gradient electric excited field induced within the plane of this element. Major electromechanical and energy characteristics of radial displacements of piezoceramic elements, which occur while approaching resonant / anti-resonant frequencies, are investigated in [16, 17]. The oscillation process in piezoceramic plates while supplying an exciting pulsed electric field was studied in [18].

Previous research on the use of equivalent schemes in the modelling of piezoelectric transducers [19, 20] has found it impossible to account mechanical processes and phenomena occurring in piezoceramics, and established the nature of the relationship between these processes and the electrical characteristics of the piezoelectric material. This ensures the adequacy of such models in terms of their application to real piezoelectric objects and the physical processes inherent in them [21].

Nonetheless, while analyzing works on the mathematical description of the processes occurring in piezoceramic transducers, we have noted that no previous study has offered systematization of the methods for calculation and modeling of these transducers, which makes it impossible to create a generalized approach to mathematical models of piezoelectric transducers of various shapes and functional purposes, which could be considered as a theoretical basis for calculating technical and operational characteristics of piezoelectric transducers.

However, it should be noted that since circular cylindrical transducers conventionally imply radial oscillations, there is an urgent need to develop a theory of harmonic radial oscillations for cylindrical piezoceramic shells of finite height.

Therefore, a topical issue of today is constructing and experimental confirmation of deterministic mathematical models of cylindrical piezoelectric transducers intended for acoustoelectric devices.

# 2 Problem statement for modelling of a cylindrical piezoelectric transducer

First, we would like to describe a cylindrical shell of finite height $h$ (Fig. 1), which is made of PZT-type (plumbum zirconate titanate $Pb[Zr_xTi_{1-x}]O_3$) piezoelectric ceramics polarized across its thickness.

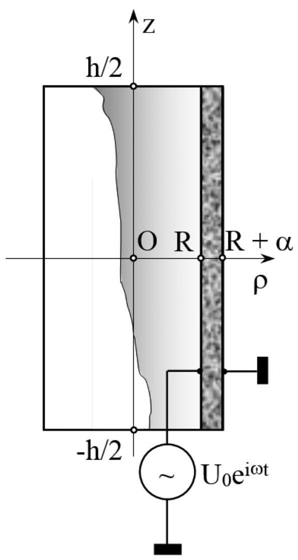


Fig. 1. Calculation scheme of a piezoceramic cylindrical shell

Shell side surfaces $\rho = R$ and $\rho = R + \alpha$ bear a thin metal plating. Electrodes are absent on the end surfaces $z = \pm h/2$. A generator, which produces an electrical potential difference $U_0 e^{i\omega t}$, is connected to the inner electroded surface $\rho = R$ of the shell. Since the shell material is radially polarized, the matrices of the shell material constants can be illustrated as follows:

a) matrix of adiabatic elastic moduli $c^E_{\beta\lambda}$ (where $\beta, \lambda = 1, 2, \ldots, 6$ are Voigt indices) in the mode of constant electric field strength

$$\|c^E_{\beta\lambda}\| = \left\| \begin{matrix} c^E_{11} & c^E_{12} & c^E_{13} & 0 & 0 & 0 \\ & c^E_{22} & c^E_{12} & 0 & 0 & 0 \\ & & c^E_{22} & 0 & 0 & 0 \\ & & & c^E_{44} & 0 & 0 \\ & & & & c^E_{55} & 0 \\ & & & & & c^E_{55} \end{matrix} \right\|, \qquad (1)$$

b) matrix of adiabatic piezoelectric moduli $e_{k\beta}$ ($k = 1, 2, 3; \beta = 1, 2, \ldots, 6$)

$$\|e_{k\beta}\| = \left\| \begin{matrix} e_{11} & e_{12} & e_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{26} \\ 0 & 0 & 0 & 0 & e_{35} & 0 \end{matrix} \right\|, \qquad (2)$$

c) adiabatic dielectric permittivity matrix $\chi^\varepsilon_{ij}$ ($i, j = 1, 2, 3$), measured under constant strain mode

$$\|\chi^\varepsilon_{ij}\| = \left\| \begin{matrix} \chi^\varepsilon_{11} & 0 & 0 \\ & \chi^\varepsilon_{22} & 0 \\ & & \chi^\varepsilon_{22} \end{matrix} \right\|. \qquad (3)$$

In matrices recordings (1)–(3), the same symbols denote elements of equal size. The relations between the reference values $c^{Eref}_{\beta\lambda}$, $e^{ref}_{k\beta}$ and $\chi^{\varepsilon ref}_{ij}$, that refer to the piezoelectric ceramics plate polarized across its thickness, and matrices elements (1)–(3) are represented as follows: $c^E_{11} = c^{Eref}_{33}$; $c^E_{22} = c^{Eref}_{11}$; $c^E_{12} = c^{Eref}_{13}$; $c^E_{13} = c^{Eref}_{12}$; $c^E_{44} = \frac{1}{2}(c^{Eref}_{11} - c^{Eref}_{12})$; $c^E_{55} = c^{Eref}_{55}$; $e_{11} = e^{ref}_{33}$; $e_{12} = e_{13} = e^{ref}_{31}$; $e_{26} = e_{35} = \frac{1}{2}(e^{ref}_{33} - e^{ref}_{31})$; $\chi^\varepsilon_{11} = \chi^{\varepsilon ref}_{33}$; $\chi^\varepsilon_{22} = \chi^{\varepsilon ref}_{11}$.

## 3 Building a mathematical model of a cylindrical piezoelectric transducer

The generator, while producing the electrical potential difference, creates an axisymmetric electric field inside the shell, the tension vector of which alters harmoniously in time and is almost completely defined by the radial component $E_\rho(t) = E_\rho e^{i\omega t}$, wherein $E_\rho$ is the peak value. Influenced by the electric field, axisymmetric deformations appear within the volume of the shell. These deformations are formed by radial $u_\rho(t)$ and axial $u_z(t)$ displacements of material particles.

Components $u_\rho(t)$ and $u_z(t)$ of the displacement vectors change in time according to the harmonic law $u_\beta(t) = u_\beta e^{i\omega t} (\beta = \rho, z)$, while their peak values $u_\rho$ and $u_z$ are determined by coordinates $\rho$ and $z$ (of the observation point (irrespectively of the circumferential coordinate $\varphi$) and satisfy the following equations of steady harmonic oscillations [8]:

$$\frac{\partial\sigma_{\rho\rho}}{\partial\rho} + \frac{\partial\sigma_{z\rho}}{\partial z} + \frac{(\sigma_{\rho\rho} - \sigma_{\varphi\varphi})}{\rho} + \rho_0\omega^2 u_\rho = 0, \qquad (4)$$

$$\frac{\partial\sigma_{\rho z}}{\partial\rho} + \frac{\partial\sigma_{zz}}{\partial z} + \frac{\sigma_{\rho z}}{\rho} + \rho_0\omega^2 u_z = 0, \qquad (5)$$

where $\sigma_{\lambda,\beta}$ ($\lambda, \beta = \rho, z$) represent peak values of resultant mechanical stresses harmonically varying in time inside the piezoceramic shell volume deformed by the electric field; $\rho_0$ marks density of the piezoceramics. Resultant stresses $\sigma_{\lambda,\beta}$ are postulated by the generalized Hooke's law for an elastic medium with complicated (piezoelectric) properties [22]. Taking into account the matrices structure for material constants (1) and (2), the relations for calculating the quantities $\sigma_{\lambda,\beta}$ will be noted as follows:

$$\sigma_{\rho\rho} = c^E_{11}\varepsilon_{\rho\rho} + c^E_{12}\varepsilon_{\varphi\varphi} + c^E_{13}\varepsilon_{zz} - e_{11}E_\rho, \qquad (6)$$

$$\sigma_{\varphi\varphi} = c^E_{12}\varepsilon_{\rho\rho} + c^E_{22}\varepsilon_{\varphi\varphi} + c^E_{12}\varepsilon_{zz} - e_{12}E_\rho, \qquad (7)$$

$$\sigma_{zz} = c^E_{13}\varepsilon_{\rho\rho} + c^E_{12}\varepsilon_{\varphi\varphi} + c^E_{22}\varepsilon_{zz} - e_{13}E_\rho, \qquad (8)$$

$$\sigma_{\rho z} = \sigma_{z\rho} = c^E_{55}\left(\frac{\partial u_\rho}{\partial z} + \frac{\partial u_z}{\partial\rho}\right) - e_{35}E_z, \qquad (9)$$

where $\varepsilon_{\beta\beta}$ ($\beta = \rho, \varphi, z$) are peak values of harmonical time-varying deformations (relative elongations) of infinitesimal segments oriented along the corresponding coordinate lines. In case of axial symmetry, the formulas to calculate deformations through the components of the displacement vector of material particles in the medium will be as follows:

$$\varepsilon_{\rho\rho} = \frac{\partial u_\rho}{\partial\rho}; \quad \varepsilon_{\varphi\varphi} = \frac{u_\rho}{\rho}; \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}. \qquad (10)$$

Let us assume that the cylindrical shell vibrates in a vacuum. In this case, as follows from Newton's third law, the following conditions must be satisfied for the shell's surface:

$$\sigma_{\rho\rho}|_{\rho=R;R+\alpha} = 0, \quad \sigma_{\rho z}|_{\rho=R;R+\alpha} = 0, \qquad (11)$$

$$\sigma_{z\rho}|_{z=\pm h/2} = 0, \quad \sigma_{zz}|_{z=\pm h/2} = 0. \qquad (12)$$

Since the stress-strain state of the thin shell does not alter within the thickness of its wall, the condition for stress $\sigma_{\rho\rho}$ to be equal to zero on the surface of the shell's wall should obviously be fulfilled inside the wall as well, that is, within the volume of the shell. If we note zero in the left-hand side of the relation (6), we obtain the expression for calculating the component of the deformation tensor $\varepsilon_{\rho\rho}$:

$$\varepsilon_{\rho\rho} = -\frac{c^E_{12}}{c^E_{11}\varepsilon_{\varphi\varphi}} - \frac{c^E_{13}}{c^E_{11}\varepsilon_{zz}} + \frac{e_{11}}{c^E_{11}E_\rho}. \qquad (13)$$

The fact that elastic moduli $c^E_{12}$ and $c^E_{13}$ are identical in magnitude enables us to assume, similarly to previous calculations, that $c^E_{12} = c^E_{13}$ and to denote the material constants with coinciding values with the same symbols while performing further calculations.

Taking into account relation (13), we can rewrite expressions (7) and (8) as follows:

$$\sigma_{\varphi\varphi} = c_{22}\varepsilon_{\varphi\varphi} + c_{12}\varepsilon_{zz} - e_{12}^{*}E_{\rho}, \quad (14)$$

$$\sigma_{zz} = c_{12}\varepsilon_{\varphi\varphi} + c_{22}\varepsilon_{zz} - e_{12}^{*}E_{\rho}, \quad (15)$$

where $c_{12} = c_{12}^{E}(1 - c_{12}^{E}/c_{11}^{E}); c_{22} = c_{22}^{E} - (c_{12}^{E})^2/c_{11}^{E})$ are elastic moduli for biaxial stress-strain state; $e_{12}^{*} = e_{12} - e_{11}c_{12}^{E}/c_{11}^{E}$ denotes piezomodule for the mode of constancy (equality to zero) of the radial stress. In this case, the equations for steady-state harmonic oscillations (4) and (5), which possess the meaning of Newton's second law in the differential form, take the following form:

$$\frac{\partial\sigma_{z\rho}}{\partial z} - \frac{\sigma_{\varphi\varphi}}{R} + \rho_0\omega^2 u_{\rho} = 0, \quad (16)$$

$$\frac{\partial\sigma_{zz}}{\partial z} + \frac{\sigma_{z\rho}}{R} + \rho_0\omega^2 u_{z} = 0. \quad (17)$$

The electrical state of a deformable cylindrical shell is defined as the sum of electrical polarization $\vec{D}^{*}$, which is induced in the volume of the shell by an external device, that is, by an electric potential difference generator (Fig. 1), and electric polarization $\vec{D}^{\Pi}$, which is generated by the direct piezoelectric effect in the process of dynamic deformation of the piezoceramic shell. Hereinafter, polarization $\vec{D}^{*}$ will be designated as external electric polarization, while component $\vec{D}^{\Pi}$ will be designated as internal polarization. The electric induction vector $\vec{D}^{*}$ is completely specified by the component of the electric field strength vector $E_{\rho}^{*}$, which is created by the generator; at the same time, it is not influenced by variable $z$. Thus, $\vec{D}^{*}\left\{D_{\rho}^{*}, 0, 0\right\}$, where the radial component constitutes

$$D_{\rho}^{*} = \chi_{11}^{\varepsilon}E_{\rho}^{*}. \quad (18)$$

Internal electric polarization is determined by the deformations of the piezoceramic shell and the internal electric field, the electric potential $\Phi^{\Pi}$ of which does not depend on the values of the circumferential coordinate $\varphi$. Hence $\vec{D}^{\Pi}\left\{D_{\rho}^{\Pi}, 0, D_{z}^{\Pi}\right\}$, where the radial and axial components of the internal electric induction vector are defined as follows:

$$D_{\rho}^{\Pi} = e_{12}^{*}(\varepsilon_{\varphi\varphi} + \varepsilon_{zz}), \quad D_{z}^{\Pi} = 2e_{35}\varepsilon_{\rho z} + \chi_{22}^{\varepsilon}E_{z}^{\Pi}, \quad (19)$$

where $E_{z}^{\Pi} = -\frac{\partial\Phi^{\Pi}}{\partial z}$. Since $\sigma_{\rho z} = 2c_{55}^{E}\varepsilon_{\rho z} - e_{35}E_{z}^{\Pi} = 0$ at any given point inside the wall of the piezoceramic shell. Next, taking into account that $\varepsilon_{\rho z} = \frac{1}{2}(\frac{\partial u_{\rho}}{\partial z} + \frac{\partial u_{z}}{\partial \rho}) = \frac{1}{2}(\frac{\partial u_{\rho}}{\partial z})$, we obtain the following definition:

$$\frac{\partial u_{\rho}}{\partial z} = \frac{e_{35}E_{z}^{\Pi}}{c_{55}^{E}}. \quad (20)$$

Having substituted relation (20) in the definition of the axial component of the internal electric polarization vector, we obtain

$$D_{z}^{\Pi} = \chi_{22}^{\sigma}E_{z}^{\Pi}, \quad (21)$$

where $\chi_{22}^{\sigma} = \chi_{22}^{\varepsilon} + e_{35}^{2}/c_{55}^{E}$ is dielectric constant in the mode of constant (equal to zero) voltage $\sigma_{\rho z}$.

Since total electric polarization $\vec{D} = \vec{D}^{*} + \vec{D}^{\Pi}$ must satisfy the condition for mobile (free) electricity carriers to be absent [9], namely, condition

$$div\,\vec{D} = 0, \quad (22)$$

then, taking into account that vector $\vec{D}^{*}$ satisfies condition $div\,\vec{D}^{*} = 0$ by definition, we come to the conclusion that vector $\vec{D}^{\Pi}$ must satisfy the condition

$$div\,\vec{D}^{\Pi} = 0. \quad (23)$$

It follows from condition $div\,\vec{D}^{*} = 0$ that:

$$1/\rho\ \partial/\partial\rho\ (\rho D_{\rho}^{*}) = 0. \quad (24)$$

Differential equation (24) enables us to identify potential $\Phi^{*}$ of the electric field generated inside the shell volume by the generator of the electric potential difference, i.e., the external electric field. The expression for calculating potential $\Phi^{*}$ will be as follows:

$$\Phi^{*} = -C_1/\chi_{11}^{\varepsilon}\ ln(\rho/R) + C_2, \quad (25)$$

where $C_1$ and $C_2$ are constants to be found in the course of further problem solving. From condition (23) under the assumption that the potential of the internal electric field does not depend on the radial coordinate $\rho$ the stress-strain state remains unchanged over the thickness of the shell wall, we derive the differential equation for the potential $\Phi^{\Pi}$ of the internal electric field:

$$\frac{\partial^2\Phi^{\Pi}}{\partial z^2} = \frac{e_{12}^{*}}{\chi_{22}^{\sigma}}\left(\frac{u_{\rho}}{R^2} + \frac{1}{R}\ \frac{\partial u_z}{\partial z}\right). \quad (26)$$

Since condition $D_{z}^{\Pi} = 0$ on the end surfaces $z = \pm h/2$ of the shell must be satisfied (the dielectric constant of piezoceramics exceeds the dielectric constant of vacuum by more than three orders of magnitude), potential $\Phi^{\Pi}$, determined by equation (26) must satisfy the following boundary conditions:

$$\left.\frac{\partial\Phi^{\Pi}}{\partial z}\right|_{z=\pm h/2} = 0. \quad (27)$$

The total electric potential $\Phi = \Phi^{*} + \Phi^{\Pi}$ or the potential of the resulting electric field must satisfy the following conditions on the electroded surfaces of the piezoceramic shell:

$$\left. C_2 + \tilde{\Phi}^{\Pi} - U_0 \right|_{\rho=R} = 0, \quad (28)$$

$$\left. -\frac{C_1}{\chi_{11}^{\varepsilon}}\ln\left(1 + \alpha/R\right) + C_2 \right|_{\rho=R+\alpha} = 0, \quad (29)$$

where $\tilde{\Phi}^{\Pi} = \frac{1}{h}\int\limits_{-h/2}^{h/2}\Phi^{\Pi}(z)\,dz$ is the potential of the polarization (internal) electric field averaged over the height, and, more generally, over the area of the electroded lateral surface of the cylindrical shell. Averaging occurs due to conduction currents in electroded surfaces. The currents equalize the polarization potential on the electroded surfaces of the shell.

It follows from condition (24) that $D^*_\rho = C_1/\rho \approx C_1/R$ consequently, taking into account relation (18), it can be justified that:

$$E^*_\rho = C_1/R\chi^\varepsilon_{11}. \tag{30}$$

Since radial component $E_\rho$ of the strength vector of the resulting electric field in the volume of the vibrating shell is completely conditioned by the component $E^*_\rho$, equations (14) and (15), incorporating the deformations definition (10), can be formed as follows:

$$\sigma_{\varphi\varphi} = c_{22}u_\rho/R + c_{12}\partial u_z/\partial z - e^*_{12}C_1/R\chi^\varepsilon_{11}, \tag{31}$$

$$\sigma_{zz} = c_{12}u_\rho/R + c_{22}\partial u_z/\partial z - e^*_{12}C_1/R\chi^\varepsilon_{11}. \tag{32}$$

By inserting expressions (31) and (32) into the equations of motion (16) and (17), we obtain:

$$-c_{22}\frac{u_\rho}{R^2} - c_{12}\frac{1}{R}\frac{\partial u_z}{\partial z} + \frac{e^*_{12}C_1}{R^2\chi^\varepsilon_{11}} + \rho_0\omega^2 u_\rho = 0, \tag{33}$$

$$c_{12}\frac{1}{R}\frac{\partial u_\rho}{\partial z} + c_{22}\frac{\partial^2 u_z}{\partial z^2} + \rho_0\omega^2 u_z = 0. \tag{34}$$

Based on equation (32), the radial component of the displacement vector of material particles in the shell is derived:

$$u_\rho = -\frac{c_{12}}{c_{22}}\frac{R}{\left[1-(kR)^2\right]}\frac{\partial u_z}{\partial z} + \frac{e^*_{12}C_1}{c_{22}\chi^\varepsilon_{11}\left[1-(kR)^2\right]}, \tag{35}$$

where $k^2 = \rho_0\omega^2/c_{22}$ represents the square wavenumber of the radial component of axisymmetric vibrations of the cylindrical shell.

By differentiating expression (35) with respect to the variable $z$ and substituting the obtained result into equation (34), we conclude that:

$$\partial^2 u_z/\partial z^2 + \gamma^2 u_z = 0, \tag{36}$$

where symbol $\gamma^2$ denotes the square wavenumber of the axial component of axisymmetric vibrations of the cylindrical shell of a finite height. Wherein:

$$(\gamma R)^2 = \frac{(kR)^2\left[1-(kR)^2\right]}{1-(kR)^2-(c_{12}/c_{22})^2}. \tag{37}$$

Obviously, the dimensionless quantity $kR$ in relation (37) can be interpreted as the dimensionless frequency $\Omega$, which, by definition, is an independent variable. In this case, the dimensionless quantity $\zeta = \gamma R$ acquires the status of a dependent variable. The physical meaning of value $\zeta$ is defined by equation (36), since it denotes the wavenumber of axial displacement of the material particles changing harmonically in time. The dependence graph $\Omega = f(\zeta)$, which has the frequency spectrum meaning of the wavenumbers of the axial harmonic vibrations of the piezoceramic shell, is shown in Fig. 2. The graph is conditioned by the qualitative content of our above-mentioned assumptions on the nature of the stress-strain state of a cylindrical shell during radial vibrations, which constitute the main content of the so-called membrane theory of elastic shells, developed in the middle of the twentieth century [23]. When constructing the curves shown in Fig. 2, the following numerical data were processed: $c^E_{11} = 106$ GPa; $c^E_{22} = 112$ GPa; $c^E_{12} = 62$ GPa. The indicated elastic moduli are inherent in piezoceramics of PZT-19 type.

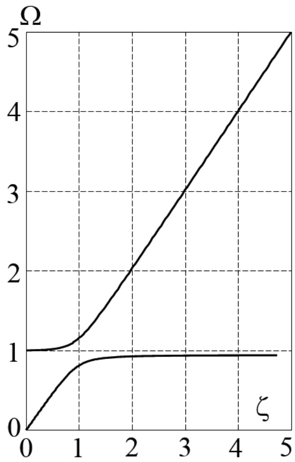


Fig. 2. Frequency spectrum of axial vibrations wavenumbers of a shell made of PZT-19 piezoelectric ceramics

The solution to equation (36), which will not contradict the physical meaning of the problem being solved, takes the following form:

$$u_z = B sin\gamma z, \tag{38}$$

where $B$ is the constant to be found.

Constant $B$ is found from the boundary conditions (12), which, under the assumptions made about the shear stress $\sigma_{\rho z}$ are reduced to the condition $\sigma_{zz}|_{z=\pm h/2} = 0$. By substituting expressions (35) and (38) into relation (32) and equating the obtained result to zero at $z = \pm h/2$, we define constant $B$ as follows:

$$B = W_0(kR)\frac{\gamma}{k}\frac{\beta(k,\gamma)}{kR}, \tag{39}$$

where $W_0(kR) = \frac{e^*_{12}C_1}{c_{22}\chi^\varepsilon_{11}\left[1-(kR)^2\right]}$; $\beta(k,\gamma) = \frac{1-(kR)^2-c_{12}/c_{22}}{\cos(\gamma h/2)}$.

After defining constant $B$ the radial component of the material particles' displacement vector can be found:

$$u_\rho = W_0(kR)[1-\varepsilon(k,\gamma)cos\gamma z], \tag{40}$$

where $\varepsilon(k,\gamma) = \frac{c_{12}\left[1-(kR)^2-c_{12}/c_{22}\right]}{c_{22}\left[1-(kR)^2-(c_{12}/c_{22})^2\right]\cos(\gamma h/2)}$.

After substituting the found values of the displacement vector components into equation (26), we obtain:

$$\frac{\partial^2\Phi^{\Pi}}{\partial z^2} = \frac{e_{12}^*}{\chi_{22}^{\sigma}R^2}W_0(kR)\left[\alpha(k,\gamma)\cos\gamma z + 1\right], \quad (41)$$

where $\alpha(k,\gamma) = (\gamma/k)^2\beta(k,\gamma) - \varepsilon(k,\gamma)$.

By integrating the left and the right sides of equation (41), we obtain:

$$\frac{\partial\Phi^{\Pi}}{\partial z} = \frac{e_{12}^*}{\chi_{22}^{\sigma}R^2}W_0(kR)\left[\frac{\alpha(k,\gamma)}{\gamma}\sin\gamma z + z\right] + C. \quad (42)$$

Obviously, constant $C = 0$, hence $\partial\Phi^{\Pi}/\partial z = E_z^{\Pi} = 0$ while $z = 0$.

Let us represent the first polarization potential derivative $\Phi^{\Pi}$, as the sum of an infinite series of trigonometric functions $\sin k_n x$, i.e.:

$$\frac{\partial\Phi^{\Pi}}{\partial z} = \sum_{n=1}^{\infty} A_n \sin k_n z, \quad (43)$$

where $A_n$ denotes expansion factors; $k_n$ denotes the dimensional parameter.

Since condition (27) is obligatory, the requirement should be observed:

$$\pm sin\left(\frac{1}{2}k_n h\right) = 0,$$

whence it follows that the expansion parameter $k_n = 2\pi n/h$, where $n = 1, 2, 3, \ldots$ is an element in a series of natural numbers. Obviously, the equality must be satisfied:

$$\frac{\partial\Phi^{\Pi}}{\partial z} = \sum_{n=1}^{\infty} A_n \sin k_n z, \quad (44)$$

where $A_n$ is the weight coefficient, or the expansion factor, to be determined.

Therefore, it is possible to prove that:

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \sin\left(\frac{2\pi n}{h}z\right)\sin\left(\frac{2\pi m}{h}z\right)dz = \begin{cases} 0\,\forall\, n \neq m, \\ \frac{h}{2}\ while\ n = m. \end{cases}$$

Taking into account the integral above, we shall determine the expansion coefficients (43) as follows:

$$A_n = \frac{(-1)^n}{\pi n}\frac{e_{12}^* h W_0(kR)}{\chi_{22}^{\sigma}R^2}\times$$
$$\times\left[\frac{2(\pi n)^2}{(\gamma h)^2 - (2\pi n)^2}\cdot\frac{\sin(\gamma h/2)}{(\gamma h/2)} - 1\right]. \quad (45)$$

Further we shall integrate the left and right sides of expansion (43) and note down the general expression for calculating the electric potential $\Phi^{\Pi}$:

$$\Phi^{\Pi} = -\frac{h}{2\pi}\sum_{n=1}^{\infty}\frac{A_n}{n}\cos\left(\frac{2\pi n}{h}z\right) + \Phi_0, \quad (46)$$

where $\Phi_0$ is a constant.

Constant $\Phi_0$ shall be derived from the fact that $\Phi^{\Pi}$ is the electric field potential, which is formed by perturbations of the stress-strain state of the cylindrical shell in the axial direction. With $z = 0$ such perturbations are absent, hence, we may assume that:

$$\Phi^{\Pi}(0) = -\frac{h}{2\pi}\sum_{n=1}^{\infty}\frac{A_n}{n} + \Phi_0 = 0.$$

From the equality above, it follows that

$$\Phi_0 = \frac{h}{2\pi}\sum_{n=1}^{\infty}\frac{A_n}{n},$$

and expression (46) will be formulated as follows

$$\Phi^{\Pi} = -\frac{h}{2\pi}\sum_{n=1}^{\infty}\frac{A_n}{n}\left[\cos\left(\frac{2\pi n}{h}z\right) - 1\right]. \quad (47)$$

The electric potential $\tilde{\Phi}^{\Pi}$, averaged over the height of the shell, is established from relation (47) as follows:

$$\tilde{\Phi}^{\Pi} = \frac{h}{2\pi}\sum_{n=1}^{\infty}\frac{A_n}{n},$$

which is followed by determining constant $C_2$ from condition (28):

$$C_2 = U_0 - \frac{h}{2\pi}\sum_{n=1}^{\infty}\frac{A_n}{n} =$$
$$= U_0 - K^2\frac{h^2 C_1}{2\pi^2 R^2\left[1-(kR)^2\right]\chi_{11}^{\varepsilon}}\times$$
$$\times\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left[\frac{(2\pi n)^2}{(\gamma h)^2 - (2\pi n)^2}\frac{\sin(\gamma h/2)}{(\gamma h/2)} - 1\right], \quad (48)$$

where $K^2 = (e_{12}^*)^2/(\chi_{22}^{\sigma}c_{22})$ is the square of the electromechanical coupling coefficient of the shell material for the axial component of axisymmetric vibrations.

By substituting expression (48) into boundary conditions (29), we shall obtain the relation for calculating the constant $C_1$. Self-evident calculations provide the following definition:

$$C_1 = \frac{U_0\chi_{11}^{\varepsilon}\left[1-(kR)^2\right]}{\ln(1+\alpha/R)\left[1-(kR)^2\right] + K^2 D(\omega,\Gamma)}, \quad (49)$$

where

$$D(\omega,\Gamma) = \frac{h^2}{2\pi^2R^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\times$$
$$\times\left[\frac{(2\pi n)^2}{(\gamma h)^2 - (2\pi n)^2}\frac{\sin(\gamma h/2)}{(\gamma h/2)} - 1\right]$$

is the additive property, which is related to the connectivity of elastic and electric fields inside the volume of an oscillating piezoceramic shell. The value of the additive property depends on frequency $\omega$ and geometric parameters of the shell. It is obvious that $|D(\omega,\Gamma)| < 1$ for any finite value of the dimensionless quantity $\gamma h$ and $h/R < 1$. Since $K^2 < 1$, product $K^2 D(\omega,\Gamma)$ always constitutes much less than one for shells of small height, when inequality $h/R < 1$, is satisfied. Taking this circumstance into account, formula (49) for calculating constant $C_1$ will be recorded as follows in the first approximation:

$$C_1 = U_0 \chi_{11}^{\varepsilon} / ln(1+\alpha/R). \tag{50}$$

After determining constants $C_1$ and $C_2$ we can say that the stress-strain and electrical state of a cylindrical shell of finite height is defined in full accordance with the fundamental mechanic principles, i.e., with the second and third Newton's laws in differential form, and satisfies the condition (22) of the absence of free carriers of electricity within the volume of the deformable piezoceramics. The latter implies that the problem of harmonic axisymmetric vibrations of a piezoelectric shell of finite height has been completely solved.

Next, we shall calculate electrical impedance $Z_{el}(\omega)$ of the vibrating shell. The incontrovertible fact is:

$$Z_{el}(\omega) = -U_0/I, \tag{51}$$

where the negative value is explained by a decrease in the potential with increasing numerical values of the radial coordinate (Fig. 1); $I$ denotes the electric current amplitude in the conductors, which is supplied to the electroded surfaces on the cylindrical shell. The amplitude of the current $I$ and the amplitude of the electric charge $Q$ on the electroded surface of the shell are related to each other:

$$I = -i\omega Q. \tag{52}$$

The amplitude of the electric charge $Q$ shall be established through the amplitude value of the radial component $D_\rho = D_\rho^* + D_\rho^\Pi$ of the electric induction vector according to the following formula:

$$Q = 2\pi R \int_{-h/2}^{h/2} \left(D_\rho^* + D_\rho^n\right) dz. \tag{53}$$

Hence $D_\rho^* = C_1/R$ and $D_\rho^\Pi = e_{12}^*(u_\rho/R + \partial u_z/\partial z)$, then, substituting these quantities into integral (53), we obtain the following relation for calculating electric charge $Q$:

$$Q = C_0^{\varepsilon} U_0 \Psi(\omega,\Pi), \tag{54}$$

where $C_0^{\varepsilon} = 2\pi h \chi_{11}^* / ln(1+\alpha/R)$ is the static electrical capacitance of a cylindrical shell with fully electroded lateral surfaces; $\Psi(\omega,\Pi)$ is the function depending on the frequency $\omega$ and the set $\Pi$ of geometric, physical and mechanical parameters, the numerical values of which are calculated by the following formula:

$$\Psi(\omega,\Pi) = \frac{K^2}{\left[1-(kR)^2\right]} \left[\alpha(k,\gamma) \frac{\sin(\gamma h/2)}{(\gamma h/2)} + 1\right] + 1. \tag{55}$$

Symbol $K^2 = (e_{12}^*)^2/(c_{22}\chi_{11}^{\varepsilon})$ denotes the square of piezoceramics electromechanical connection for the radial component of axisymmetric vibrations of a cylindrical shell.

After calculating electric charge $Q$, we shall record the final form of the expression for calculating the electric impedance of a cylindrical piezoceramic shell of finite height:

$$Z_{el}(\omega) = 1/[i\omega C_0^{\varepsilon} \Psi(\omega,\Pi)]. \tag{56}$$

It follows from definition (56) that at $e_{12}^* = 0$, i.e., when the dielectric between the cylindrical electrodes does not possess piezoelectric properties, function $\Psi(\omega,\Pi)$ =1 and expression (56) both take the trivial form $Z_{el}(\omega) = 1/(i\omega C_0^{\varepsilon})$, that is, the electrical impedance of the shell is equal to the capacitive reactance of a capacitor with electrical capacitance $C_0^{\varepsilon}$. When the shell material possesses piezoelectric properties, the mechanism through which its electrical impedance depends on the frequency is rather complex.

## 4 Discussion and experimental confirmation of simulation results

Figure 3 demonstrates frequency-dependent deviations in the modulus of the electrical impedance of a vacuum-vibrating shell made of PZT-19 piezoelectric ceramics with merit of $Q = 80$.

While plotting the curves shown in Fig. 3, the following numerical data [24, 25] were used: $c_{11}^E = 106$ GPa; $c_{12}^E = 62$ GPa; $c_{22}^E = 112$ GPa; $e_{11} = 18$ C/m$^2$; $e_{12} =$ -7 C/m$^2$; $\chi_{11}^{\varepsilon} = 1000\ \chi_0$; $\chi_0 = 8,85 \cdot 10^{-12}$ F/m. Shell dimensions: $\alpha = 1$ mm; $R = 14$ mm and $h = 20$ mm. Electrical potential difference constitutes $U_0 = 1$ V. The solid bold curve shows the results of calculating modulus $Z_{el}(\omega)$ based on formula (56). Thin curves show frequency-dependent dynamics in radial (solid line; $z = 0$) and axial (dashed line; $z = h/2$) displacements of material particles in a piezoceramic shell.

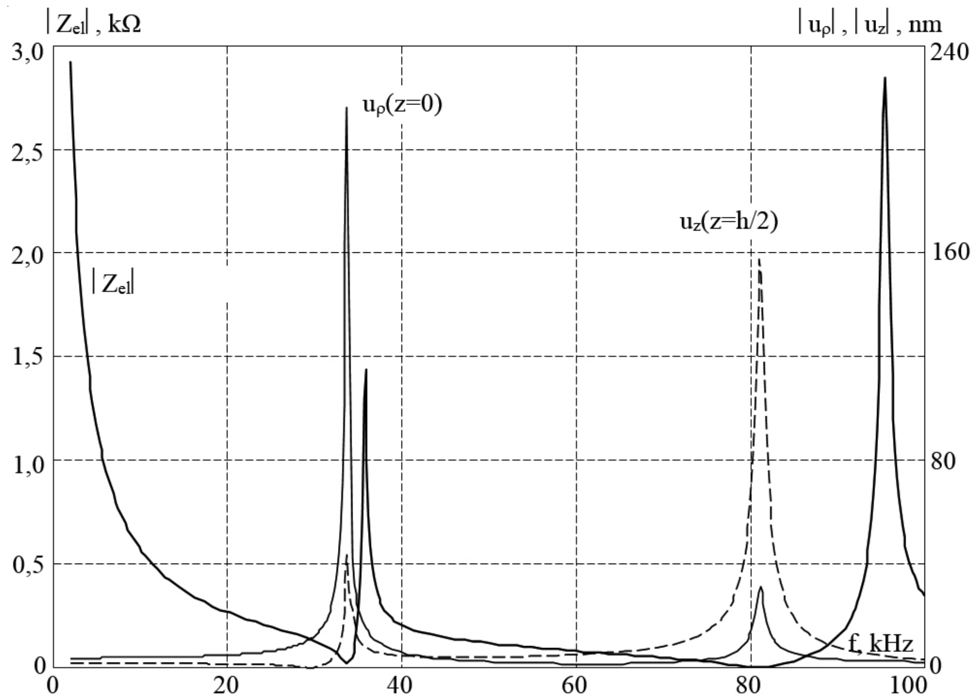


Fig. 3. Frequency-dependent dynamics of electrical impedance (bold solid curve) and components of the material particle displacement vector (solid and dashed thin curves) of an oscillating piezoceramic shell made of PZT-19 piezoceramics

With resonance frequencies, the displacements reach maximum values and, as a consequence, maximum values are taken by the deformations and the radial component of the electric induction vector, which is equivalent to a sharp increase in the levels of polarization charges. The latter induces a sharp increase in the current of free electricity carriers in the conductors, that connect the generator of the electrical potential difference and the electroded surfaces of the piezoceramic shell. While the amplitude of the electrical potential difference is constant, an increase in the electric current only occurs when the electrical impedance in the electric current circuit decreases. Thus, within the electromechanical resonance frequencies, a sharp decrease in the electrical impedance of both the vibrating piezoelectric element and the cylinder shell is observed. When passing the resonance frequency in the direction of increasing frequencies, the deformation indications alter along the height of the shell. In this case, mutual compensation of electrical and elastic components of the electrical induction within the volume of the deformable piezoelectric is observed. Consequently, the level of the resulting electrical polarization subsides sharply and the electric current in the conductors lessens. The latter corresponds to a sharp increase in the electrical impedance of the oscillating piezoelectric sample. Since the piezoceramic element consumes the maximum amount of energy from the oscillation source within the resonance frequency, which constitutes the main physical content of the resonance phenomenon, at frequencies where the impedance rises sharply, the consumption drops to minimum. The described phenomenon provides the grounds for the frequencies to be considered electromechanical antiresonance frequencies.

Figure 4 illustrates the calculations (solid line) and experimentally obtained data (dashed line) curves indicating frequency dependence of the electrical impedance modulus of the oscillating piezoelectric shell.

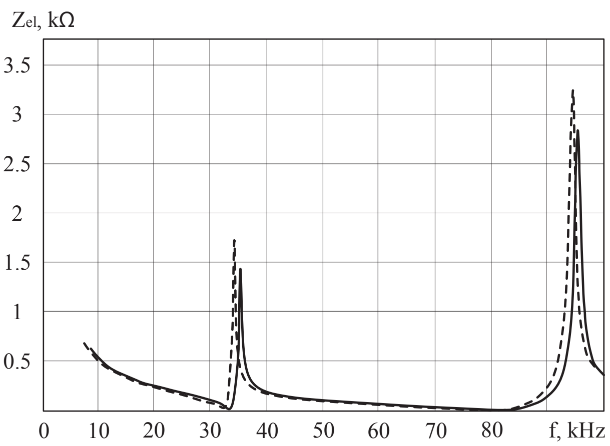


Fig. 4. Calculations (solid line) and experimentally obtained (dashed line) data for the electrical impedance of the oscillating piezoelectric shell

The calculations are based on the parameters that have been previously used for calculating curve $|Z_{el}(\omega)|$, can be found in Fig. 3. Identical shell sizes have been selected both for the calculations and the experiment, namely $\alpha = 10^{-3}$ m; $R = 14{\cdot}10^{-3}$ m and

$h = 20{\cdot}10^{-3}$ m. The electrical impedance values of the piezoceramic shell in kiloohms are plotted along the ordinate axis, and the current frequency normalized to the frequency of the first resonance is plotted along the abscissa axis.

The nature of dynamics in both curves in Fig. 4 coincides with an accuracy of 13-17% within a sufficient range of frequencies. In other words, expression (56) is a mathematical model of the electrical impedance of the oscillating shell, the obtained mathematical expression sufficiently corresponds to the real object and the processes occurring in it.

Thus, the evidence from this study suggests that the approach we have proposed here to mathematical modelling the physical state of a cylindrical shell of finite length provides reliable results that correspond adequately to the experimental data. Therefore, this method can be recommended for calculating the transmission characteristics of electroacoustic transducers in the modes of emitting and reception of ultrasonic waves.

# Conclusions

The problem of building a mathematical model of cylindrical piezoelectric transducers is proposed in the research, the solution of which proves interdependence between the electrical impedance of the shell, the cyclic frequency and the set of geometric, physical, mechanical, and electrical transducer parameters.

The obtained analytical dependences, which can be applied to determine the electrical impedance and amplitude values of electric current and electric charge on the electroded surface of a cylindrical piezoelectric shell under the inverse piezoelectric effect, enable us to completely solve the problem of harmonic axisymmetric oscillations in the piezoelectric shell of finite height.

The current study has found evidence of a frequency-dependent change of electric impedance and components of the displacement vector for material particles in the oscillating piezoelectric transducer. That allowed us to detect a sharp decrease in such impedance (2.6-5 times), both for the oscillating piezoelectric element of PZT-19 piezoceramics in general, and cylindrical shells in particular, with frequencies of electromechanical resonances ranging within 33-35 kHz and 82 kHz respectively.

Another important finding was that comparing mathematically calculated and experimentally obtained values of the electrical impedance of the oscillating cylindrical piezoceramic shell indicated a high convergence between the data. Hence, the discrepancy between the simulation results and the experimentally obtained data at the same values of the operating frequency within the frequency range up to 100 kHz did not exceed 17%.

The proposed paper presents the results related to the experimental scientific and technical project "Developing a highly efficient mobile ultrasonic system to intensify the extraction process while manufacturing concentrated functional beverages for combatants", that is being implemented by the authors (state registration entry number: 0121U109660, entry date: 12.03.2021).

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## Математичне моделювання циліндричних п'єзоелектричних перетворювачів для електроакустичних пристроїв

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В статті наводяться послідовність та результати розроблення і дослідження математичних моделей циліндричних п'єзоелектричних перетворювачів, які активно використовуються в електро- та гідроакустиці (наприклад, у пристроях для випромінювання та прийому акустичних коливань у повітряному чи водному середовищі). Відмінними особливостями розроблюваних моделей є те, що встановлені за їх допомогою залежності, представляють собою математичний опис електроакустичного зв'язку між хвильовими полями на різних ділянках порожнистого п'єзокерамічного перетворювача циліндричної форми.

Отримані внаслідок проведеного моделювання аналітичні залежності дозволяють встановити електричний імпеданс та амплітудні значення струму і електричного заряду на електродованій поверхні п'єзоелектричного перетворювача (циліндричної п'єзоелектричної оболонки кінцевої висоти) за умов зворотного п'єзоелектричного ефекту, і тим самим отримати повний розв'язок задачі про гармонійні вісесиметричні коливання такого перетворювача.

Приведені результати отримані з використанням розробленої математичної моделі для циліндричних оболонкових перетворювачів із п'єзоелектричної кераміки типу ЦТС. Встановлена частотно-залежна зміна електричного імпедансу і компонентів вектора зміщення матеріальних частинок п'єзоперетворювача, що коливається: так, на частотах електромеханічних резонансів (порядку 33-35 кГц та 82 кГц) спостерігається різке зменшення такого імпедансу (у 2,6-5 разів). Проведені порівняння математично розрахованих та експериментально отриманих значень електричного імпедансу циліндричної п'єзокерамічної оболонки, що коливається, показали високу збіжність між ними (розбіжність між результатами моделювання та експериментально отриманими даними за однакових значень робочої частоти в діапазоні до 100 кГц не перевищувала 17%).

*Ключові слова:* п'єзоелектричний перетворювач; акустоелектроніка; математична модель; імпеданс; циліндрична оболонка

## Математическое моделирование цилиндрических пьезоэлектрических преобразователей для электроакустических устройств

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В статье приводятся последовательность и результаты разработки и исследования математических моделей цилиндрических пьезоэлектрических преобразователей, активно используемых в электро- и гидроакустике (например, в устройствах для излучения и приема акустических колебаний в воздушной или водной среде). Отличительными особенностями разрабатываемых моделей является то, что установленные с их помощью зависимости представляют собой математическое описание электроакустической связи между волновыми полями на разных участках полого пьезокерамического преобразователя цилиндрической формы.

Полученные в результате проведенного моделирования аналитические зависимости позволяют установить электрический импеданс и амплитудные значения тока и электрического заряда на электродированной поверхности пьезоэлектрического преобразователя (цилиндрической пьезоэлектрической оболочки конечной высоты) в условиях обратного пьезоэлектрического эффекта, и тем самым обеспечить полное решение задачи о гармонических осесимметричных колебаниях такого преобразователя.

Приведенные результаты получены с использованием разработанной математической модели для цилиндрических оболочечных преобразователей из пьезоэлектрической керамики типа ЦТС. Установлено частотнозависимое изменение электрического импеданса и компонентов вектора смещения материальных частиц колеблющегося пьезопреобразователя: так, на частотах электромеханических резонансов (порядка 33-35 кГц и 82 кГц) наблюдается резкое уменьшение такого импеданса (в 2,6-5 раз). Проведенные сравнения математически рассчитанных и экспериментально полученных значений электрического импеданса колеблющейся цилиндрической пьезокерамической оболочки показали высокую сходимость между ними (расхождение между результатами моделирования и экспериментально полученными данными при одинаковых значениях рабочей частоты в диапазоне до 100 кГц не превышало 17%).

*Ключевые слова:* пьезоэлектрический преобразователь; акустоэлектроника; математическая модель; импеданс; цилиндрическая оболочка