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Error Probability of a Multipath Communication Channel With Inaccurate Estimation of the Impulse Characteristic of Such Channel

Pochernyaev V. N.[1], *Syvkova N. M.*[1], *Mahomedova M. S.*[2]

[1]National Academy of the Security Service of Ukraine, Kyiv, Ukraine
[2]Kyiv Professional College of Communications, Kyiv, Ukraine

E-mail: *natsivonat@gmail.com*

The possibility of error in the case of imprecise estimation of the impulse characteristic of a multipath channel is investigated in the article. The study was carried out for a multipath communication channel of discrete channel models, which corresponds to the mapping of a continuous two-path channel onto a discrete channel with an impulse characteristic. Numerical results of calculations are obtained, which can be used to calculate the error probability in the cases indicated in the article, which differ in the ratio of the amplitudes of the interfering beams. Formulas for calculating probability integrals are presented in the article. The influence of the accuracy of estimating the components of the impulse characteristic vector on the error probability in a two-beam channel with constant parameters is studied. The results of a study of the influence of the communication channel model on the error probability for different models of the communication channel for 8PSK modulation are also presented. With the "deterioration" of the type of the channel impulse characteristic (an increase in the number of channel amplitude-frequency characteristic dips in the signal band and an increase in their depth), the decrease in the error probability characteristic due to begins at lower estimation error values. The results of studying the error probability of 8PSK and 64QAM signals in a single-beam channel with Rayleigh fading are presented. It is determined that the influence of errors becomes more noticeable with an increase in the signal-to-noise ratio in the channel and with an increase in the number of dips in the amplitude-frequency characteristic of the channel in the signal band and an increase in their depth.

Keywords: error probability; impulse characteristic; normalized standard deviation; communication channel model; multipath communication channel; one-path channel with Rayleigh fading; two-path channel with constant parameters

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Introduction

The components of the sampling vector of the impulse characteristic (IC) of the multipath channel are independent random variables, and the errors of their estimates are considered to be mutually independent of each other and of the present values. This assumption is valid if the optimal filter matched with the distorted multipath channel is used at the receiver input, with subsequent decoration of original samples. Such a filter is very difficult to implement, and instead of it, in practice, a filter is used that is consistent not with the received signal, the distorted channel, but with the transmitted one. Usually a raised cosine filter or some kind of it is used [1, 2].

1 Literature analysis

In the article [1], two options for combating intersymbol interference in multipath communication channels were studied: the use of an equalizer and time orthogonal multiplexing. The use of equalizers and orthogonal time multiplexing is compared, and the possibility of using equalizers to create controlled intersymbol interference as a way to deal with such interference that occurs in the troposcatter communication channel is shown.

The article [2] considers the system of control, monitoring and diagnostics of a combined radio engineering system from the standpoint of the theory of complex systems. Specific examples of promising combined radio engineering systems operating via multipath communication channels with the use of equalizers built according to the minimum standard deviation (MSD) criterion are given.

The expediency of using combined radio engineering systems has been repeatedly noted at the annual international conferences MILCOM [5–7].

An analysis of bibliographic sources showed that the closest in terms of the method of solving such problems are studies conducted in [4, 8–12].

In all considered cases, the readings of the IC vector of a discrete communication channel, formed by mapping a continuous multipath channel onto a discrete channel, cannot be considered independent values. Analytical calculation of the dependence of characteristics on the accuracy of estimating the channel parameters in this case, which is of practical interest, is extremely difficult due to the lack of analytical expressions.

The purpose of the article is to determine P_{error} in the case of inaccurate estimation of the IC of a multipath channel.

2 The problematic part

When considering multipath channels with Rician fading for arbitrary signals with incoherent reception, the error probability is determined through integrals of the form [3]. Arbitrary signals mean orthogonal, opposite, simplex, and others. The problem is that there are no general functions for P_{error} for arbitrary signals. Therefore, one should consider integrals of functions of the form:

$$\xi^n[\pm 1 - erf(\alpha\xi + \beta)]^m,$$

where $n = 0, 1, 2, \ldots$, $m = 1, 2, 3, \ldots$, $\alpha \neq 0$ and β – continuous parameters, ξ – independent variable, $erf(u) = \frac{2}{\sqrt{\pi}} \int_0^u exp(-t^2)dt$ – integral of probability (α, β and ξ can be both real and complex independently of each other). The differences $[\pm 1 - erf(\alpha\xi + \beta)]$ are the complements of $erf(\alpha\xi + \beta)$ to its possible boundary values ± 1 in accordance with the sign choice rule for the cases $R_e(\alpha\xi) \to \pm\infty$ as ξ approaches infinity of the given trajectory on the complex planes.

3 Main part

The accuracy of estimates of the IC communication channel will be characterized by the value of the normalized MSD of the estimate ε_h^2.

Outchannel is carried based on the model of a discrete channel corresponding to the mapping of a continuous two-path channel onto a discrete channel with an IC of the form:

$$h_n = \alpha \left[a_1\delta_n + a_2 e^{j\varphi} \sum_{k=-1}^{K} \sin(n - k - \tau/T) \right], \qquad (1)$$

where $n = 1, \ldots$; a_1 and a_2 – real ray amplitudes; φ – random phase shift between beams; τ/T – relative delay in units of symbol length T between beams; $K \gg 1$. The value of K characterizes the accuracy of mapping a continuous channel to a discrete. The value α is a normalization constant chosen from the condition:

$$\sum_{n=1}^{L} |h_n|^2 = 1,$$

where L – number of rays.

We will specifically consider two options that differ in the ratio of the amplitudes of the interfering beams: 1a) $a_1 = a_2 = 1/\sqrt{2}$ and 1b) $a_1 = 2a_2 = 2/\sqrt{3}$. These options are among the most difficult to evaluate, for example, option 1a due to the equality of the amplitudes of the interfering beams, leading to channel frequency response dips to zero.

Let's move on to the analytical part of the work.

We present the formula for indefinite integrals of the above functions for arbitrary $n \geq 0$ and its separate forms for $\beta = 0$ for even $n = 2k$ and odd $n = 2k + 1$ ($m = 0, 1, 2, \ldots$):

$$\begin{aligned} \int \xi^n[\pm 1 - erf(\alpha\xi + \beta)]d\xi &= \pm \frac{\xi^{n+1}}{n+1} - \int \xi^n erf(\alpha\xi + \beta)d\xi = \\ &= [\pm 1 - erf(\alpha\xi + \beta)] \left[\frac{\xi^{n+1}}{n+1} - \frac{n!}{(-\alpha)^{n+1}} \sum_{k=0}^{[n/2]} \frac{\beta^{n+1-2k}}{4^k k!(n+1-2k)!} \right] + O\left(N^{\frac{3}{2}}\right), \end{aligned} \qquad (2)$$

$$\int \xi^{2k}[\pm 1 - erf(\alpha\xi)]d\xi = \frac{\xi^{2k+1}}{2k+1}[\pm 1 - erf(\alpha\xi)] - \frac{k! exp(-\alpha^2\xi^2)}{(2k+1)\sqrt{\pi}} \sum_{j=0}^{k} \frac{\xi^{2j}}{j!\alpha^{2k+1-2j}} + O\left(N^{\frac{3}{2}}\right), \qquad (3)$$

$$\begin{aligned} \int \xi^{2k+1}[\pm 1 - erf(\alpha\xi)]dz &= \frac{\xi^{2k+2}}{2k+2}[\pm 1 - erf(\alpha\xi)] + \frac{(2k+1)!}{(k+1)!} \times \\ &\times \left[\frac{erf(\alpha\xi)}{(2\alpha)^{2k+2}} - \frac{exp(-\alpha^2\xi^2)}{\sqrt{\pi}} \times \sum_{k=0}^{m} \frac{(j+1)!\xi^{2j+1}}{4^{k-j}(2j+2)!\alpha^{2k+1-2j}} \right] + O\left(N^{\frac{3}{2}}\right). \end{aligned} \qquad (4)$$

In formula (2), $[n/2]$ is the integer part of the number $n/2$. These expressions are obtained using formulas for integrating the probability integral itself and its derivatives with a power function. The general formula (2) is written here (for convenience) with the addition of an integral constant equal to $\mp n!F(n,\beta)/(-\alpha)^{n+1}$. The simpler formulas (3) and (4) are given, taking into account the greatest practical interest in the corresponding cases.

Let us pass to integrals of the form $\xi^n[\pm 1 - erf(\alpha\xi+\beta)]^{m_l}$ for $m_l > 1$.

These expressions are easily obtained using integrals of derivatives of statistics functions with square and cube of the probability integral. As a result, we arrive at cumbersome formulas, which, taking into account the physical conditions of the problem, can be represented as:

$$\int \xi^n[\pm 1 - erf(\alpha\xi+\beta)]^2 d\xi = [\pm 1 - erf(\alpha\xi+\beta)]^2 \times \left[\frac{\xi^{n+1}}{n+1} - \frac{n!}{(-\alpha)^{n+1}} \sum_{k=0}^{[n/2]} \frac{\beta^{n+1-2k}}{4^k k!(n+1-2k)!}\right] + O\left(N^{\frac{3}{2}}\right), \quad (5)$$

$$\int \xi^{2m}[\pm 1 - erf(\alpha\xi)]^2 d\xi = \frac{\xi^{2m+1}}{2m+1}[\pm 1 - erf(\alpha\xi)]^2 - \frac{m!}{(2m+1)\sqrt{\pi}} \left\{ 2exp(-\alpha^2\xi^2)[\pm 1 - erf(\alpha\xi)] \times \sum_{k=0}^{m} \frac{\xi^{2k}}{k!\alpha^{2m+1-2k}} + \right.$$
$$\left. + \frac{\sqrt{2}}{\alpha^{2m+1}} erf(\sqrt{2}\alpha\xi) \sum_{k=0}^{m} \frac{(2k)!}{8^k (k!)^2} - \frac{exp(-2\alpha^2\xi^2)}{2\sqrt{\pi}} \sum_{k=1}^{m} \frac{(2k)!}{(k!)^2} \sum_{l=1}^{k} \frac{(l-1)!\xi^{2l-1}}{8^{k-l}(2l-1)!\alpha^{2m+2-2l}} \right\} + O\left(N^{\frac{3}{2}}\right), \quad (6)$$

$$\int \xi^{2m+1}[\pm 1 - erf(\alpha\xi)]^2 d\xi = [\pm 1 - erf(\alpha\xi)]^2 \left[\frac{\xi^{2m+2}}{2m+2} - \frac{(2m+1)!}{(m+1)!(2\alpha)^{2m+2}}\right] -$$
$$- \frac{(2m+1)!}{(m+1)!\sqrt{\pi}} \left\{ exp[-(\alpha^2\xi^2)][\pm 1 - erf(\alpha\xi)] \times \sum_{k=0}^{m} \frac{k!\xi^{n-2k}}{4^{m-k}(2k+1)!\alpha^{2m+1-2k}} \right\} + O\left(N^{\frac{3}{2}}\right). \quad (7)$$

The results obtained in the form of formulas (1)-(7) can be used to calculate P_{error} in the indicated cases. Numerical calculation results are shown in Figures 1, 2 and 3.

The error probability curves are shown in Fig. 1 for an inaccurate estimate of the component of the vector IC.

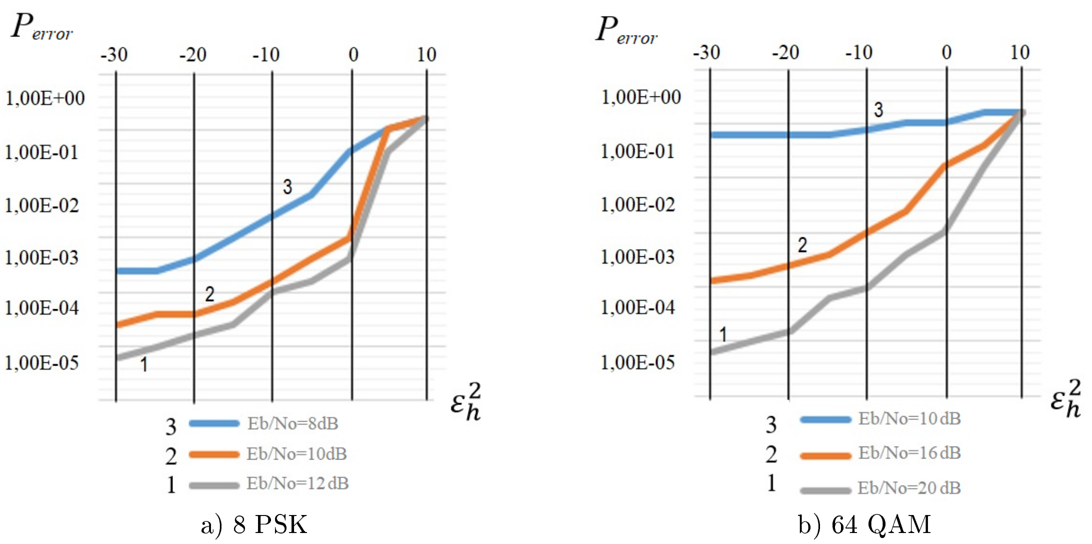

Fig. 1. Influence of the accuracy of estimating the components of the vector IC on P_{error} in a two-beam channel with constant parameters (model 1b)

Curves P_{error} for different communication channel models for 8PSK modulation are shown in Fig. 2. It can be seen that with the "deterioration" of the type of channel IC (an increase in the number of channel frequency response dips in the signal band and an increase in their depth), the decrease in the characteristic P_{error} due to estimation errors begins at lower error values.

Figure 3 shows the curves P_{error} of 8PSK and 64QAM signals in a single-path channel with Rayleigh fading. As can be seen from Fig. 3, there is a certain zone of estimation errors, within which their influence on the characteristics of immunity to interference is insignificant, and when leaving it, a sharp increase in the value of P_{error} is observed. Moreover, the errors in estimating ε_h^2 in the larger direction are more critical.

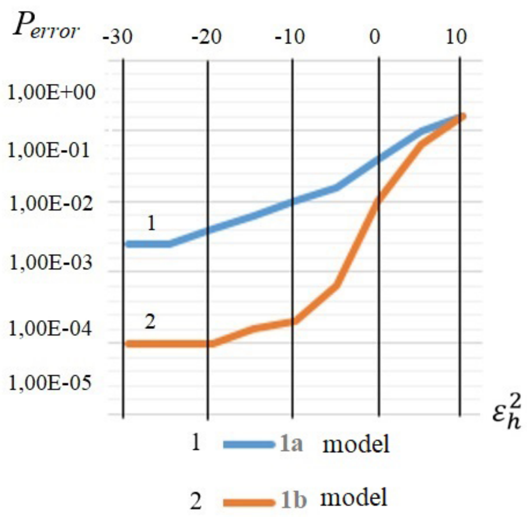

Fig. 2. The impact of the communication channel model on P_{error}

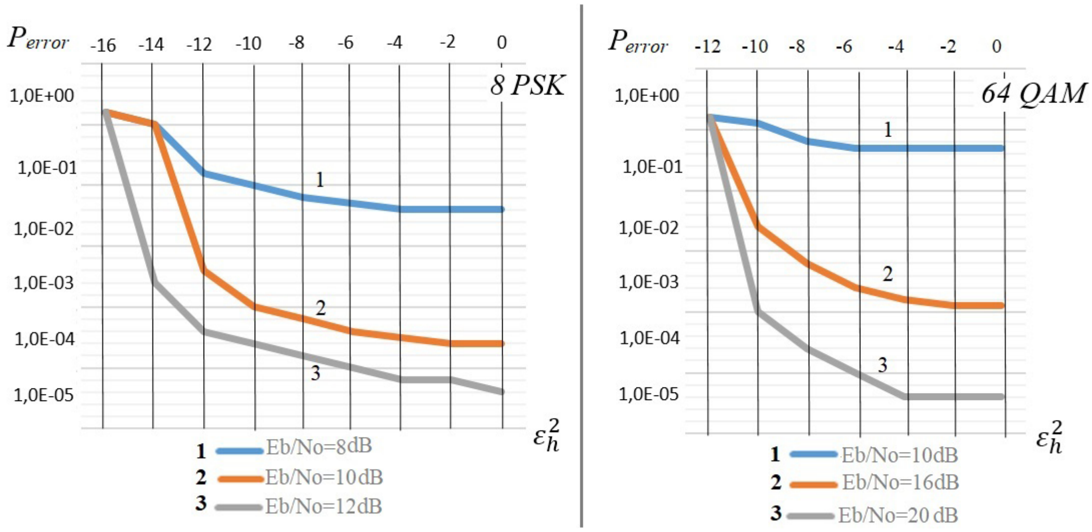

Fig. 3. Influence of the estimation accuracy of the vector component IC on P_{error} in a single-beam channel with Rayleigh fading

Conclusion

Dual-use communication systems, which include troposcatter systems, operate over a multipath communication channel. Such a troposcatter communication channel is characterized by signal fading and intersymbol interference. Note that the impact of errors becomes more noticeable when: a) increasing the signal-to-noise ratio in the channel; b) "deterioration" of the type of channel IC (an increase in the number of dips in the frequency response of the channel in the signal band and an increase in their depth).

Modems in combinations of troposcatter-radiorelay communication systems use various types of PSK and QAM. The requirements for the troposcatter components of combined systems in terms of error probability are very high, at the level $P_{error} = 10^{-5}$ degree. Therefore, the graphs in Fig. 1, 2, 3 have practical applications. Since the requirements for troposcatter communication systems in terms of error probability increase, theoretically obtaining the results (formulas 1-7) will allow determining the error probability for more complex modulation types than those indicated in the article.

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Ймовірність помилки багатопроменевого каналу зв'язку при неточному оцінюванні імпульсної характеристики такого каналу

Почерняєв В. М., Сивкова Н. М., Магомедова М. С.

У статті досліджується ймовірність помилки у випадку неточного оцінювання імпульсної характеристики багатопроменевого каналу. Дослідження проведено для багатопроменевого каналу зв'язку на базі моделі дискретного каналу, що відповідає відображенню безперервного двопроменевого каналу на дискретний канал з імпульсною характеристикою. Отримані результати розрахунків, які можна використовувати для розрахунку ймовірності помилки у зазначених в роботі випадках, що відрізняються співвідношенням амплітуд інтерферуючих променів. В роботі наведені формули для розрахунків інтегралів ймовірності та проведено дослідження впливу точності оцінки компонентів вектора імпульсної характеристики на ймовірність помилки у двопроменевому каналі з постійними параметрами. Також наведено результати дослідження впливу моделі каналу зв'язку на ймовірність помилки при різних моделях каналу зв'язку для модуляції 8PSK та величині відношення сигнал/шум 8 дБ. При «погіршенні» виду імпульсної характеристики каналу (збільшенні кількості провалів амплітудно-частотної характетристики каналу у смузі сигналу та зростанні їх глибини) зниження характеристики ймовірності помилки починається при менших значеннях оцінки помилок. Надані результати дослідження ймовірності помилки сигналів 8PSK та 64QAM в однопроменевому каналі з релеєвськими завмираннями. Визначено, що вплив помилок стає помітнішим при збільшенні відношення сигнал/шум у каналі та при збільшенні кількості провалів амплітудно-частотної характеристики каналу у смузі сигналу та зростанні їх глибини.

Ключові слова: ймовірність помилки; імпульсна характеристика; інтеграл; нормоване середньоквадратичне відхилення; модель каналу зв'язку; багатопроменевий канал зв'язку; однопроменевий канал з релеєвськими завмираннями; двопроменевий канал з постійними параметрами