Transverse Resonance Technique for Analysis of Symmetrical Stub in Microstrip Transmission Line

Rassokhina Yu. V., Krizhanovski V. G.

Vasyl' Stus Donetsk National University, Vinnitsa, Ukraine

E-mail: *yu.rassokhina@donnu.edu.ua*,*v.krizhanovski@donnu.edu.ua*

Open or short-circuit stubs in a strip (microstrip) transmission line are one of the most common elements of planar circuits used in numerous devices in the microwave frequency range: various types of filters, couplers, power amplifiers, antennas, sensors, wireless energy transfer systems, etc. Modern planar circuits in the microwave frequency range already contain stubs of a complex shape and a complex pattern inside the microstrip line. Therefore, an urgent problem is to develop an analyzing method for discontinuities in form of the closed or open stub in a microstrip transmission line at frequencies at which the transmission line theory already has significant errors and high-frequency effects must be considered. In paper a technique of scattering characteristics calculating on a symmetrical microstrip open stub by transverse resonance method is presented. Boundary value problems for a rectangular volume resonator based on a microstrip transmission line with a symmetric open stub are solved for three different boundary conditions in the plane of symmetry and on the longitudinal boundaries. To algebraize the boundary value problems for the resonator's eigen frequencies with discontinuity, the corresponding two-dimensional functions of the magnetic potential are constructed, through which the components of the current density on the strip are calculated. The magnetic potential functions were written by decomposing them into series by orthogonal Chebyshev polynomials, which consider the behavior of the field on a thin edge and ensure fast convergence of the series and the algorithm. The developed algorithms were tested by calculating the scattering characteristics of a microstrip open stub using the transverse resonance method on the example of open stub in a microstrip transmission line with a resonant frequency of about 3.0 GHz. In addition, the method was tested on the example of numerical calculations of the dependence of resonant reflection frequencies of an open stub on its width.

Keywords: microstrip line; open stub; transverse resonance method; resonance frequencies; scattering matrix

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Introduction

Open or short-circuit stubs in a strip (microstrip) transmission line are one of the most common elements of planar circuits used in numerous devices in the microwave frequency range: various types of filters, couplers, power amplifiers, antennas, sensors, wireless energy transfer systems, etc. Modern planar circuits in the microwave frequency range already contain stubs of a complex shape and a complex pattern inside the microstrip line [1–8].

The scattering characteristics of ordinary rectangular stubs in a microstrip line are easily determined by transmission line theory by which calculates the input admittance of the stub. A more accurate analysis of such discontinuity, which considers edge and other effects of a microwave circuit with an open or shorted stub, is already a difficult problem of applied electrodynamics. Given the computing capabilities of modern computer technology, complex planar circuits are analyzed using commercial programs by numerical methods, mostly by the moments method followed by the construction of an equivalent discontinuity circuit [1–8]. Rigorous analysis of stub discontinuities in strip and microstrip lines can be carried out using the mode matching method, which is based on the decomposition method and describes the field in them by the eigenwaves of each partial region. But that is a cumbersome method.

More promising for rigorous analysis of such discontinuities, in our opinion, is the transverse resonance method, which was introduced by Sorrentino and Itoh [9] and allows analyzing complex structures without breaking the microwave circuit into small elements. The idea of the method is that there is a relationship between the eigenfrequencies of the volume resonator, in which the discontinuity is located, and the scattering matrix elements on this discontinuity. The transverse resonance method is a universal method for analyzing waveguide and planar circuits, which calculates both the dispersion characteristics of regular transmission lines and the scattering characteristics of unregular distributed circuits [10–17]. Using the example of the periodic structures scattering characteristics [18], it was shown that for symmetrical in the transverse direction discontinuities, the intersection points of the eigenfrequency spectra obtained from the solutions of boundary value problems with two different conditions in the symmetry plane directly indicate the zeros or poles of the scattering characteristics. We are talking about the conditions of the electric and magnetic walls (*e.w.* and *m.w.*) in the symmetry plane and on the longitudinal boundaries of the resonator, according to which the boundary value problems with such boundary conditions will be called "**electric**" and "**magnetic**" boundary value problems, respectively.

The application of the transverse resonance method for the analysis of discontinuities in planar transmission lines (including those of complex shape) requires the development of algorithms for calculating the resonance frequencies of a volume resonator with discontinuity and finding an effective basis for the series expansion of the current density on the strip or the electromagnetic field distribution in the slot resonator [19]. In [20], the transverse resonance method was developed for analysis of step discontinuity in a microstrip line, where a basis of orthogonal Chebyshev polynomials was used to algebraize the boundary value problem for describing the current density in the transverse direction. In the case of stub discontinuity, this approximation is not sufficient, and it is necessary to build basis functions considering the distribution of the current density on the open or shorted sections of the transmission line. Thus, to solve the boundary value problem for the microstrip stub, the current density function must be described through the series expansion in two mutually perpendicular directions.

The work aims to develop an analyzing technique for distributed discontinuity in the form of an open or shorted stub in a strip (microstrip) transmission line by the transverse resonance method, using orthogonal polynomials to describe the current density on the strip.

1 Formulation and solution of boundary value problems

The topology of the two-layer planar structure under consideration is provided in Fig. 1, which shows a symmetrical open stub in a microstrip transmission line. According to the transverse resonance method, to determine the resonant interaction frequencies of the fed transmission line 1 with discontinuity 2-3, the two boundary value problems with electric and magnetic wall conditions (e.w. or m.w.) in the plane of symmetry $z = 0$ must be solved. At the resonator boundary $z = L$ the conditions of an electric or magnetic wall must also be fulfilled.

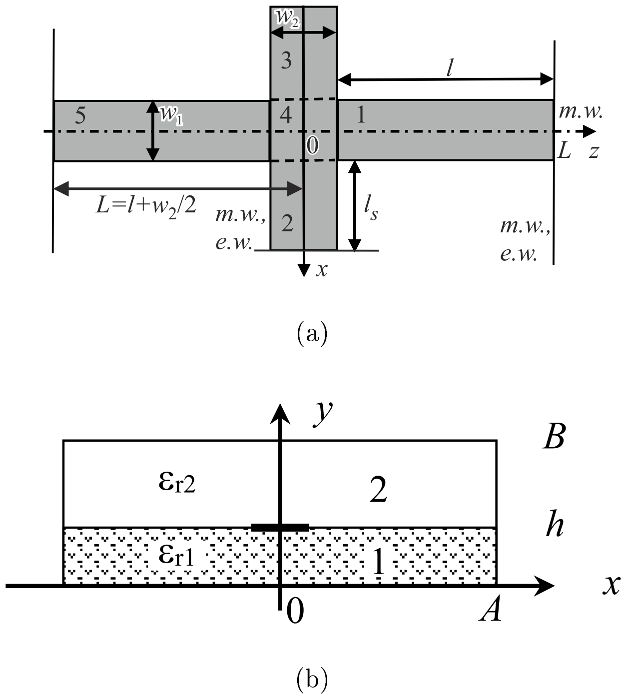


Fig. 1. Topology of a symmetrical open stub in a microstrip line (a) and cross section of a microstrip line (b)

Consider the solution of the boundary value problem for the current density $\vec{J}_\tau$ of a microstrip resonator expressed in terms of magnetic type vector potentials $J_{h,n}(x,z)$:

$$\vec{J}_\tau\,(x,z) = -\frac{1}{j\cdot k_0}\sum_{n=1}^{P}\nabla J_{h,n}(x,z)\,C_{h,n}, \qquad (1)$$

where $k_0 = \omega_0/c$, $J_{h,n}$ are eigenfunctions of the magnetic vector potential for the current density, $C_{h,n}$ is unknown expansion coefficient, P is the order of series reducing.

The electromagnetic field components in the shielded structure satisfy the Helmholtz equation, that is, the wave equation in Cartesian coordinates. However, the current density function in a microstrip line has singularity at the thin edges of the strip, so Chebyshev polynomials $T_n(x) = \cos(n \arccos x)$ of the first kind are used to describe it, which have a weight function $1/\sqrt{1-x^2}$ that corresponds to the singularity of the field behavior on the thin edge and satisfies the proper differential equation. Chebyshev polynomials of even order $T_{2n}(x)$ correspond to the symmetry of the fundamental wave of the microstrip line (the condition of the magnetic wall at $x = 0$), and polynomials of odd order $T_{2n+1}(z)$ (the condition of the electric wall at $z = 0$) correspond to waves of a higher type, which are usually reactive in the microwave frequency range, i.e. do not propagate. A detailed description of the formulation and solution of boundary value problems for vector potentials with corresponding boundary conditions can be found in [20].

Considering the above, the two-dimensional function for the magnetic vector potential $J_{h,n}(x,z)$ of the "**electric**" boundary value problem in partial regions 1-4 can be presented in the form:

$$J_{h1}(x,z)=\sum_{k=0}^{M}A_{1k}N_{1k}T_{2k}\left(\frac{x}{w_1/2}\right)\frac{\sin k_{z1k}(L-z)}{k_{z1k}\cos k_{z1k}l},$$

$$L=l+w_2/2,\ \ |x|\le w_1/2,\ \ w_2/2\le z\le L,\ \ N_{1k}=2\sqrt{2-\delta_{k0}}/\sqrt{\pi w_1},$$

$$J_{h2}(x,z)=\sum_{k=0}^{M}A_{2k}N_{2k}T_{2k+1}\left(\frac{z}{w_2/2}\right)\frac{\cos k_{x1k}(L_s-x)}{k_{x1k}\sin k_{x1k}l_s},$$

$$L_s=l_s+w_1/2,\ \ |z|\le\ w_2/2,\ \ w_1/2\le x\le L_s,\ \ N_{2k}=2\sqrt{2}/\sqrt{\pi w_2},$$

$$J_{h3}(x,z)=\sum_{k=0}A_{3k}N_{2k}T_{2k+1}\left(\frac{z}{w_2/2}\right)\frac{\cos k_{x1k}(L_s+x)}{k_{x1k}\sin k_{x1k}l_s},$$

$$-w_1/2\le x\le -L_s,$$

$$J_{h4}(x,z)=\sum_{k=0}^{M}A_{41k}N_{1k}T_{2k}\left(\frac{x}{w_1/2}\right)\frac{\sin k_{z1k}z}{k_{z1k}\cos(k_{z1k}w_2/2)}+\sum_{k=0}A_{42k}N_{2k}T_{2k+1}\left(\frac{z}{w_2/2}\right)\frac{\cos k_{x1k}x}{k_{x1k}\sin(k_{x1k}w_1/2)}, \quad (2)$$

where $|x|\le w_1/2$, $|z|\le w_2/2$, $k_{z1,k}^2=\chi_{hn}^2-\left(\frac{2k}{w_1/2}\right)^2$, $k_{x1,k}^2=\chi_{hn}^2-\left(\frac{2k+1}{w_2/2}\right)^2$, χ_{hn}^2 are eigenvalues of the eigenfunction $J_{h,n}(x,z)$, which is found from the solution of the boundary value problem.

From the continuity conditions of the functions on the partial domains boundaries and considering the basis functions singularity, a system of linear algebraic equations (SLAE) is obtained in the form:

$$\sum_{m=0}A_{41m}\left[F_{1k}(k_{z1k})\,\delta_{km}-\sum_{n=0}\frac{1}{F_{2n}}S_{1,kn}S_{2,nm}\right]=0. \quad (3)$$

Equating the determinant of SLAE (3) to zero, we obtain a spectrum of eigenvalues χ_{hn}^2 and, accordingly, eigenfunctions for the magnetic vector potential $J_{h,n}(x,z)$, which determines the components of the current density on the strip. Expressions for matrix elements in (3) have the form:

$$F_{1k}(k_{z1k})=\frac{\tan k_{z1k}l}{k_{z1k}}+\frac{\tan(k_{z1k}w_2/2)}{k_{z1k}},$$

$$F_{2n}(k_{x1n})=\frac{\cot k_{x1n}l_s}{k_{x1n}}+\frac{\cot(k_{x1n}w_1/2)}{k_{x1n}},$$

$$S_{1,kn}=\begin{cases}\sqrt{\frac{w_1}{w_2}}\sqrt{4-2\delta_{k0}}\frac{(-1)^kJ_{2k}(k_{x1n}w_1/2)}{k_{x1n}\sin(k_{x1n}w_1/2)},\chi_{hn}>\frac{2n+1}{w_2/2}\\ \sqrt{\frac{w_1}{w_2}}\sqrt{4-2\delta_{k0}}\frac{I_{2k}(|k_{x1n}|w_1/2)}{|k_{x1n}|\sinh(|k_{x1n}|w_1/2)},\chi_{hn}<\frac{2n+1}{w_2/2}\end{cases},$$

$$S_{2,kn}=\begin{cases}\sqrt{\frac{w_2}{w_1}}\sqrt{4-2\delta_{n0}}\frac{(-1)^kJ_{2k+1}(k_{z1n}w_2/2)}{k_{z1n}\cos(k_{z1n}w_2/2)},\ \chi_{hn}>\frac{2n}{w_1/2}\\ \sqrt{\frac{w_2}{w_1}}\sqrt{4-2\delta_{n0}}\frac{I_{2k+1}(|k_{z1n}|w_2/2)}{|k_{z1n}|\cosh(|k_{z1n}|w_2/2)},\chi_{hn}<\frac{2n}{w_1/2}\end{cases},$$

where $J_k(x)$ are ordinary Bessel functions of the 1st kind, $I_k(x)$ are modified Bessel functions of the 1st kind, which emerge from the relation $J_{2k}(ix)=(-1)^kI_k(x)$. The expansion coefficients A_{41m}, A_{42m} of the functions according to the polynomial basis are calculated with accuracy up to some constant factor, which is determined from the normalization condition of the magnetic potential basis functions:

$$\int_{S_{MSL}}[\nabla J_{h,n}(x,z)]^2\,dS=\chi_{h,n}^2\int_{S_{MSL}}J_{h,n}^2(x,z)\,dS=1.$$

It is worth noting that the "electrical" boundary value problem also has a solution by $\chi_{h,n}=0$, which must be considered by rigorous solving the problem for the rectangular volume resonator eigenfrequencies.

For the "**magnetic-electric**" boundary value problem, that is, under the condition of a magnetic wall in the symmetry plane $z=0$ and an electric wall on the longitudinal boundary $z=L$, the magnetic potential eigenfunctions in partial regions 1-4 can be determined as:

$$J_{h1}(x,z)=\sum_{k=0}A_{1k}\sqrt{\frac{2}{w_1}}\sqrt{\frac{4-2\delta_{k0}}{\pi}}T_{2k}\left(\frac{x}{w_1/2}\right)\frac{\sin k_{z1k}(L-z)}{k_{z1k}\cos k_{z1k}l},$$

$$J_{h2}(x,z)=\sum_{k=0}A_{2k}\sqrt{\frac{2}{w_2}}\sqrt{\frac{4-2\delta_{k0}}{\pi}}T_{2k}\left(\frac{z}{w_2/2}\right)\frac{\cos k_{x1k}(L_s-x)}{k_{x1k}\sin k_{x1k}l_s},$$

$$J_{h3}(x,z)=\sum_{k=0}A_{3k}\sqrt{\frac{2}{w_2}}\sqrt{\frac{4-2\delta_{k0}}{\pi}}T_{2k}\left(\frac{z}{w_2/2}\right)\frac{\cos k_{x1k}(L_s+x)}{k_{x1k}\sin k_{x1k}l_s},$$

$$J_{h4}(x,z)=\sum_{k=0}A_{41k}\sqrt{\frac{2}{w_1}}\sqrt{\frac{4-2\delta_{k0}}{\pi}}T_{2k}\left(\frac{x}{w_1/2}\right)\frac{\cos k_{z1k}z}{k_{z1k}\sin(k_{z1k}w_2/2)}+$$
$$+\sum_{k=0}A_{42k}\sqrt{\frac{2}{w_2}}\sqrt{\frac{4-2\cdot\delta_{k0}}{\pi}}T_{2k}\left(\frac{z}{w_2/2}\right)\frac{\cos k_{x1k}x}{k_{x1k}\sin(k_{x1k}w_1/2)},$$

where $k_{z1,k}^2=\chi_{hn}^2-\left(\frac{2k}{w_1/2}\right)^2$, $k_{x1,k}^2=\chi_{hn}^2-\left(\frac{2k}{w_2/2}\right)^2$.

The SLAE for determining the eigenvalues and coefficients of the expansion into series of the magnetic potential has the form:

$$\sum_{m=0}A_{42m}\left[F_2(k_{x1k})\,\delta_{km}+\sum_{n=0}\frac{1}{F_{1n}(k_{z1n})}S_{2kn}S_{1nm}\right]=0, \tag{4}$$

where, by analogy with the “electrical” problem,

$$F_{1k}(k_{z1k})=\frac{\tan k_{z1k}l}{k_{z1k}}-\frac{\cot(k_{z1k}w_2/2)}{k_{z1k}},$$

$$F_{2n}(k_{x1n})=\frac{\cot k_{x1n}l_s}{k_{x1n}}+\frac{\cot(k_{x1n}w_1/2)}{k_{x1n}},$$

$$S_{1kn}(k_{x1n})=\sqrt{\frac{w_1}{w_2}}\sqrt{2-\delta_{k0}}\sqrt{2-\delta_{n0}}\times$$
$$\times(-1)^k\frac{J_{2k}(k_{x1n}w_1/2)}{k_{x1n}\sin(k_{x1n}w_1/2)},$$

$$S_{2kn}(k_{z1n})=\sqrt{\frac{w_2}{w_1}}\sqrt{(2-\delta_{n0})(2-\delta_{k0})}\times$$
$$\times(-1)^k\frac{J_{2k}(k_{z1n}w_2/2)}{k_{z1n}\sin(k_{z1n}w_2/2)}.$$

In the same way, the two-dimensional function of the magnetic potential is defined for the boundary value problem with boundary conditions of the magnetic wall in the plane of symmetry and on the longitudinal boundary of the volume resonator (“**magnetic**” boundary problem).

Boundary value problems solving for current density eigenfunctions in an irregular microstrip line is used for problem solving for rectangular volume resonators with discontinuity. In this case, the discontinuity is an open capacitive stub in the microstrip transmission line. According to the transverse resonance method, the points of spectral curves intersection, corresponding to the solutions of the electric and magnetic-electric boundary value problem, determine the minimum transmission coefficient points, that is, the rejecting frequencies of the main signal. And the points of intersection of these spectral curves determine the minimum points of the reflection coefficient.

The electric and magnetic vector potentials of a rectangular volume resonator are presented in the form of double Fourier series:

$$\begin{aligned}A_{ey,i}&=\sum_{m=1}^{N}\sum_{n=1(0)}^{N}\phi_{mn}(x,z)\,F_{ei,mn}(k_{yi,mn}),\\ A_{hy,i}&=\sum_{m=1}^{N}\sum_{n=0(1)}^{N}\psi_{mn}(x,z)\,F_{hi,mn}(k_{yi,mn}),\end{aligned} \tag{5}$$

where $k_{yi,mn}^2=k_0^2\varepsilon_{ri}-\chi_{mn}^2$, $i=1,2$ is a partial area number, N is order of series reduction.

The coupling integrals $\alpha_{h,q,mn}^m$, $\beta_{h,q,mn}^m$ between a strip resonator with discontinuity and a volume resonator are calculated by the formulas [20]:

$$\begin{aligned}\alpha_{h,q,mn}^m&=\int_{S_{MSL}}\nabla J_{h,q}(x,z)\left[\nabla\psi_{mn}(x,z)\times e_y\right]dS,\\ \beta_{h,q,mn}^m&=\int_{S_{MSL}}\nabla J_{h,q}(x,z)\,\nabla\phi_{mn}(x,z)\,dS,\end{aligned} \tag{6}$$

where ψ_{mn}, ϕ_{mn} are basis functions of the electric and magnetic vector potential of a volume resonator ($k_{xm}=\pi(2m-1)/2A$, $k_{zn}=\pi n/L$, for the “electric” and “magnetic” boundary value problem or $k_{zn}=\pi(2n-1)/2L$ for the magnetic-electric problem):

$$\phi_{mn}(x,z)=\begin{cases}P_{mn}\cos k_{xm}x\sin k_{zn}z, & ew-ew\\ P_{mn}\cos k_{xm}x\cos k_{zn}z, & mw-mw\end{cases},$$

$$\psi_{mn}(x,z)=\begin{cases}P_{mn}\sin k_{xm}x\cos k_{zn}z, & ew-ew\\ P_{mn}\sin k_{xm}x\sin k_{zn}z, & mw-mw\end{cases},$$

$$P_{mn}=\sqrt{\frac{2}{A}}\sqrt{\frac{2-\delta_{n0}}{L}}\frac{1}{\chi_{mn}},\quad \chi_{mn}^2=k_{xm}^2+k_{zn}^2.$$

According to the transverse resonance technique, the scattering matrix elements on a symmetric discontinuity are calculated from the solutions of two boundary value problems of *e.w.–e.w.* and *m.w.–e.w.* for the resonator with respect to its longitudinal dimension l_i, $i=1,2$, by the formulas:

$$\begin{aligned}S_{11}&=-(\Gamma_1+\Gamma_2)/2,\\ S_{11}&=(\Gamma_1-\Gamma_2)/2,\end{aligned} \tag{7}$$

where $\Gamma_{1(2)}=\exp\left(2j\beta_z l_{1(2)}\right)$, β_z is the propagation constant of the fundamental wave of a regular microstrip transmission line. By module, the scattering matrix elements are determined by the difference in the longitudinal dimensions of the volume resonator with discontinuity:

$$|S_{11}| = |\cos \beta_z (l_1 - l_2)|,$$

$$|S_{12}| = |\sin \beta_z (l_1 - l_2)|.$$

2 Algorithm testing and results of symmetric open stub analysis

The algorithms were developed and tested on the example of a two-dimensional planar structure on a Ro3010 laminate (Rogers RO3010 advanced circuit materials are ceramic-filled PTFE composites that offer a higher dielectric constant with excellent stability) with a thickness of h=0.635 mm with dielectric constant $\varepsilon_r = 10.2$, the width and height of the grounding volume resonator are equal, respectively $A = 15.0$ and $b_1 = 8.0$ mm, other parameters of the structure: $w_1 = w_2 = w = 0.58$ mm (the characteristic impedance of the main transmission line is $Z_0 = 50\,\text{Ohm}$). With a constant number M of basis functions from orthogonal polynomials (2) considered and reduction of series (1) by eigenfunctions of vector potentials up to P=3, sufficient algorithm convergence is observed when reduction of series (1) up to N=300.

Eigenfunction numbers of a strip resonator with a symmetric open stub of length $l_s = 10.5 - w/2$ mm, which were obtained from solutions of three boundary value problems, are shown in Fig. 2. In the first approximation, the wave numbers of the "electric" resonator correspond to the values $\chi_{h,n}^{(e.w.)} = \pi n/L$, for the magnetic-electric problem $\chi_{h,n}^{(m.w.-e.w.)} = \pi n/2\,(L + l_s)$ and for the magnetic problem $\chi_{h,n}^{(m.w.)} = \pi n/\,(L + l_s)$.

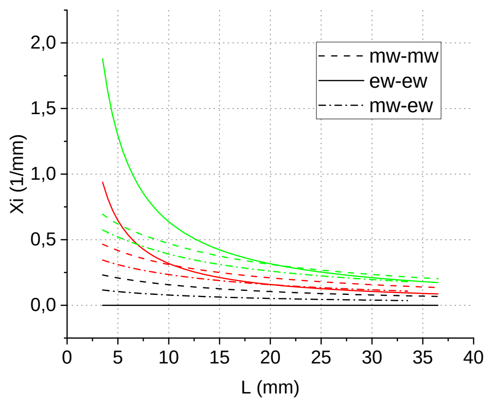
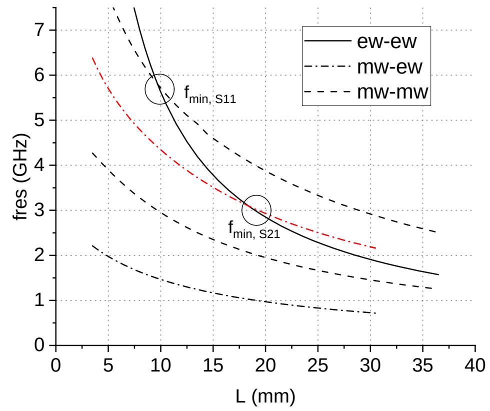


Fig. 2. Eigenvalues $\chi_{h,n}$ of magnetic potential basic functions $J_{h,n}$ for a strip resonator with a symmetrical open stub, obtained from the solutions of the "electrical", "magnetic-electrical" and "magnetic" boundary value problems

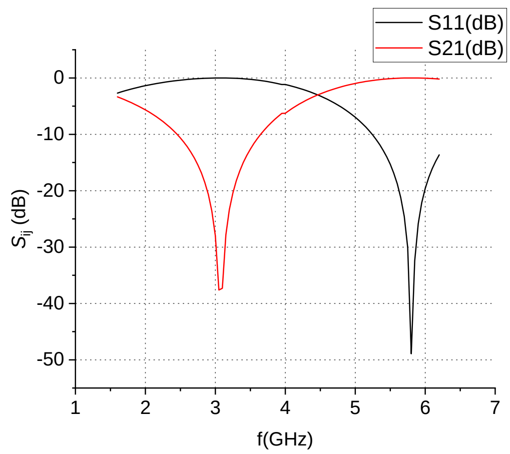
According to the transmission lines theory, the input conductivity of a symmetrical open stub is equal to:

$$Y_{in} = 2jY_0 \tan \theta_s,$$

where $Y_0 = 1/Z_0$, $\theta_s = \frac{\omega}{c} l_s \chi$, χ is the wave delay factor, which for this material is equal to about $\chi \approx 2.62$. Resonant frequency of the stub with length l_s (that is, the frequency at which the electric length is $\theta_s = \pi/2$) calculated by transmission lines theory is $f_{res} = 2.85\,\text{GHz}$.



(a)



(b)

Fig. 3. Spectrum of eigenfrequencies of a three-dimensional rectangular resonator based on an microstrip line with a symmetrical open stub, obtained from the solutions of boundary value problems with parameters (in mm): $w = 0.58$, $l_s = 10.2$ (a) and its corresponding scattering characteristics on discontinuity (b)

Figure 3a shows the spectra of the resonator's eigenfrequencies obtained from solutions of three boundary value problems for a volume resonator with discontinuity in the form of a symmetric open stub in a microstrip transmission line. The intersection point of the spectral curves of the electric and magnetic-electric boundary value problems corresponds to the frequency at which the minimum of the transmission coefficient is observed S_{21} (about 3.08 GHz), and the point of intersection of the spectral curves of the "electric" and "magnetic" boundary value problems corresponds to the minimum of the reflection coefficient S_{11} at frequency about 5.8 GHz. The corresponding scattering characteristics on a symmetrical open stub, calculated by the transverse resonance method, are shown in Fig. 3b. To obtain the scattering matrix elements, the spectral curves were approximated by a rational function of the form $f_{res}(x) = 1/Q_m(x)$, when $Q_m(x)$ is a m-order polynomial, m=9.

Thus, according to the results of numerical calculation, a physically correct result was obtained for the scattering characteristics on a symmetrical stub in a microstrip transmission line, considering high-frequency effects, namely dispersion and marginal capacitance of the open stub.

In Fig. 4 the dependence of the resonance frequency and the derivative of the spectral characteristic on the stub width is shown. As expected from physical considerations, the frequency of resonance reflection increases with the ratio w_1/w_2 increase, and the Q-factor of the resonance characteristic, on the contrary, decreases.

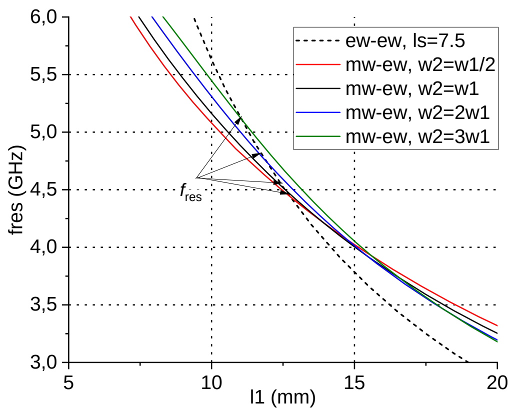


Fig. 4. The eigen frequency spectrum of a three-dimensional rectangular resonator based on a microstrip line with a symmetrical open stub, obtained from the solutions of "electrical" and "magnetic-electrical" boundary value problems with parameters (in mm): $w_1 = 0.58$, $l_s = 7.5$ depending on the stub width w_2

Conclusion

A technique for analyzing of open stubs in a microstrip transmission line by transverse resonance method is proposed. To implement the method, the problems for the eigenfunctions of the strip resonator's current density with a symmetrical open stub were previously solved under the condition of an electric and magnetic wall in the symmetry plane and on the longitudinal boundary. To determine the eigenfunctions of the current density, a basis of orthogonal polynomials was used, which ensures fast convergence of algorithms for the numerical calculation of eigenfunctions. The obtained solutions were used for the algebraization of boundary value problems on the resonance frequencies of a volume resonator with discontinuity and, accordingly, for the calculation of the scattering matrix elements on a symmetric open stub by the transverse resonance technique. The algorithm was tested by calculating the scattering characteristic on a symmetrical microstrip open stub with a resonant reflection frequency of about 3 GHz. The obtained algorithms can be applied to the analysis and development of civil and special-purpose devices in microwave frequency range.

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Метод поперечного резонансу для аналізу симетричного шлейфу у смужковій лінії передачі

Рассохіна Ю. В., Крижановський В. Г.

Розімкнені або короткозамкнені шлейфи у смужковій (або мікросмужковій) лінії передачі є одними з найпоширеніших елементів планарних схем, що використовуються у численних пристроях мікрохвильового діапазону частот. Сучасні планарні схеми мікрохвильового діапазону містять вже шлейфи складної форми та складний рисунок (pattern) всередині самої мікросмужкової лінії. Тому актуальною задачею є розробка методу аналізу неоднорідностей типу замкненого або розімкненого шлейфу у мікросмужковій лінії передачі на частотах, на яких теорія довгих ліній вже має суттєві похибки і вже треба враховувати високочастотні (крайові) ефекти. Серед існуючих методів аналізу неоднорідностей виділяється метод поперечного резонансу, за яким неоднорідності у планарних схемах аналізуються цілком, без розбиття вихідної області на часткові області.

В роботі наведено методику розрахунку характеристик розсіяння на симетричному мікросмужковому шлейфі за методом поперечного резонансу. Розв'язано крайові задачі для прямокутного об'ємного резонатору на базі мікросмужкової лінії передачі із симетричним розімкненим шлейфом для трьох різних граничних умов у площині симетрії та на поздовжніх границях. Для алгебраїзації крайових задач на власні частоти резонатору із неоднорідністю побудовано відповідні двовимірні функції магнітного потенціалу, через які розраховуються компоненти густини струму на смужці. Функції магнітного потенціалу були записані через розкладання їх у ряди за ортогональними поліномами Чебишова, які враховують поведінку поля на тонкому ребрі та забезпечують швидку збіжність самих рядів та алгоритму в цілому. Побудовані алгоритми протестовані за допомогою розрахунку характеристик розсіяння мікросмужкового шлейфу за методом поперечного резонансу на прикладі планарної структури із симетричним розімкненим шлейфом у мікросмужковій лінії передачі із частотою резонансного відбиття близько 3.0 ГГц. Крім того, метод був протестований на прикладі чисельних розрахунків залежності частот резонансного відбиття розімкненого шлейфу від його ширини.

Ключові слова: мікросмужкова лінія; розімкнений шлейф; метод поперечного резонансу; резонансні частоти; матриця розсіяння