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Mathematical Modeling of Piezoelectric Ceramic Ring Transducers for Functional Instrumentation

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The article reviews a mathematical model of piezoelectric ceramic ring transducers which are functional, highly effective, and applicable as components of functional instrumentation devices, such as sensors, automatic control devices, measuring devices, data collection devices, electronic control systems, etc. The main distinctive characteristic of the mathematical model developed in this study is the ability to establish analytical dependencies for determining such electromechanical characteristics of a piezoceramic ring as: electrical impedance, quality factor, elastic modulus, piezo modulus, dielectric constant, as well as the amplitude values of the electric charge and electric current on the electroded surfaces of the piezoceramic ring, thus significantly expanding the range of these products and determine their operational characteristics at the design stage. The key research question of this study is frequency dependence of the change in electrical impedance for a ring made of PZT-type (plumbum zirconate titanate) piezoelectric ceramics, which significantly depends on the values of mechanical and geometric parameters, the wave number of elastic oscillations, as well as the corresponding Bessel and Neumann functions of the first order, according to which a sharp decrease in the electrical impedance from 4900 to 10 Ohms is observed when the quasi-wave number increases from 0 to 2. Also, this study has established a high degree of convergence between the theoretically obtained and experimentally determined electrical impedance modules for ring transducers made of PZT-type piezoelectric ceramics (the discrepancy between the impedance values in these cases did not exceed 16%).

Keywords: piezoelectric transducer; functional instrumentation; mathematical model; ring element; impedance

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Introduction

Developing highly efficient functional devices and systems is an extremely important task in modern science and technology due to enabling creation of new technical tools (sensors, automatic control devices, measuring devices, data collection devices, electronic control systems, and much more) that provide implementation of various functions [1]. According to the analysis performed by "Yole Développement", a well-known analytical company [2], one of the most practical materials for functional device construction is piezoelectric ceramics, which is associated with such advantages as high accuracy, reliability, manufacturability, multifunctionality, as well as the possibility to miniaturize products made of it and to further integrate them into microcircuit engineering devices.

Among the entire nomenclature of piezoceramic transducers for functional instrument construction, ring-shaped elements hold a special place due to several specific characteristics that are advantageous as compared with piezoceramic elements of other shapes [3], namely, higher efficiency, resistance to the influence of external factors, broadband and lower energy consumption during their operation. However, to achieve enhanced functional performance of such piezoelectric transducers, it is necessary to improve the scientific and technical foundations of piezoelectric engineering, which includes improving manufacturing methods and technologies, as well as the mathematical apparatus of such transducers. To this end, there is a need to study the features of mathematical modeling for ring piezoelectric transducers used for functional instrumentation. Analyzing results of such modeling will define ways to improve the quality and efficiency of devices that use piezoelectric transducers [4].

1 The relevance of the research based on analysis of publications

By researching various aspects of mathematical modeling of ring piezoelectric transducers, we determine their main properties and parameters, together with the optimal parameters of their design to achieve the best results in functional instrumentation. Hence, mathematical modeling of products made of piezoelectric ceramics attracts researchers and scientists from various disciplines, such as mathematics, physics, electrical engineering, and mechanics. The leading research groups in this field include such institutions as the Institute of Piezotechnics and Electromechanics at the National Academy of Sciences of Ukraine, the Institute of Mechanical Problems at the National Academy of Sciences of Ukraine, the Institute of Electrodynamics at the National Academy of Sciences of Ukraine, and others [5–7]. Also, research groups at universities and scientific centers in a number of countries are engaged in the above issues [8–10].

There have been a number of studies on various aspects of mathematical modeling for products made of piezoelectric ceramics (including ring-shaped products) conducted by Ukrainian researchers, for example, Dmitry Balandin, Alexander Cherpakov, Andrey Nasedkin, Pavel Oganesyan, Ivan Parinov, Egor Petrakov, Oleg Petrishchev, Vladislav Yakovlev, Arcady Soloviev [11–14] and others. Among foreign scientists, we would like to mention Jian Chen, Edward Grant, Hong Hu, Hui Jin, Fred Livingston, Wei Liu, Guoxiang Peng, Guangqing Wang, Jin Yao [15–18] and others.

Thus, having analyzed the aforementioned scientific literature devoted to the mathematical description of physical processes in piezoelectric transducers, the authors of this article have concluded that unified mathematical models of ring piezoelectric transducers are absent. In addition, little research has been found that surveyed the specifics of calculating their electrical impedance.

Establishing the key aspects of mathematical modeling of ring piezoelectric transducers and experimental confirmation of the results obtained with this model is a complex task, since piezoelectric transducers have a complex geometry and mechanical structure, and their properties depend on a number of factors. Therefore, developing such a mathematical model is an urgent task for functional instrumentation, hence the purpose of this study is to solve it.

2 Mathematical modelling of a ring-shaped piezoceramic transducer

In this research, a washer is a disk with a hole, radial dimensions $R_1 < R_2$ and thickness α which satisfies the strong inequality $\frac{\alpha}{R_2-R_1} \ll 1$.

It is assumed that the washer (Fig. 1) is made of polarized through the thickness piezoceramics with electroded surfaces $z = 0$ and $z = -\alpha$ being connected to a source of a harmonically time-varying electric potential difference. If the circular frequency ω of the sign change in the potential difference is chosen such that the scale of the spatial stress-strain state inhomogeneity is commensurate with the radius R_2, the radial oscillations mode is realized in the piezoelectric element.

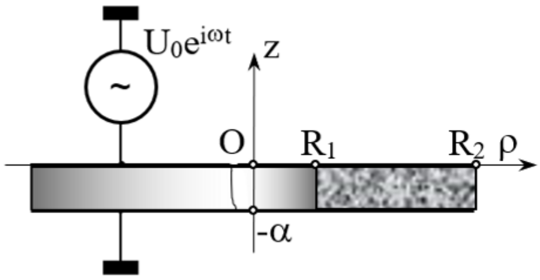

Fig. 1. A calculation scheme for a piezoceramic washer

The overall scheme for constructing a relation to calculate the radial component of the material particles' displacement vector in a piezoceramic washer is described in [19] and without any reservations corresponds to the situation under consideration.

The problem solved in [20] is different since the point $\rho = 0$ does not belong to the domain of existing solution for the equation of harmonic oscillations in the direction of the radial axis. For this reason, the notation for the general solution of this equation is

$$u_\rho = AJ_1(\gamma\rho) + BN_1(\gamma\rho), \tag{1}$$

where A and B are the constants to be defined; $J_1(\gamma\rho)$ and $N_1(\gamma\rho)$ are Bessel and Neumann functions of the first order; γ is the wave number defined. The constants A and B are determined from the boundary conditions

$$\sigma_{\rho\rho}|_{\rho=R_1} = 0, \quad \sigma_{\rho\rho}|_{\rho=R_2} = 0, \tag{2}$$

where $\sigma_{\rho\rho}$ denotes surface $\rho = const$ normal resulting mechanical stress, which is determined by the relation (1) and is calculated by the following formula

$$\sigma_{\rho\rho} = \gamma c_{11}\left\{A\left[J_0(\gamma\rho) - \frac{1-k}{(\gamma\rho)}J_1(\gamma\rho)\right] + \right.$$
$$\left. +B\left[N_0(\gamma\rho) - \frac{1-k}{(\gamma\rho)}N_1(\gamma\rho)\right] + \frac{e^*_{31}U_0}{\gamma\alpha c_{11}}\right\}, \tag{3}$$

where $k = \frac{c_{12}}{c_{11}}$.

By substituting the values $\rho = R_1$ and $\rho = R_2$ into the relation (3) and equating the results obtained to zero, as required by conditions (2), we obtain a system of algebraic equations

$$\begin{aligned} Aa\left(\gamma R_1\right) + Bb\left(\gamma R_1\right) &= -\frac{e_{31}^{*}U_0}{\gamma\alpha c_{11}}, \\ Aa\left(\gamma R_2\right) + Bb\left(\gamma R_2\right) &= -\frac{e_{31}^{*}U_0}{\gamma\alpha c_{11}}, \end{aligned} \tag{4}$$

where

$$a\left(\gamma R_m\right) = J_0\left(\gamma R_m\right) - (1-k)\,\frac{J_1\left(\gamma R_m\right)}{\gamma R_m};$$

$$b\left(\gamma R_m\right) = N_0\left(\gamma R_m\right) - (1-k)\,\frac{N_1\left(\gamma R_m\right)}{\gamma R_m}; \quad m = 1, 2.$$

The solution for the system of equations (4) with respect to the required constants A and B is obvious:

$$\begin{aligned} A &= -\frac{e_{31}^{*}U_0}{\gamma\alpha c_{11}^{*}} \cdot \frac{b\left(\gamma R_2\right) - b\left(\gamma R_1\right)}{\Delta\left(R_1, R_2\right)}; \\ B &= -\frac{e_{31}^{*}U_0}{\gamma\alpha c_{11}^{*}} \cdot \frac{a\left(\gamma R_1\right) - a\left(\gamma R_2\right)}{\Delta\left(R_1, R_2\right)}, \end{aligned} \tag{5}$$

where $\Delta\left(R_1, R_2\right) = a\left(\gamma R_1\right) b\left(\gamma R_2\right) - a\left(\gamma R_2\right) b\left(\gamma R_1\right)$ is the determinant in the system of equations (4).

Substituting the expressions (5) into the definition (1) produces the final formula for calculating the values of the radial component u_ρ for the elastic displacements vector of the washer's material particles

It follows from expression (6) that the zeros of $\Delta(R_1, R_2)$ function determine the frequencies, namely, the frequencies of electromechanical resonances at which u_ρ displacements increase indefinitely. Indeed, the numerical values of these frequencies depend on two parameters, namely, on the mechanical parameter $k = \frac{c_{12}}{c_{11}}$ and the geometric parameter $r_{12} = \frac{R_1}{R_2}$.

Table 1 shows the numerical values of the first square root $x_1 (x = \gamma R_2)$ in the equation $\Delta(R_1, R_2) = 0$ and the ratio of the second square root to the first one $x_{12} = \frac{x_2}{x_1}$, calculated for various values of parameters k and r_{12} under the assumption that there are no energy losses in the volume of the deformable washer, i.e., the quality factor of the material is $Q_0 \to \infty$.

$$u_\rho = -\frac{e_{31}^{*}U_0 R_2}{\alpha c_{11}^{*}} \cdot \left\{ \frac{\left[b\left(\gamma R_2\right) - b\left(\gamma R_1\right)\right] J_1\left(\gamma\rho\right) + \left[a\left(\gamma R_1\right) - a\left(\gamma R_2\right)\right] N_1\left(\gamma\rho\right)}{\left(\gamma R_2\right)\Delta\left(R_1, R_2\right)} \right\}. \tag{6}$$

Table 1 Numerical values for the first square roots x_1 of equation $\Delta(R_1, R_2) = 0$ and the ratios of the second square root x_2 to the first (value x_{21}) for various mechanical and geometric parameters of the piezoceramic washer

k	$R_1/R_2 = 0{,}15$		$R_1/R_2 = 0{,}30$		$R_1/R_2 = 0{,}45$		$R_1/R_2 = 0{,}60$	
	x_1	x_{21}	x_1	x_{21}	x_1	x_{21}	x_1	x_{21}
0,30	1,895940	2,540777	1,612594	3,075509	1,384092	4,297248	1,219665	6,541829
0,31	1,898681	2,532418	1,611038	3,074348	1,380930	4,304336	1,216103	6,559255
0,32	1,901281	2,524167	1,609288	3,073527	1,377594	4,311996	1,212389	6,577605
0,33	1,903736	2,516023	1,607339	3,073055	1,374081	4,320242	1,208522	6,596739
0,34	1,906043	2,507987	1,605188	3,072939	1,370388	4,329091	1,204499	6,617181
0,35	1,908198	2,500060	1,602831	3,073189	1,366513	4,338560	1,200319	6,638462
0,36	1,910196	2,492243	1,600264	3,073815	1,362453	4,348665	1,195979	6,660779
0,37	1,912033	2,484537	1,597483	3,074828	1,358205	4,359427	1,191478	6,684164
0,38	1,913703	2,476944	1,594482	3,076239	1,353767	4,370865	1,186812	6,708650
0,39	1,915202	2,469466	1,591258	3,078059	1,349135	4,383001	1,181981	6,734275
0,40	1,916524	2,462105	1,587805	3,080302	1,344306	4,395859	1,176981	6,761077
0,41	1,917663	2,454864	1,584118	3,082982	1,339277	4,409462	1,171810	6,789099
0,42	1,918613	2,447747	1,580192	3,086114	1,334045	4,423837	1,166464	6,818383
0,43	1,919367	2,440757	1,576022	3,089713	1,328604	4,439012	1,160943	6,848978
0,44	1,919918	2,433898	1,571601	3,093796	1,322953	4,455018	1,155241	6,880934
0,45	1,920380	2,427174	1,566923	3,098383	1,317086	4,471886	1,149357	6,914304
0,46	1,920380	2,420592	1,561983	3,103492	1,311000	4,489650	1,143287	6,949146
0,47	1,920275	2,414156	1,556773	3,109145	1,304690	4,508348	1,137028	6,985522
0,48	1,919934	2,407874	1,551286	3,115365	1,298151	4,258017	1,130576	7,023498
0,49	1,919346	2,401753	1,545516	3,122175	1,291379	4,548704	1,123927	7,063144
0,50	1,918503	2,395801	1,539454	3,129603	1,284369	4,570451	1,117077	7,104536

Compare the results shown in tables 1 and 2.

The data in Table 2 illustrate that the numerical values of the square root ratio x_{21} decrease gradually when the values of the mechanical parameter k increase. With an increase of 0.3 to 0.5, i.e., by a factor of 1.67 in parameter k, the ratio of square roots x_{21} for a solid disk decreases from 2.630434 to 2.505889, i.e., by 4.74% of the original value. The data in Table 1 demonstrate that the growth of parameter k for washers with a large cavity diameter ($r_{12} \geq 0,45$) causes an increase in the numerical values of the square root ratio x_{21}. Hence, for the value $r_{12} = 0.60$, the square root ratio x_{21} increases from 6.541829 when $k = 0.30$ to 7.104536 when the resolution of $k = 0.50$, i.e., by 8.6% of the original value.

Table 2 First two roots of the equation $xJ_0(x) - (1-k)J_1(x) = 0$

k	x_1	x_2	ξ_{21}
0,00	1,841184	5,331443	2,895660
0,05	1,878980	5,341153	2,842582
0,10	1,915393	5,350843	2,793601
0,15	1,950511	5,360511	2,748259
0,20	1,984414	5,370155	2,706167
0,25	2,017172	5,379773	2,666988
0,30	2,048850	5,389364	2,630434
0,35	2,079508	5,398928	2,596253
0,40	2,109198	5,408462	2,564226
0,45	2,137971	5,417963	2,534162
0,50	2,165871	5,427433	2,505889
0,55	2,192942	5,436869	2,479259
0,60	2,219221	5,446270	2,454137
0,65	2,244744	5,455635	2,430404
0,70	2,269547	5,464962	2,407953
0,75	2,293658	5,474251	2,386690
0,80	2,317109	5,483500	2,366527
0,85	2,339926	5,492708	2,347385
0,90	2,362135	5,501874	2,329195
0,95	2,383761	5,510998	2,311892
1,00	2,404826	5,520078	2,295417

For a washer with the radii ratio $r_{12} = 0.45$ a similar increase in the x_{21} values constitutes 6.36%. With the radii ratio $r_{12} = 0.30$ within the range $0.30 \leq k \leq 0.34$, we observe a slight decrease in the numerical values of the square roots ratio x_{21}, and then, starting from the value $k = 0.34$, the values increase gradually from 3.072939 to 3.129603, i.e., by only 1.84% of the initial minimum values. With a smaller radii ratio ($r_{12} = 0.15$), the numerical value of the square roots ratio tends to decrease with increasing values of parameter k. This ratio decreases from 2.540777 to 2.395801, i.e., by 6.1% compared to the original value.

The arguments above infer that the radii r_{12} ratio values significantly affect how sensitive the frequency ratio of the first two electromechanical resonances induced by the piezoceramic washer's radial vibrations is to a change in the parameter $k = \frac{c_{12}}{c_{11}}$, which, consequently, affects the reliability of determining the piezoceramic elasticity moduli. For small ($r_{12} < 0.12$) and large ($r_{12} > 0.5$) values, the ratio of the square roots x_{21} in the frequency equation $\Delta(R_1, R_2) = 0$, which is exactly equal to the ratio of the electromechanical resonance frequencies, depends quite strongly on the parameter k values. This makes it possible to reliably determine the values of the elastic moduli according to the method described in [21]. If the geometric parameter r_{12} ranges within $0.2 \leq r_{12} \leq 0.4$, situations may arise when two values of the parameter k correspond to one value x_{21} of the square roots ratio, which entails an ambiguity in determining the elastic moduli. The described situation makes applying the method represented in [19] fundamentally impossible for finding the piezoceramic material constants.

3 Electrical impedance of a flat piezoceramic washer that performs axisymmetric radial vibrations

Here we determine the electrical impedance $Z_{el}(\omega)$ of the piezoceramic washer.

Note that the electric induction vector in the volume and on the surface of the washer is completely determined by the axial component D_Z. Indeed, the circumferential component D_Φ is zero due to the axial symmetry of the piezoceramic washer's physical state. The radial component D_ρ goes to zero on the electroded surfaces $z = 0$ and $z = -\alpha$. In addition, the condition $D_\rho \approx 0$ must be satisfied on the side surfaces $\rho = (R_1, R_2)$ [22, 23]. Since the physical state of a thin washer does not depend on the axial coordinate z values, we necessarily accept that $D_\rho = 0 \; \forall \; x_k \in V$, where V denotes the washer's volume.

Since the condition $div\vec{D} = 0$ must be satisfied in the volume of the piezoceramic washer, this situation is equivalent to the condition $\frac{\partial D_z}{\partial z} = 0$.

Whence

$$D_z = e_{31}^* \left(\frac{\partial u_\rho}{\partial \rho} + \frac{u_\rho}{\rho} \right) - \chi_{33}^\sigma \frac{U_0}{\alpha}. \qquad (7)$$

The relation (7) is obtained by integrating the expression for calculating the axial component D_Z along the coordinate z considering the conditions $\frac{\partial D_z}{\partial z} = 0$ and $E_Z = -\frac{\Phi}{\partial z}$, where Φ is the electric potential in the volume of an oscillating piezoceramic washer.

The amplitude of the electric current I in the conductors that connect the electroded washer surfaces

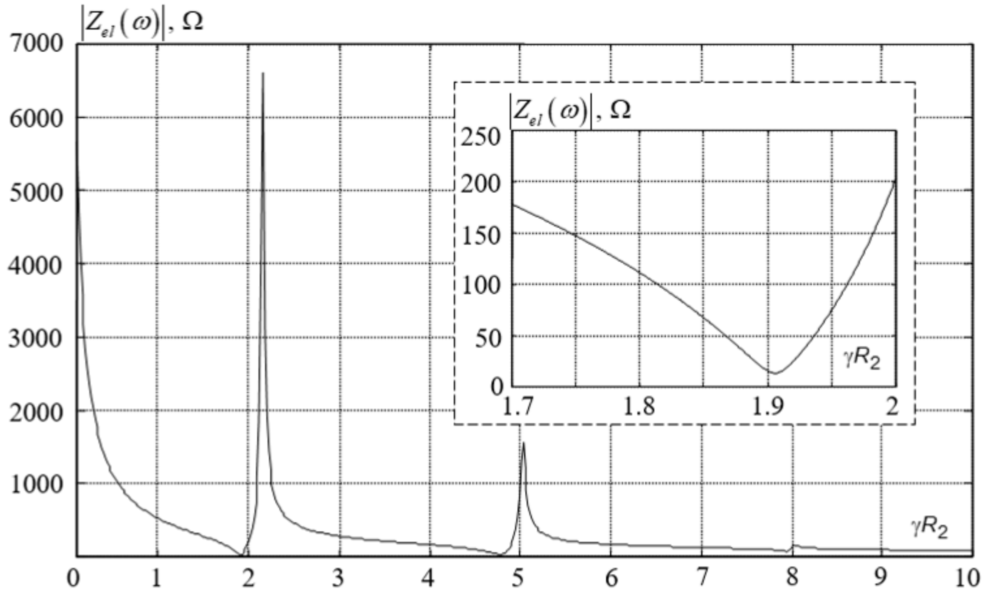

Fig. 2. The electrical impedance of a piezoceramic washer made of PZT-19

and the terminals of the electric potential difference generator (Fig. 1) is to be determined through the resulting electric charge Q by the formula $I = -i\omega Q$, that is

$$I = -i\omega 2\pi \int_{R_1}^{R_2} \rho D_z(\rho)\, d\rho. \tag{8}$$

Thus, by substituting expression (7) into formula (8) and performing integration accounting for definition (6), we obtain

$$I = i\omega C_0^\sigma U_0 \left[1 \rightleftarrows\rightleftarrows + K_{31}^2 \Xi(R_1, R_2)\right], \tag{9}$$

where $C_0^\sigma = \frac{\pi(R_2^2 - R_1^2)\chi_{33}^\sigma}{\alpha}$ is the static electric capacitance of the piezoceramic washer; $K_{31}^2 = \frac{(e_{31}^*)^2}{\chi_{33}^\sigma c_{11}}$ is the squared coefficient of piezoceramic electromechanical coupling in the planar radial vibrations mode. The frequency-dependent function (wavenumber γ) is defined by the expression

$$\Xi(R_1, R_2) = \frac{2R_2^2}{(R_2^2 - R_1^2)\gamma R_2 \Delta(R_1, R_2)} \times$$
$$\times \{[b(\gamma R_2) - b(\gamma R_1)] \cdot [J_1(\gamma R_2) - r_{12} J_1(\gamma R_1)] +$$
$$+ \ [a(\gamma R_1) - a(\gamma R_2)] \cdot [N_1(\gamma R_2) - r_{12} N_1(\gamma R_1)]\}.$$

The above leads us to the final notation of the formula calculating electrical impedance $Z_{el}(\omega)$:

$$Z_{el}(\omega) = \frac{U_0}{I} = \frac{1}{i\omega C_0^\sigma [1 + K_{31}^2 \Xi(R_1, R_2)]}. \tag{10}$$

4 Discussion of simulation results

Figure 2 shows calculation results for the electrical impedance module of a piezoceramic washer made of PZT-19 with the physical and mechanical parameters: $c_{11}^E = 112\ GPa$; $c_{12}^E = 62\ GPa$; $c_{33}^E = 106\ GPa$; $\rho_0 = 7400\ kg/m^3$; $e_{33} = 18\ C/m^2$; $e_{31} = -7\ C/m^2$; $\chi_{33}^\varepsilon = 1000 \cdot \chi_0$; $\chi_0 = 8,85 \cdot 10^{-12}\ F/m$ quality factor of the ceramics is $Q_0 = 100$. The disc's thickness is $\alpha = 1$ mm. The disc's radius is $R = 10\alpha$. The parameter $r_{12} = 0,15$. The inset in the Figure 2 features the change in the module of electrical impedance appearing in the vicinity of the first electromechanical resonance frequency.

Similarly to radial oscillations of a thin piezoceramic disk, it can be argued that the frequencies of the first two electromechanical resonances and antiresonances can easily and reliably be determined using standard measuring instruments. The numerical values of these frequencies at certain values of the parameter are processed according to the method described in [21], as a result of which the main physical and mechanical parameters of piezoelectric ceramics are reliably determined.

Summarizing the above, this study has found that if the frequencies of electromechanical resonance and anti-resonance are determined, it becomes possible to estimate the numerical values of the modulus of elasticity, piezoelectric modulus e_{33} and dielectric constant χ_{33}^ε. In addition, based on the results of measuring the electrical impedance of a piezoceramic ring, we can determine the Q factor of such a product at the resonance frequency, which allows to significantly expand the nomenclature of the piezoceramic physical and mechanical parameters that can be determined experimentally.

Conclusions

In summary, this research has proposed a mathematical model of ring-shaped ceramic piezoelectric transducers, which allows determining their electrical impedance depending on the operating frequency of these transducers, as well as their physical and mechanical parameters.

Analytical dependencies have been established that allow determining the electrical impedance, Q factor, modulus of elasticity, piezoelectric modulus, dielectric constant, and the amplitude values of the electric charge and electric current on the electroded surfaces of the piezoceramic ring. As a result, the calculation for the problem of a piezoelectric ring's harmonic radial oscillations was obtained, which allows to significantly expand the nomenclature of piezoceramic physical and mechanical characteristics, which are otherwise determined experimentally.

We have calculated the numerical values for the first square root of the electromechanical resonance frequency equation, as well as the ratio of the second square root to the first one for a wide range of mechanical and geometric parameters of the piezoceramic ring.

The frequency dependence of the change in electrical impedance for a ring made of piezoelectric ceramics PZT-19 has been determined, which significantly depends on the frequency-dependent function $\Xi(R_1, R_2)$, which, in turn, depends on the values of mechanical and geometric parameters, the wave number of elastic oscillations, and the corresponding Bessel and Neumann functions of the first order. The research has shown that when the value of the quasi-wave number γR_2 increases from 0 to 2, there is a sharp decrease in the electrical impedance $Z_{el}(\omega)$ from 4900 to 10 Ohm. Another major finding was that at the frequencies of the first resonance ($\gamma R_2 = 2,18$; $Z_{el}(\omega) = 6640$ Ohm) and the second resonance ($\gamma R_2 = 5,03$; $Z_{el}(\omega) = 1495$ Ohm), the oscillating piezoelectric ring has the maximum energy consumption from the electrical signal generator.

A comparison of the electrical impedance modules of a PZT-19 piezoelectric ceramic ring was performed, which determined the frequencies of electromechanical resonance and anti-resonance, based on the results of which the numerical values of the modulus of elasticity, the piezoelectric modulus e_{33} and the dielectric constant χ_{33}^{ε} were assessed. The comparative analysis showed a high degree of agreement between the theoretically obtained data and the experimentally determined results (the discrepancy did not exceed 16%).

The paper presents data obtained as a result of the experimental scientific and technical project "Creation of ultrasonic highly efficient systems for agro-industrial complex, food industry and medicine" thanks to the named scholarship of the Verkhovna Rada of Ukraine for young scientists – doctors of science for the year 2023.

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Математичне моделювання п'єзоелектричних керамічних кільцевих перетворювачів для функціонального приладобудування

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У статті розглянуто математичну модель п'єзоелектричних керамічних кільцевих перетворювачів, які є функціональними, високоефективними та виступають компонентами приладів функціонального призначення, таких як датчики, пристрої автоматичного керування, вимірювальні пристрої, пристрої збору даних, електронні системи керування тощо. Основною особливістю розробленої в даному дослідженні математичної моделі є можливість встановлення аналітичних залежностей для визначення таких електромеханічних характеристик п'єзокерамічного кільця, як: електричний опір, добротність, модуль пружності, п'єзомодуль, діелектрична проникність, а також амплітудні значення електричного заряду та електричного струму на електродованих поверхнях п'єзокерамічного кільця, що значно розширює номенклатуру цих виробів та визначає їх експлуатаційні характеристики на етапі проектування.

Основним дослідницьким питанням даної роботи є частотна залежність зміни електричного опору кільця з п'єзокераміки типу PZT (цирконат титанат свинцю), яка істотно залежить від значень механічних і геометричних параметрів, хвильового числа пружних коливань, а також відповідні функції Бесселя і Неймана першого порядку, згідно з якими при збільшенні квазіхвильового числа від 0 до 2 спостерігається різке зменшення електричного опору від 4900 до 10 Ом. При цьому спостерігається високий ступінь збіжності між теоретично отриманими та експериментально визначеними модулями електричного імпедансу для кільцевих перетворювачів з п'єзокераміки типу PZT (розбіжність значень імпедансу в цих випадках не перевищувала 16%).

Ключові слова: п'єзоелектричний перетворювач; функціональне приладобудування; математична модель; кільцевий елемент; імпеданс