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Mathematical Model of Two-Fragment Signal with Non-Linear Frequency Modulation in Current Period of Time

Kostyria O. O., Hryzo A. A., Dodukh O. M., Nariezhnii O. P.

Ivan Kozhedub Kharkiv National Air Force University, Kharkiv, Ukraine

E-mail: *oleksandr.kostyria@nure.ua*

Advantages of using frequency-modulated signals for locating objects are the possibility of using long-term probing pulses. Such signals provide the required radiated power while maintaining the desired discriminating power from range. One such signal that has found wide application is a signal with linear frequency modulation. An undesirable effect of the matched filtering of such a radio pulse is a sufficiently large level of side lobes of the compressed signal at the output of the processing device, the maximum level of which is approximately minus 13 dB. Such an effect may lead to an increase in the probability of false detection or masking of less powerful signals by side lobes of signals with greater power. One method of reducing the level of side lobes is the use of signals with non-linear frequency modulation. An example of such signals is a known two-fragment signal consisting of linearly-frequency modulated fragments in time. However, the mathematical models used to describe such a signal do not fully reflect the effects that occur at the moment of transition from one fragment of the signal to the second. These effects are manifested in a sudden change in frequency and phase, which leads to distortion of the signal spectrum, an increase in the level of the side lobes of the autocorrelation function and sharp changes in their level. Such effects have not been studied in known works, as evidenced by the results of the analysis of studies and publications given in the first section of the article. In the second section of the work, the research task is formulated. The third section of the work is devoted to the development of a mechanism for compensating for the manifestation of detected effects and its mathematical description, which is verified by modeling. Taking into account the detected effects, a new mathematical model of a non-linear frequency modulated signal has been developed. In contrast to those known in the proposed model, instantaneous frequency and phase jumps are compensated for, which occur at the moments when the frequency modulation rate changes during the transition from one signal fragment to another. Further studies should be focused on the peculiarities of compensation for the manifestation of detected effects for signals with a large number of fragments, as well as combinations of fragments with different types of modulation, as indicated in the conclusions on the work.

Keywords: non-linear frequency modulation; mathematical model; autocorrelation function; sidelobe level; frequency and phase jumps

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Statement of the research task

Signals with non-linear frequency modulation (NLFM) traditionally enjoy the attention of specialists in the development of radar technology due to the possibility of obtaining signals of the maximum peak of side lobes level (MPSLL, SLL) of the autocorrelation function (ACF) smaller than linearly frequency-modulated (LFM) signals. The possibility of a significant decrease in MPSLL without the use of weight processing (WP) in the time or frequency domain is theoretically substantiated, confirmed experimentally and proved by modeling by many authors, for example, [1–4]. These studies are fundamental and have found further development both in relation to LFM and in the case of using NLFM signals. In [3, 4], NLFM signals are proposed in which the power spectral density is less at the edges of the spectrum compared to its middle, which ensures rounding of the spectrum and, as a consequence, a decrease in the MPSLL of their ACF. The simplest implementation of such a signal is the combination in time of two LFM fragments, one of which has a different value for the frequency modulation rate (FM), that is, the ratio of its spectrum width (frequency deviation) to the duration of the fragment.

The studies carried out by the authors showed that the process of formation of the NLFM signal, which consists of two LFM fragments, according to common mathematical models (MM) is accompanied by the appearance of effects that are not reflected in known publications, which necessitated the development of a new MM two-fragment NLFM signal.

1 Analysis of studies and publications

A lot of work is devoted to research on the introduction of NLFM signals in many areas of use, such as airspace control systems [5], weather location [6–9], sonar [10], aviation and space land survey [11–14], communications [15], medical ultrasound diagnostics [16], radio-frequency surveillance and electronic reconnaissance, electronic warfare [17–21], etc.

A number of scientific papers are focused on the problems of forming and processing signals from LFM and NLFM [6, 22–30]. The problem of reducing the MPSLL due to the use of only NLFM signals is devoted to the [7, 31–34] in works [5, 28, 35, 36], the joint use of WP with the help of window functions is additionally considered, the impact of Doppler frequency shift on MPSLL is also analyzed [37–40].

Widely used are NLFM signals, which consist of two or more LFM fragments, as well as combinations of LFM fragments and fragments with a different frequency modulation [13, 22, 30, 32, 34, 38, 39, 41–43].

The considered studies of NLFM signals are based on the use of MM with the current time, for example, [24, 26, 38, 42], with a time symmetrical with respect to the middle of the radio pulse [25, 31, 38, 39], there are those in which for all fragments of the NLFM signal the amplitude calculation begins with a zero time reference by shifting the timeline [22, 30, 32, 34]. In MM of two-fragment NLFM signals, as a rule, the shifted time is used. The authors of the work found that such MMs do not fully reflect the processes occurring during the formation of the NLFM signal at the time when the transition from the first fragment to the second.

2 Formulation of the research task

The purpose of the work is to develop and study the MM of the current time of the NLFM signal, which consists of two LFM fragments, which takes into account frequency and phase jumps when switching from one fragment to another.

3 Presentation of the research material

3.1 Justification of the need to develop a mathematical model of the NLFM signal in the current time

Consider a common MM NLFM signal, which consists of two LFM fragments [22, 30, 32, 34]. Fragments differ in the values of the speed of FM and the shift in time relative to each other:

$$\dot{U}(t)=\left|\dot{U}\right|\begin{cases}\exp\left(j\varphi_1(t)\right), & 0\le t\le T_1\\ \exp\left(j\varphi_2(t-T_1)\right), & T_1\le t\le T_1+T_2\end{cases}, \tag{1}$$

where $\dot{U}(t)$ is the complex signal amplitude; $\left|\dot{U}\right|$ is the complex signal amplitude module; $\varphi_1(t)=2\pi\int f_1(t)dt$, $\varphi_2(t)=2\pi\int f_2(t)dt$ are the instantaneous phases of the first and second signal fragments, respectively; $f_1(t), f_2(t)$ are the instantaneous frequencies of the corresponding signal fragment; T_1, T_2 are the durations of the first and second signal fragments.

To simplify the recording, we isolate from (1) the components of the instantaneous phase and in the future, we will operate on them and the components of the instantaneous frequency obtained by differentiating them.

Define the instantaneous phase of the LFM fragments as [22, 30, 32, 34]:

$$\varphi_n(t)=2\pi\begin{cases}f_0t+\dfrac{\beta_1}{2}t^2, & 0\le t\le T_1;\\ (f_0+\beta_1T_1)(t-T_1)+\dfrac{\beta_2}{2}\left(t^2-T_1t\right), & \\ \qquad T_1\le t\le T_1+T_2, & \end{cases} \tag{2}$$

where f_0 is the initial frequency of NLFM signal; β_1, β_2 are the frequency modulation rates of the first and second FM fragments, which are equal to:

$$\beta_1=\frac{\Delta f_1}{T_1};\quad \beta_2=\frac{\Delta f_2}{T_2},$$

where Δf_1, Δf_2 are the frequency deviations (the difference between the upper and the lower frequencies) of the corresponding LFM fragment.

The instantaneous signal frequency varies Eq. (3) as:

$$f(t)=\begin{cases}f_0+\beta_1t, & 0\le t\le T_1;\\ f_0+\beta_1T_1+\beta_2(t-T_1), & T_1\le t\le T_1+T_2.\end{cases} \tag{3}$$

A plot of the signal frequency change over time (3) is shown in Fig. 1. From the analysis (3) and Fig. 1, it turns out that in the MM under consideration, the calculation of the parameters of the second fragment of the NLFM signal begins with a zero time reference, that is, at, $t=T_1$, $t=(t-T_1)=0$, the actual graph of the instantaneous frequency change is displayed in Fig. 1 with a dotted line, which then shifts in time by a value T_1 (shown by arrows). Therefore, model (2), (3) will be considered to use shifted time.

Regardless of what physical processes occur at the moment of changing the value β_1 by, β_2 the value is taken as the initial frequency of the second fragment $f_0+\Delta f_1$ when it is actually equal f_{02}.

The initial phase of the second fragment has zero value, despite the fact that the final phase of the first fragment is equal to $2\pi(f_0+\Delta f_1)T_1$. This means that at the moment of changing the speed of the FM during the transition to the second fragment of the signal, the instantaneous frequency and phase jump occurs, which is not taken into account in the considered MM with a time shift. This model is generally insensitive to frequency jumps.

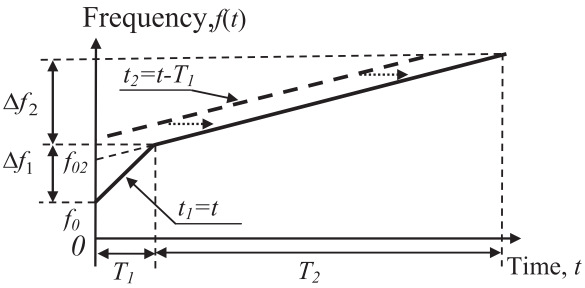

Fig. 1. Diagram of frequency change of two-segment NLFM signal

Let's consider in more detail how the phase of the signal changes at the moment of transition from the first LFM fragment to the second in the current time. From Fig. 1 it turns out that the final phase of the first fragment φ_{E1} is defined as:

$$\varphi_{E1} = 2\pi\left(f_0 T_1 + \beta_1 \frac{T_1^2}{2}\right),$$

and the initial phase of the second fragment φ_{02} is:

$$\varphi_{02} = 2\pi\left(f_{02} T_1 + \beta_2 \frac{T_1^2}{2}\right),$$

where $f_{02} = \Delta f_1 - \beta_2 T_1 = (\beta_1 - \beta_2)T_1$.

Determine the phase $\delta\varphi_{12}$ jump at the time of change β_1 to β_2:

$$\delta\varphi_{12} = \varphi_{02} - \varphi_{E1}. \quad (4)$$

After substituting the values φ_{02}, φ_{E1} in (4) and performing simple transformations, we get:

$$\delta\varphi_{12} = \pi(\beta_2 - \beta_1)T_1^2, \quad (5)$$

by further differentiation we find that the frequency jump δf_{12} at this very moment in time is:

$$\delta f_{12} = (\beta_2 - \beta_1)T_1. \quad (6)$$

Thus, based on (5) and (6), we state that in the process of formation of the NLFM signal at the moment of transition from the first to the second LFM fragment due to a change in the value of the speed of the FM, a sudden change in the instantaneous frequency occurs, which causes a jump in the instantaneous phase of the signal.

The following subsection of the article is devoted to the development of the MM of the current time, which will take into account these changes.

3.2 Development of mathematical model of two-fragment NLFM signal in current time

As a basis, unlike (2), for MM of the current time we will use another known model of instantaneous values of phase $\varphi(t)$ (7) (expressions for instantaneous frequency $f(t)$ are obtained by differentiation) of a two-fragment NLFM signal [27]:

$$\varphi(t)=\begin{cases}2\pi\left(f_0 t+\beta_1\dfrac{t^2}{2}\right), & 0\le t\le T;\\[2mm] 2\pi\left((f_0+\beta_1 T_1)t+\beta_2\dfrac{t^2}{2}\right), & T_1\le t\le T_1+T_2.\end{cases} \quad (7)$$

To determine the effect of a frequency jump, we introduce an auxiliary model, which we obtain by compensating for (7) the frequency jump (6), we get:

$$\varphi(t)=\begin{cases}2\pi\left(f_0 t+\beta_1\dfrac{t^2}{2}\right), 0\le t\le T_1;\\[2mm] 2\pi\left([f_0-(\beta_2-\beta_1)T_1]\,t+\beta_2\dfrac{t^2}{2}\right),\\ \qquad T_1\le t\le T_1+T_2.\end{cases} \quad (8)$$

We add in (8) the compensating phase component (5) and obtain the MM of the instantaneous phase of the two-fragment NLFM signal in the current time:

$$\varphi(t)=\begin{cases}2\pi\left(f_0 t+\dfrac{\beta_1 t^2}{2}\right), 0\le t\le T_1;\\[2mm] 2\pi\left([f_0-(\beta_2-\beta_1)T_1]\,t+\dfrac{\beta_2 t^2}{2}+\dfrac{(\beta_2-\beta_1)T_1^2}{2}\right),\\ \qquad T_1\le t\le T_1+T_2.\end{cases} \quad (9)$$

Thus, a new MM of a two-fragment NLFM signal (9) is obtained, which includes LFM fragments, it takes into account the abrupt change in instantaneous frequency and phase at the moments of change in the value of the speed of the FM. The developed model was tested using the MATLAB software package.

3.3 Results of mathematical modelling

Further analysis will be carried out by comparing the results of modelling. The results obtained using the considered MM with fixed parameters of the two-fragment NLFM signal were compared: $\Delta f_1 = \Delta f_2 =$ 200 kHz, $T_1 = 20$ µs, $T_2 = 100$ µs.

Figure 2 shows the results of using the current time MM without compensating for frequency and phase jumps (7). For better clarity, the results are set $f_0 = 0$. At the moment of transition to the second fragment of the signal on the oscillogram of Fig. 2a, a phase jump is observed, on the graph of Fig. 2b at this very moment we see a sudden change in frequency, the signal spectrum of Fig. 2c is distorted – it has a significant dip at a frequency of 200 kHz and ripple on the slopes, the ACF MPSLL is -14.4 dB, the width of ACF main lobe at zero level is 10.8 µs (Fig. 2d).

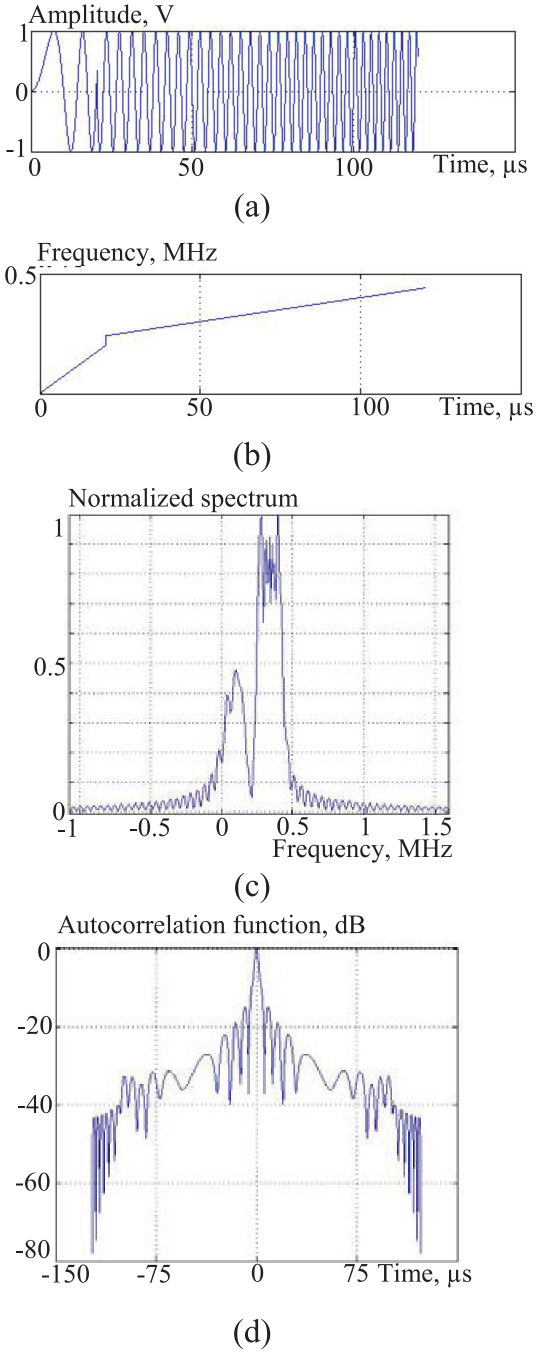

Fig. 2. Oscillogram (a), the graph of instantaneous frequency change (b), the spectrum (c), ACF (d) of the NLFM signal according to model (7)

For the current time model with frequency jump compensation (8) in Fig. 3, we see that the phase jump of the signal in Fig. 3a has been preserved, the frequency jump in Fig. 3b, compensated, the dip in the spectrum of Fig. 3c has become significantly smaller, however, the ripples on its slopes remained, the MPSLL decreased to a level of -15.08 dB, and the width of the main lobe at the zero level increased to 11.46 µs (Fig. 3d). At the edges of the graphs of Fig. 2d and Fig. 3d, there is a sharp drop in the level of the ACF side lobes and a sharp increase in the pulsation frequency, which corresponds to a larger value of the FM speed and is a characteristic sign of the presence of a jump in the signal phase at the time of transition to the second fragment. Next, we compare the models of shifted time (2), (3) and current time with the compensation of frequency and phase jumps (9). The simulation results are shown in Fig. 4 and Fig. 5.

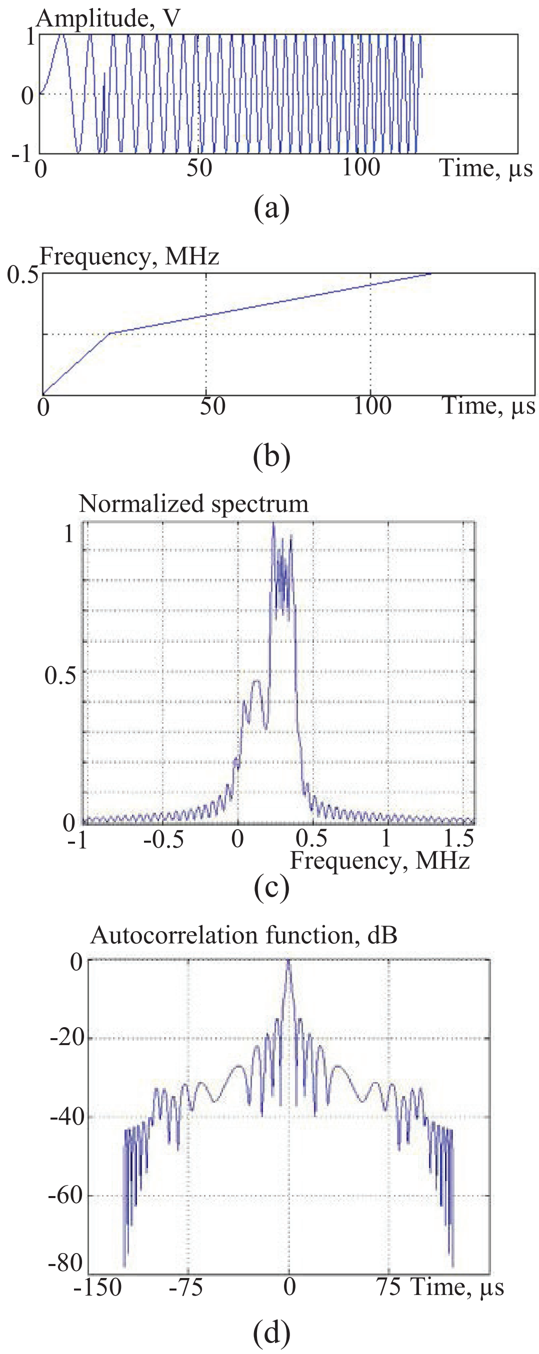

Fig. 3. Oscillogram (a), the graph of the instantaneous frequency change (b), spectrum (c), ACF (d) of the NLFM signal according to model (8)

In the oscillogram of the signal in Fig. 4a, a phase jump is observed at the moment of change in the speed of the FM, the frequency change graph for both MMs is identical to Fig. 3b, the signal spectrum also has a dip.

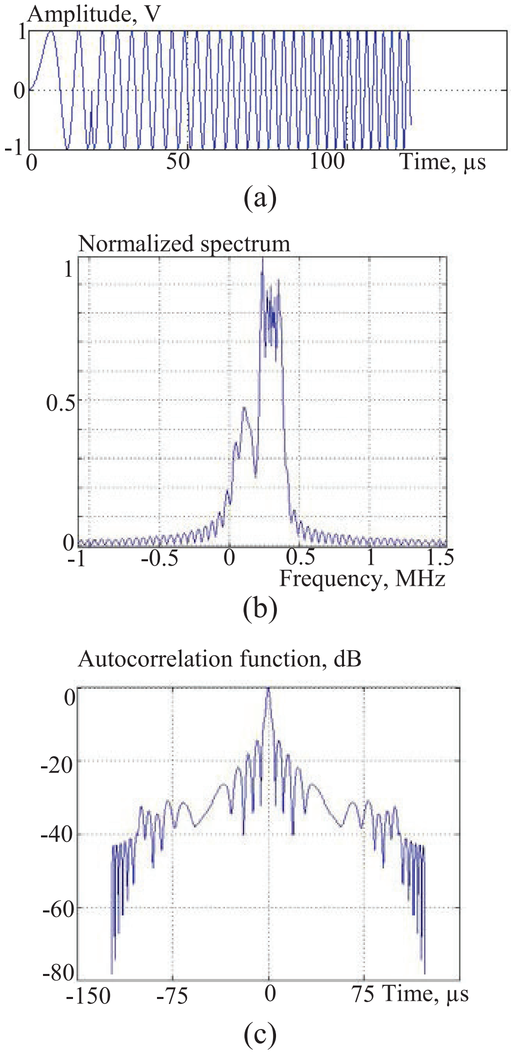

Fig. 4. Oscillogram (a), spectrum (b), ACF (c) NLFM signalas model observed in the models (2)-(3)

Another nature of the change in frequency and phase at the time of transition to the next fragment of the signal causes a two-way distortion of the top of the spectrum, which explains the decrease in ACF MPSLL compared to the usual LFM signal, the pulsations on the spectrum slopes are similar to Fig. 2 and Fig. 3. Autocorrelation function MPSLL is -14.57 dB with the width of the main lobe at a zero level of 11.4 μs. There is also a difference in the SLL ACF Fig. 4c, similar to Figs. 2d and 3d. Simulation results for the current time MM with simultaneous compensation of frequency and phase jumps (9) are shown in Fig. 5. They indicate that after the introduction of the compensation component (5) in (9), the jump in the signal phase disappeared (Fig. 5a). The signal spectrum of Fig. 5b was noticeably rounded on one side, and the higher frequency part remained unchanged, there were no pulsations on the slopes. The MPSLL value of the ACF signal is −15.77 dB, while the width of the main lobe at the zero level is 10.52 μs, unlike previous cases, there is a smooth change in the level and frequency of pulsations of the side lobes of ACP Fig. 5c.

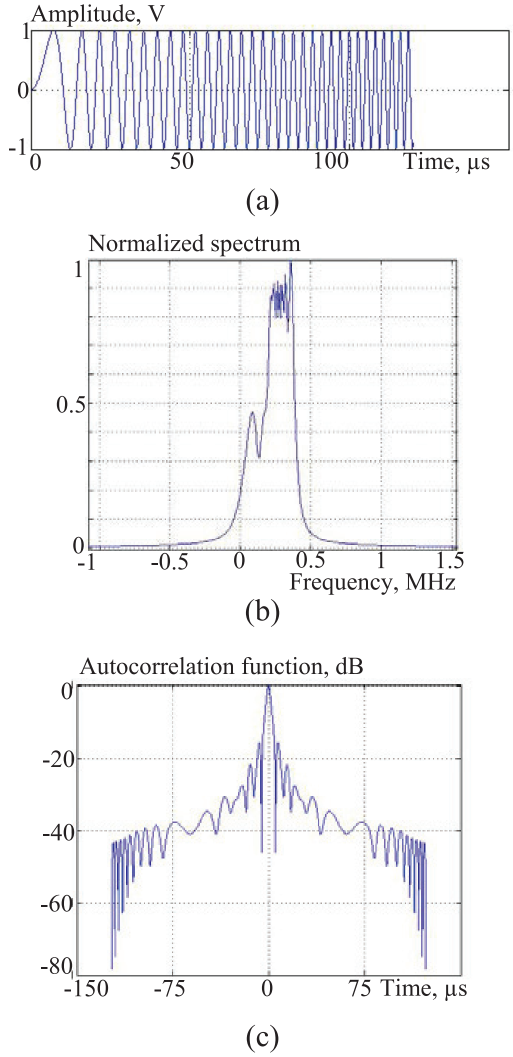

Fig. 5. Oscillogram (a), spectrum (b), ACF (c) of NLPM signal according to model (9)

Analysis of the results of mathematical modelling Fig. 2-Fig. 5 states that the use of the NLFM model of the signal in the current time (9) ensures that there are no instantaneous frequency and phase jumps at the moment of change in the speed value of the FM. The type of signal spectrum takes the expected form – it is noticeably rounded at low frequencies, the ripples of the stingrays disappeared in it, which led to a decrease in the maximum level of the side lobes of the ACF, a decrease in the SLL occurs smoothly, which is a characteristic sign of the absence of frequency and phase jumps of the resulting NLFM signal. Compared to the known MM in shifted time, due to compensation for the phase jump, the MPSLL decreased by 1.2 dB with a narrowing of the width of the ACF main lobe by about 8%, which indicates an increase in the effective spectrum width of the resulting NLFM signal.

The rate of decline of the average SLL for models (2), (7), (8) is almost the same and is 15 dB/dec. For the model (9) developed by the authors, this figure is 17.5 dB/dec.

Conclusions

The paper considers the most common MM signals with non-linear frequency modulation, which consist of two LFM fragments.

For the first time, it was shown that the models in question are characterized by significant shortcomings, namely, the presence of frequency and phase jumps (or only phase) of the signal when moving to the next fragment. In addition, such an abrupt change in frequency and phase (or only phase) leads to the appearance of symmetrical significant differences in the SLL relative to the center and an increase in the frequency of pulsations of the ACF signal at a time that corresponds to the beginning of a new fragment.

Taking into account the detected effects, a new MM NLFM signal was developed. Unlike those known in the proposed model, instantaneous frequency and phase jumps are compensated for, which occur at the moment of change in the speed of the FM when switching from one signal fragment to another.

It is shown that using the developed model provides the best spectral characteristics of the resulting two-fragment NLFM signal, shape distortion disappears and the effective spectrum width increases. This causes a decrease in the MPSLL, an increase in the rate of decline of the average SLL and a decrease in the width of the main lobe of the ACF signal. The considered MM is quite illustrative, the proposed approach can be extended to NLFM signals with a large number of LFM fragments, as well as combinations of fragments with linear and other types of modulation. The model can be useful for developers and researchers of systems for the formation and processing of NLFM signals. In the future, based on the results obtained, it is planned to develop MM of the current time of three-fragment NLFM signals that can potentially be used in practical application.

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Математична модель двофрагментного сигналу з нелінійною частотною модуляцією у поточному часі

Костиря О. О., Гризо А. А., Додух О. М., Нарежній О. П.

Переваги використання частотно-модульованих сигналів для локації об'єктів полягають у можливості застосування довготривалих зондувальних імпульсів. Такі сигнали забезпечують необхідну випромінювану потужність з одночасним збереженням потрібної розрізнювальної здатності з дальності. Одним з таких сигналів, що знайшов широке прикладне використання, є сигнал з лінійною частотною модуляцією. Небажаним ефектом узгодженої фільтрації такого радіоімпульсу є достатньо великий рівень бічних пелюсток стисненого сигналу на виході пристрою обробки, максимальний рівень яких складає приблизно мінус 13 дБ. Такий ефект може призвести до збільшення ймовірності хибного виявлення або маскування менш потужних сигналів бічними пелюстками сигналів з більшою потужністю. Одним з методів зниження рівня бічних пелюсток є застосування сигналів з нелінійною частотною модуляцією. Прикладом таких сигналів є відомий двофрагментний сигнал, що складається з поєднаних у часі лінійно-частотно модульованих фрагментів. Однак математичні моделі, які використовуються для опису такого сигналу, не в повній мірі відображають ефекти, що виникають у момент переходу від одного фрагменту сигналу до другого. Ці ефекти проявляються у стрибкоподібній зміні частоти та фази, що призводить до спотворення спектру сигналу, підвищення рівня бічних пелюсток автокореляційної функції та різких перепадів їх рівня. Такі ефекти не досліджувалися у відомих роботах, про що свідчать наведені у першому розділі статті результати аналізу досліджень і публікацій. У другому розділі роботи сформульовано завдання дослідження. Третій розділ роботи присвячено розробці механізму компенсації прояву виявлених ефектів та його математичному опису, що перевірено шляхом моделювання. З урахуванням виявлених ефектів розроблено нову математичну модель нелінійно-частотно модульованого сигналу. На відміну від відомих у запропонованій моделі компенсуються стрибки миттєвої частоти та фази, які виникають у моменти зміни швидкості частотної модуляції при переході від одного фрагменту сигналу до іншого.

Подальші дослідження доцільно зосередити на особливостях компенсації прояву виявлених ефектів для сигналів з більшою кількістю фрагментів, а також комбінацій фрагментів з різними видами модуляції, про що вказано у висновках до роботи.

Ключові слова: нелінійна частотна модуляція; математична модель; автокореляційна функція; рівень бічних пелюсток; стрибки частоти та фази