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Scattering of Plane Electromagnetic Waves by Lattices of Spherical Dielectric Resonators with Degenerate Lower Types of Natural Oscillations

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The problem of scattering of plane electromagnetic waves on lattices of spherical dielectric resonators (DRs) with low magnetic oscillations is considered. The results of theoretical calculations of complex coefficients of mutual coupling of spherical dielectric resonators in open space for cases of excitation of degenerate types of oscillations are presented. The expressions found coincide with those obtained earlier for the special case of oscillations of resonators excited along or perpendicular to the line connecting their centers. The main regularities of the change in the coupling coefficients with variations in the coordinates of the resonators in the transverse plane are considered. Analytical expressions for c-functions are found for the field of fundamental magnetic oscillations of a resonator and a plane wave in open space. On the basis of the obtained formulas, with the help of the perturbation theory, the characteristics of the scattering of plane waves on a square lattice of spherical DRs with basic degenerate magnetic oscillation types are calculated and studied. The distribution of the scattering field in the wave zone of the grating is studied for different angles of incidence. The regions of variation of the angles of incidence are determined, in which the scattering amplitude of a lattice constructed on the basis of spherical DRs differs most noticeably from DR lattices of other shapes with nondegenerate types of oscillations. The polarization characteristics of scattered waves in the far zone of the lattice are calculated. It is noted that, in contrast to the lattices of pseudorotating cylindrical DRs with the main magnetic types of oscillations, lattices based on spherical resonators are characterized by a more complex distribution of the polarization of scattered waves. In the wave zone of the lattice, scattered waves of all three types of polarization, linear, circular, elliptical, can be observed. The obtained results significantly expand the possibilities of developers, since allow us to create electrodynamic models of lattices, as well as other devices in the millimeter and infrared ranges, built on the basis of the use of spherical resonators with oscillations of the main types. Such lattices can be used in antennas, passive reflectors, and other devices of modern optical communication systems.

Keywords: dielectric resonator; lattice; coupling coefficient; c-function; scattering amplitude

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Introduction

Today, spherical dielectric resonators (DRs) are being actively studied as one of the main resonant elements of various devices in the optical and infrared wavelength ranges. The reason for this is the existence of relatively simple methods for manufacturing controlled dielectric samples of nanometer dimensions. The optical properties of metamaterials based on spherical microparticles [1–5, 14], various lattices [6, 12, 16, 18], and small spatial clusters known as photonic molecules [7, 15, 19] are studied. Spherical dielectric resonators with whispering gallery modes are already being used as sensors [9], in optical delay lines, in multiplexers, lasers [10,11,13,17,20], etc. In connection with the foregoing, a wide class of problems on the scattering of electromagnetic waves by spherical DR lattices remains relevant [8, 21].

Despite the fact that the fields of spherical DRs are described by relatively simple analytical expressions, the calculation of more complex structures, based on them, faces significant computational difficulties. At the same time, obtaining exact analytical solutions usually leads to cumber some computational structures [21]. Analytical modeling using perturbation theory is also associated with a number of difficulties arising from the high sensitivity of the output parameters to the relative frequencies of partial resonators in the structure, as well as the uncertainty in the choice of basis functions. However, the construction of electrodynamic models provides valuable information about the behavior of systems of coupled resonators under different scattering conditions.

The purpose of the article is to derive the necessary analytical relationships for constructing an electrodynamic model of the scattering of plane electromagnetic waves on the lattices of spherical DRs with the main magnetic types of oscillations H_{1m1} ($m = 0, \pm 1$). Analysis of wave scattering on a planar lattice of spherical DRs.

To carry out calculations using perturbation theory [23], the coefficients of mutual coupling of resonators with different types of degenerate oscillations were generalized. Calculations of c-functions describing the degree of interaction of natural oscillations with the field of a plane wave are carried out. An electrodynamic model of the process of scattering by a planar lattice of spherical DRs is constructed.

1 Coupling coefficients calculation for main oscillations of the spherical dielectric resonators

For expanding the scattered field of the lattice in terms of natural oscillations of the resonator system, it's necessary to know the coupling coefficients for arbitrary coordinates and types of oscillations [22]. Mutual coupling coefficients of the spherical dielectric microresonators in the open space are not studied in full detail. On the basis of the analytical relations [25], we have obtained formulas for the most general cases of excitation for magnetic oscillations H_{1m1}, which we will give a more symmetrical form:

$$\begin{aligned}
&\text{for } \Delta z > 2r_0: \\
&\kappa_{xx} = i\alpha_1^H(p,q)\times \\
&\times\left\{2h_0^{(2)}(k_0\Delta r)+\left[\frac{2\Delta x^2-\Delta y^2-\Delta z^2}{\Delta r^2}\right]h_2^{(2)}(k_0\Delta r)\right\}; \\
&\kappa_{yy} = i\alpha_1^H(p,q)\times \\
&\times\left\{2h_0^{(2)}(k_0\Delta r)+\left[\frac{2\Delta y^2-\Delta x^2-\Delta z^2}{\Delta r^2}\right]h_2^{(2)}(k_0\Delta r)\right\}; \\
&\kappa_{zz} = i\alpha_1^H(p,q)\times \\
&\times\left\{2h_0^{(2)}(k_0\Delta r)+\left[\frac{2\Delta z^2-\Delta x^2-\Delta y^2}{\Delta r^2}\right]h_2^{(2)}(k_0\Delta r)\right\}; \\
&\kappa_{xy} = 3i\alpha_1^H(p,q)\times\frac{\Delta x\Delta y}{\Delta r^2}h_2^{(2)}(k_0\Delta r); \\
&\kappa_{xz} = 3i\alpha_1^H(p,q)\times\frac{\Delta x\Delta z}{\Delta r^2}h_2^{(2)}(k_0\Delta r); \\
&\kappa_{yz} = 3i\alpha_1^H(p,q)\times\frac{\Delta y\Delta z}{\Delta r^2}h_2^{(2)}(k_0\Delta r).
\end{aligned} \tag{1}$$

Here the function $\alpha_n^H(p,q)$ determining coupling dependence on the dielectric parameters for a given type of resonator oscillations [22], also $p = k_1 r_0$ and $q = k_0 r_0$ are the characteristic parameters; r_0 – radius; $k_0 = \omega/c$; $k_1 = \sqrt{\varepsilon_{1r}}k_0$ are the wave numbers; ω – circular frequency; c – speed of light; ε_{1r} – dielectric permittivity of the resonator; $\Delta x = x_1 - x_2$; $\Delta y = y_1 - y_2$; $\Delta z = z_1 - z_2$; $\Delta r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$; (x_s, y_s, z_s) coordinates of the resonator centers (see Fig. 1, a); $h_n^{(2)}(z) = (\pi/2z)^{1/2}H_{n+1/2}^{(2)}(z)$ is the spherical Hankel functions of the second kind [23]. Indices in κ_{uv} denote the direction of the magnetic field at the center of each of the partial resonators in a given coordinate system, which is characteristic of one its degenerate oscillations of the magnetic type H_{1m1} $(u, v = x, y, z)$.

In a particular case $\Delta x = \Delta y = 0$, the obtained relations coincide with those found earlier [22]. As follows from (1), the formulas actually coincide with each other with an appropriate permutation of the coordinates. For example:

$$\kappa_{yy}(\Delta x, \Delta y, \Delta z) = \kappa_{xx}(\Delta y, \Delta x, \Delta z);$$

$$\kappa_{zz}(\Delta x, \Delta y, \Delta z) = \kappa_{xx}(\Delta z, \Delta y, \Delta x);$$

$$\kappa_{xz}(\Delta x, \Delta y, \Delta z) = \kappa_{xy}(\Delta x, \Delta z, \Delta y);$$

$$\kappa_{yz}(\Delta x, \Delta y, \Delta z) = \kappa_{xy}(\Delta y, \Delta z, \Delta x).$$

This becomes obvious if we take into account the maximum spatial symmetry of the shape of both resonators.

The calculated dependences of the coupling on the coordinates of the resonators are shown in Figs. 1 (b-k) for the relative permittivity of the resonators $\varepsilon_{1r} = 36$ and $k_0\Delta z = 2q$. Note here that these coupling functions are even on a plane $(k_0\Delta x, k_0\Delta y)$ for oscillations κ_{uu} (Fig. 1, b-g) and odd for oscillations κ_{uv} (see (1) and Fig. 1, h-k).

2 Calculation and analysis of c-functions

C-functions determine the degree of influence of an external field on a partial dielectric resonator. Such an interaction is determined by the distribution of the external field

$$\begin{aligned}
&\vec{E}^+ = (\vec{x}_0\cos\alpha_1 + \vec{y}_0\cos\alpha_2 + \vec{z}_0\cos\alpha_3)Ae^{-i\vec{k}_0\vec{r}}, \\
&\vec{H}^+ = (\vec{x}_0\cos\beta_1 + \vec{y}_0\cos\beta_2 + \vec{z}_0\cos\beta_3)\frac{A}{w_0}e^{-i\vec{k}_0\vec{r}}
\end{aligned} \tag{2}$$

onto the field of natural oscillations of the resonator. Here $\vec{x}_0$, $\vec{y}_0$, $\vec{z}_0$ are unit vectors directed along the axis x, y, z, respectively; $w_0 = 120\pi$ – wave resistance of open space; A – amplitude; $\vec{k}_0 = (\vec{x}_0\cos\gamma_1 + \vec{y}_0\cos\gamma_2 + \vec{z}_0\cos\gamma_3)k_0$ – wave vector; $\vec{r} = (\vec{x}_0 x + \vec{y}_0 y + \vec{z}_0 z)$.

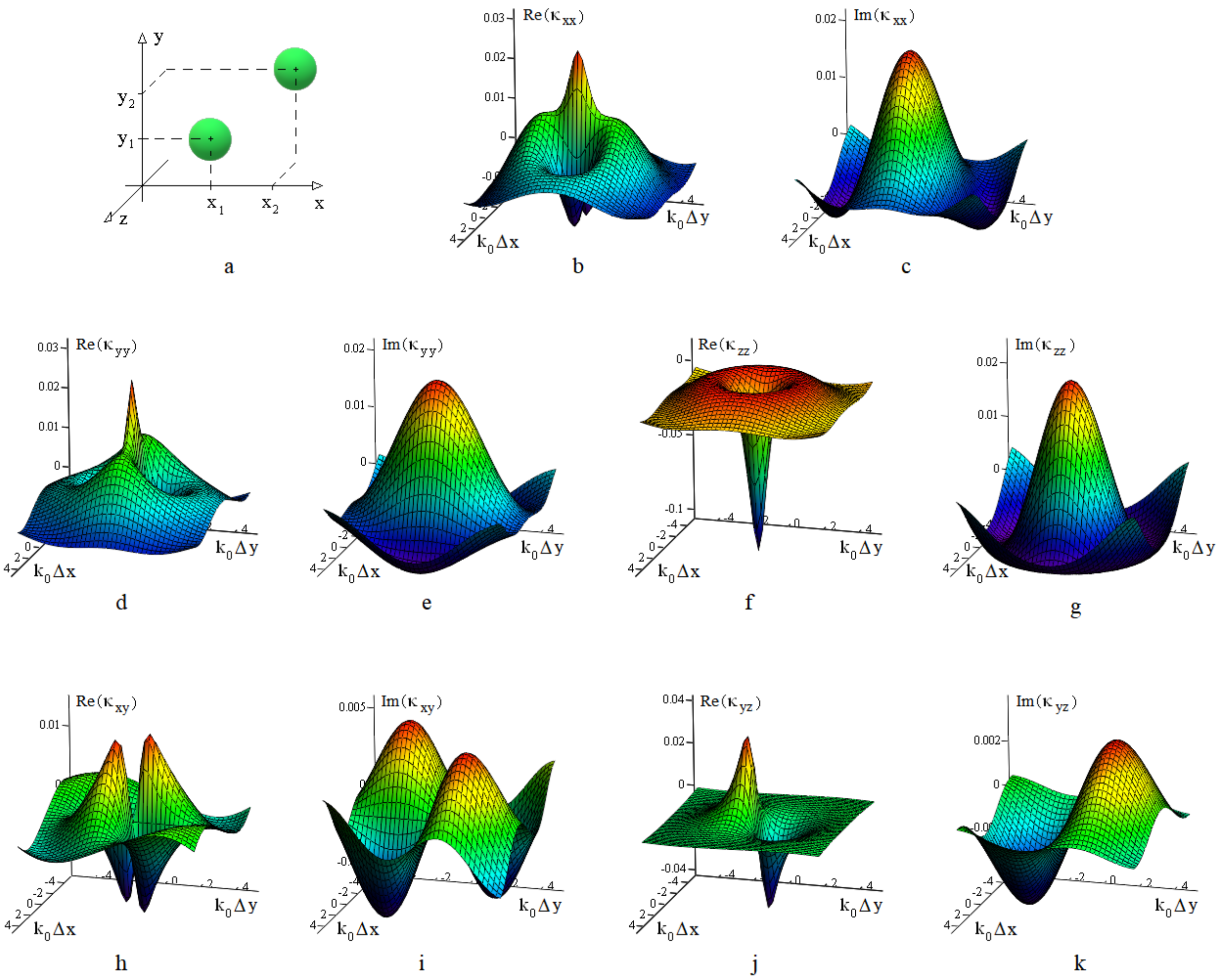


Fig. 1. Two spherical DR in a given rectangular coordinate system (x, y, z) (a). Dependences of the coupling coefficients of the magnetic oscillations H_{1m1} of a spherical DR on the mutual distance between there centers for $k_0\Delta z = 2q$ (b-k) ($k_0 = \omega/c$; $\Delta x = x_1 - x_2$; $\Delta y = y_1 - y_2$). The indices in κ_{uv} denote the direction of the magnetic field at the center of each resonator $(u, v = x, y, z)$

We have found an analytical expression for the c-function for the simple case of excitation of oscillations of magnetic types H_{1m1} in a spherical resonator in a coordinate system(x', y', z') whose z'-axis is directed along the wave vector $\vec{k}_0$:

$$c^+ = 2\pi i r_0 \delta_{m1} A^* \times$$
$$\times \left\{ j_1(p) \frac{d}{dq}\left[q j_1(q)\right] - j_1(q) \frac{d}{dp}\left[p j_1(p)\right] \right\} \begin{Bmatrix} \sin\beta_1 \\ \cos\beta_1 \end{Bmatrix} e^{ik_0 z'}. \quad (3)$$

Here $(0, 0, z')$ is the resonator center coordinate; $m = 0, \pm 1$ – azimuthal number of fundamental natural oscillations of the DR; δ_{mn} – Kronecker symbol; $j_n(z) = (\pi/2z)^{1/2} J_{n+1/2}(z)$ is the spherical Bessel function [23]; $*$ – complex conjugate symbol. If the magnetic field of natural oscillations in the center of the resonator is directed along the y axis, this case corresponds to the upper value in the curly bracket, and if it is directed along the x axis, to the lower one.

As can be seen from (3), the function c^+ is nonzero only for oscillations H_{1m1} with $m = 1$. The c^+-function is proportional to the projection of the magnetic field of the incident plane wave on the direction of the magnetic field of natural oscillations in the center of the resonator.

3 Construction of a model for the scattering of plane waves by an lattice of spherical DRs

As an example, let's build a scattering model on a 10×10 square lattice shown in Fig. 2, a, b. To simplify the calculation of the amplitudes of natural oscillations of the DR system, we pass to the coordinate system, the axis z of which is directed along the vector $\vec{k}_0$ and use (3). The amplitudes of the resonators are found using the perturbation theory [22], after which the field scattered by the grating is calculated by going back to the coordinate system of the lattice. In this case, we take into account all three types of degenerate oscillations of each of the resonators that arise in the lattice, both due to interaction with the incident wave and due to the coupling between the resonators.

Figure 2 shows the angular dependences of the squared modulus of scattering amplitude $|f\langle\theta_k,\varphi_k|\theta,\varphi\rangle|^2$ for different cases. Here (θ_k,φ_k) is the direction of the vector $\vec{k}_0$ in the lattice coordinate system.

The obtained data showed that lattices built on the basis of spherical DRs differ from lattices of other resonator shapes in terms of scattering characteristics, primarily at angles of incidence close to $\pi/4<\theta_k<3\pi/4$ and $\varphi_k\neq 0$ (Fig. 2, f-l). In cases where the incident wave is directed at $\theta_k>3\pi/4$ and $\varphi_k=0$ to the lattice plane, these differences are minimal (Fig. 2, c-e).

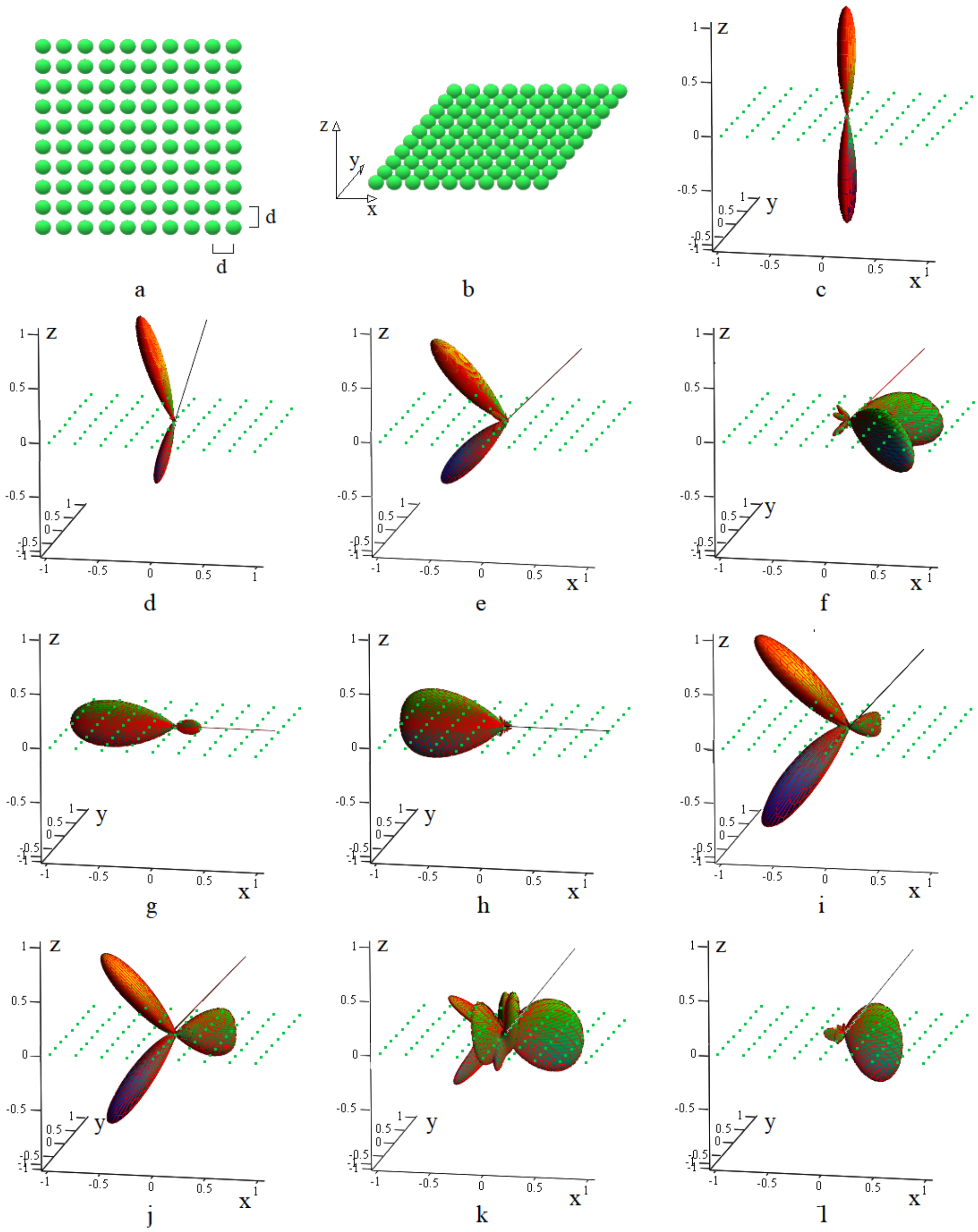


Fig. 2. Square lattices 10×10 of spherical DRs – a, b. Angular dependences of the squared modulus of scattering amplitude $|f\langle\theta_k,\varphi_k|\theta,\varphi\rangle|^2$ for $d=\lambda_0/4$ $(\lambda_0=2\pi/k_0)$; $\theta_k=\pi$; $\varphi_k=0$ (c); $\theta_k=0,9\pi$; $\varphi_k=0$ (d); $\theta_k=0,75\pi$; $\varphi_k=0$ (e, f); $\theta_k=0,5\pi$; $\varphi_k=0$ (g, h); and for $\theta_k=0.75\pi$; $\varphi_k=0,1\pi$ (i, j); $\theta_k=0.75\pi$; $\varphi_k=0,2\pi$ (k, l); for s-scattering – (c-e, g, i, k) for p-scattering (f, h, j, l); straight lines show the direction of the incident wave

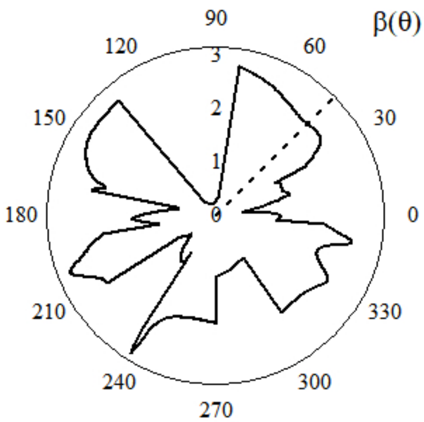
4 Polarization of scattered waves by DR lattices

In this work, we also studied the general regularities of the change in the polarization of the scattered waves. For this purpose, the electric component of the field in the wave zone was represented as the Jones vector in the spherical coordinate system (r, θ, φ), with a minimum number of independent parameters [24]:

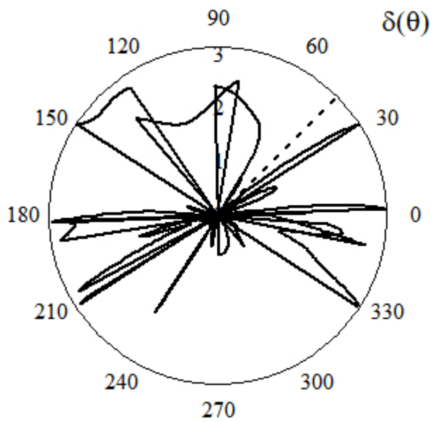
$$\vec{e}^{\infty} = (\cos\beta \cdot \vec{n}_{0\varphi} + e^{i\delta}\sin\beta \cdot \vec{n}_{0\theta}) \cdot e^{i(\omega t - k_0 r)}/r,$$

here $\vec{n}_{0\varphi}$, $\vec{n}_{0\theta}$ – the unit vectors are oriented in the directions φ, θ, respectively.

Dependencies calculated by us $\beta(\theta)$, $\delta(\theta)$, for a particular case of scattering (Fig. 2,e) are shown in Figs. 3. As it can be seen from the obtained data, spherical DR lattices differ from DR gratings with no degenerate oscillations in the general case by the presence of all three types of polarization: linear, circular, and elliptical. Only in a few special cases of incidence the scattered wave remain linearly polarized. In the general case, we can formulate the statement that for arrays of pseudo-rotating cylindrical DRs [26] with fundamental magnetic oscillations, the scattered waves will be linearly polarized.



(a)



(b)

Fig. 3. Angular dependencies $\beta(\theta)$ (a); $\delta(\theta)$ (b) for p-scattering in the plane $\varphi_k = 0, \pi$

Conclusions

An analytical expressions for the coupling coefficients of the spherical microresonator in the open space has been obtained. Expressions for c-functions are found for spherical DRs with the main magnetic types of degenerate natural oscillations and a plane wave. Based on the perturbation theory, the scattering characteristics of a flat square 10×10 DR lattice are calculated. Thus, the data obtained show that the characteristics of scattering on lattices of spherical DRs have a more complex structure associated with the excitation of several types of degenerate oscillations of this type of resonators.

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Розсіювання плоских електромагнітних хвиль решітками сферичних діелектричних резонаторів з виродженими нижчими типами власних коливань

Трубін О. О.

Розглядається задача розсіювання плоских електромагнітних хвиль на решітках діелектричних резонаторів (ДР) сферичної форми із нижчими коливаннями магнітного типу. Наведено результати теоретичних розрахунків комплексних коефіцієнтів взаємного зв'язку сферичних діелектричних резонаторів у відкритому просторі для випадків порушення вироджених типів коливань. Знайдені вирази збігаються з отриманими раніше для окремого випадку коливань резонаторів, які збуджуються вздовж або перпендикулярно прямій, яка з'єднує їх центри. Розглянуто основні закономірності зміни коефіцієнтів зв'язку під час варіації координат резонаторів у поперечній площині. Знайдені нові аналітичні вирази c-функцій (3) для поля основних магнітних коливань резонатора і плоскої хвилі у відкритому просторі. На підставі отриманих формул, за допомогою теорії збурень розраховано та досліджено характеристики розсіювання плоских хвиль на квадратних решітках сферичних ДР з основними виродженими магнітними типами коливань. Досліджено розподіл поля розсіювання у хвильовій зоні решітки для різних кутів падіння. Визначено області зміни кутів падіння, в яких амплітуда розсіювання решітки, побудованої на основі сферичних ДР, найбільш помітно відрізняється від решіток ДР інших форм із невиродженими типами коливань. Розраховано характеристики поляризації розсіяних хвиль у дальній зоні решітки. Зазначено, що на відміну від ґрат псевдообертальних ДР циліндричної форми з основними магнітними типами коливань, решітки, побудовані на основі сферичних резонаторів, характеризуються більш складнішим розподілом поляризації розсіяних хвиль. У хвильовій зоні решітки можуть спостерігатися розсіяні хвилі всіх трьох типів поляризації – лінійної, кругової, еліптичної. Отримані результати значно розширюють можливості розробників, оскільки дозволяють створювати електродинамічні моделі решіток, а також інших пристроїв міліметрового та інфрачервоного діапазонів, побудованих на основі застосування сферичних резонаторів з коливаннями основних типів. Такі решітки можуть бути використані в антенах, пасивних відбивачах, а також в інших пристроях сучасних оптичних систем зв'язку.

Ключові слова: діелектричний резонатор; решітка; коефіцієнт зв'язку; c-функція; амплітуда розсіювання