

# Energy Detector of Stochastic Signals in Noise Uncertainty

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Wide use of software-defined radio has led to a significant sophistication of electronic environment. This is mainly due to ability of generation signals of almost any shape. To detect signals with an unknown dynamic frequency-time structure, it is advisable to use advanced energy detector algorithms. The purpose of this article is to automate processes of stochastic signals detection and time parameters estimation under the conditions of unknown frequency-time structure of signals and noise power. The essence of proposed method is to detect and track temporal energy changes averaged over  $L$  samples of received signal in selected frequency channel. Threshold value for a given probability of false alarm is calculated using current estimates of signal power. This threshold is dynamic and is refined only in time intervals free from the signals. In those time windows where energy exceeds threshold, a decision is made about the presence of a signal. An algorithm for detecting stochastic signals is proposed. If a signal is present at the initial moment of time, proposed algorithm can detect only its end by a sharp decrease of signal energy. After that, new noise level is estimated and threshold value is refined. Detection curves of proposed algorithm are obtained. It is shown that when number of samples  $L$  is increased by an order, the gain in signal-to-noise ratio in signal detection is about 4 dB. The maximum value of correct detection probability of a pulse signal is achieved with the same pulse duration and the length of the integration interval. Compared to method of signal smoothing with moving average window, proposed method has less computational complexity, simplifies the search for signal time boundaries, and gives smaller errors in signal duration estimates. Recommendations for the implementation of developed algorithm are formulated.

*Keywords:* stochastic signal; energy detector; time parameters; frequency channel; noise level; threshold; integration interval

DOI: [10.20535/RADAP.2023.94.32-40](https://doi.org/10.20535/RADAP.2023.94.32-40)

## Introduction

Due to rapid development and implementation of software-defined radio technology in modern radio systems, it has become possible to generate signals of almost any shape [1, 2]. This fact leads to sophistication of electronic environment and stimulates theoretical and applied research for improving methods and algorithms of stochastic signals detection. For signal detection with an unknown structure that can change dynamically, it is advisable to use advanced energy detectors, since such detectors do not require information about signal structure, and only use difference in signal energy levels and background noise. This approach will provide fast estimation of frequency channel occupancy for cognitive radio systems [3], as well as signal detection in radio monitoring systems.

entific papers. An overview of classical approaches to stochastic radio signals detection is given in [4]. Energy algorithm for signals processing from multiple antennas is proposed in [5]. Presented in [6] method compares energies of filtered and unfiltered signal samples. In [7], Hough transform from a spectrogram is used to detect stochastic pulse signals, and in [8], it is proposed to perform a sequential analysis of signal envelope. To avoid the influence of signal envelope fluctuations, a two-threshold method of signal detection was proposed in [9]. Proposed in [10] algorithm uses rank filtering of smoothed signal energy and provides a probability of correct detection of 0.87 at a signal-to-noise ratio (SNR) of 0 dB. Proposed in [11] method is based on utilization of statistical characteristics of signal energy ratios in subbands and does not require information about the noise power or signal structure.

## 1 Related works

Problems of stochastic signals detection and time parameters estimation are considered in numerous sci-

Analysis of existing methods and algorithms for stochastic signals detection demonstrates their relatively high computational complexity and inability of time parameters estimation. Therefore, implementation

of a fast method for stochastic signals detection and time parameters estimation is a current challenge.

## 2 Problem statement

The aim of the article is to automate processes of stochastic signals detection and its time parameters estimation under the conditions of unknown frequency-temporal structure of signals and noise power.

## 3 System model

The essence of proposed method is to detect and track changes of temporal integral energy characteristics of received signal in given frequency channel.

The initial conditions for the development of an advanced energy detector are following assumptions [12]:

1) occurrence of a signal with a random structure in analyzed frequency channel results in energy level increasing in analyzed channel;

2) noise power changes much more slower and have a smaller dynamic range comparing to the signal power.

If the noise level is unknown, it must be estimated for threshold calculation. This estimate is an average energy over the integration interval. In those time windows, where energy exceeds threshold, a decision about the presence of a signal is made. In such time window, if necessary, we can search for more accurate signal boundaries.

Estimates of noise power are calculated with its  $L$  samples. Using this estimate, the threshold for a given false alarm probability is calculated, threshold processing is performed, and time parameters (time boundaries, duration) of detected signals are estimated.

Vector of complex received signal in analyzed frequency channel may be written in the following form:

$$\mathbf{x} = \mathbf{s}_I + j\mathbf{s}_Q + \xi_I + j\xi_Q, \quad (1)$$

where  $\mathbf{s}_I$ ,  $\mathbf{s}_Q$  – samples vectors of in-phase and quadrature signal components;

$\xi_I$ ,  $\xi_Q$  – samples vectors of in-phase and quadrature noise components;

$j = \sqrt{-1}$  – imaginary unit.

Vectors  $\xi_I$  and  $\xi_Q$  are normally distributed with zero mean and equal standard deviations (SD)  $\sigma_{\xi_I} = \sigma_{\xi_Q}$ . Then complex noise vector will also have a zero mean, and its SD is calculated by the following expression:

$$\sigma_{\xi} = \sqrt{\sigma_{\xi_I}^2 + \sigma_{\xi_Q}^2} = \sqrt{2}\sigma_{\xi_I} = \sqrt{2}\sigma_{\xi_Q}. \quad (2)$$

To detect stochastic signals and determine their time boundaries in analyzes frequency channel, we will

calculate average energy of received signal over  $L$  samples according to the following expression:

$$E_L = \frac{1}{L} \sum_{i=1}^L (x_I^2(i) + x_Q^2(i)). \quad (3)$$

Sampled values of energy  $E_L$  are gamma distributed. Threshold value  $\gamma$  for  $E_L$  can be calculated as the quantile of the gamma distribution of  $p = 1 - P_F$  level for a given probability of false alarm  $P_F$ . To accomplish this, we use an approximation of gamma distribution quantile  $\gamma_p$  using chi-square distribution  $\chi^2$  [13]:

$$\gamma_p = \chi_p^2(2(L+1)), \quad (4)$$

where  $\chi_p^2(2(L+1))$  is a value of quantile of  $p$  level for  $\chi^2$  distribution with  $K = 2(L+1)$  degrees of freedom.

Using the Wilson-Hilferty approximation of  $\chi^2$  distribution quantiles [13] threshold value  $\gamma$  can be calculated using the following expression:

$$\gamma = C \left( 1 - \frac{2}{9K} + u \sqrt{\frac{2}{9K}} \right)^3, \quad (5)$$

where  $C$  is a refined value of noise energy and

$$u = \frac{1,24 + 0,85H^{0,657}}{1 + 0,0001H^{-3} + \frac{2,38}{H}}, \quad (6)$$

$$H = -0,96 \ln \left( \frac{P_F}{1 - P_F} \right).$$

Calculated in this way threshold will be dynamic, as it tracks noise power changes in given frequency channel.

Value of  $C$  is calculated as arithmetic mean of energy  $E_L$  for those integration intervals where threshold is not exceeded. Where threshold is exceeded,  $C$  is not refined.

Since there are no estimates of noise power in analyzed frequency channel at the initial stage of observation, two possible cases must be considered: signal is present or absent in analyzed channel.

In the first case, due to uncertainty of noise power, we can only detect the end of signal by a sharp decrease of energy in given channel. A sharp change of channel energy is considered to be a change that exceeds current value of power estimation SD  $\sigma_E$  by a factor of  $\alpha$ . The coefficient  $\alpha$  determines probability that a random local energy drop will be accepted as signal end and depends on length of integration window  $L$ . When  $L > 30$  probability density function of  $E_L$  can be approximately considered as normal. Then, in the case of a rectangular signal envelope, for a value of the coefficient  $\alpha = 6$ , it can be assumed with a probability of 0.997 that a sharp change in energy has occurred and a signal end has been detected. Then noise level may be estimated and threshold recalculated.

If there is no signal at initial time, noise level in frequency channel is estimated, threshold is calculated,

and if it is exceeded, a decision of signal detection is made. As long as the calculated energy values exceed the threshold, its value does not change and the noise power is not refined.

In sophisticated electronic environment with sharp changes of power in analyzed channel, as well as a significant number of such levels, it is advisable to estimate noise power at the time interval with the lowest power level.

The choice of  $L$  value is compromised. To increase the accuracy of noise power estimates,  $L$  should be as large as possible. On the other hand, to reduce errors of signals time parameters estimates,  $L$  should be as small as possible.

The error in determining signal begin  $\Delta t$  caused by a mismatch between signal begin and integration interval start. This error is a uniformly distributed random variable and varies from 0 (start of integration interval coincides with signal begin) to  $L$ .

Error in estimating signal duration  $\varepsilon_t$  depends on  $L$ , error in determining signal begin  $\Delta t$ , and SNR (at low values). Value of this error for different signal durations can be estimated by the following expression:

$$\varepsilon_t = \begin{cases} \frac{L}{N_s} - 1, & k \geq 1 \\ \frac{pL - N_s}{N_s}, & k < 1 \end{cases}, \quad (7)$$

where  $N_s$  – pulse length in samples;

$k = L/N_s$  – ratio of integration interval and signal duration;

$p = \lceil N_s/L \rceil$  – rounded to the nearest integer ratio of the signal duration and window length.

When  $\Delta t \approx 0.5L/F_s$  for  $k \geq 1$ , value of  $\varepsilon_t$  can be twice larger as that calculated by expression (7), and for  $k < 1$  it can be  $\varepsilon_t + 2L$ . The error value  $\varepsilon_t$  can be zero even when  $\Delta t \neq 0$ , for cases when  $k \leq 1$  and  $N_s/L$  is an integer.

## 4 Algorithm of stochastic signal detection

Figure 1 shows block diagram of stochastic signals detecting algorithm in a given frequency channel. This algorithm requires following input data: vector of received signal samples  $\mathbf{x}$ , value of false alarm probability  $P_F$ , length of integration interval  $L$  and coefficient  $\alpha$ , as well as auxiliary variables  $i$ ,  $K$ . Variable  $i$  corresponds to number of integration interval of length  $L$  and is used only at initial stage of algorithm. Variable  $K$  is used to store the number of integration intervals in cases where there is a sharp decrease of signal power.

In block 2, average value of signal energy  $E_L(i)$  contained in  $L$  signal samples is calculated in accordance with expression (3) for the  $i$ -th integration interval. If  $i = 0$  (block 3), then value of coefficient  $C$  (refined energy value) is equated to energy value

$E_L$  (block 4) and threshold is calculated (block 5) in accordance with expression (5).

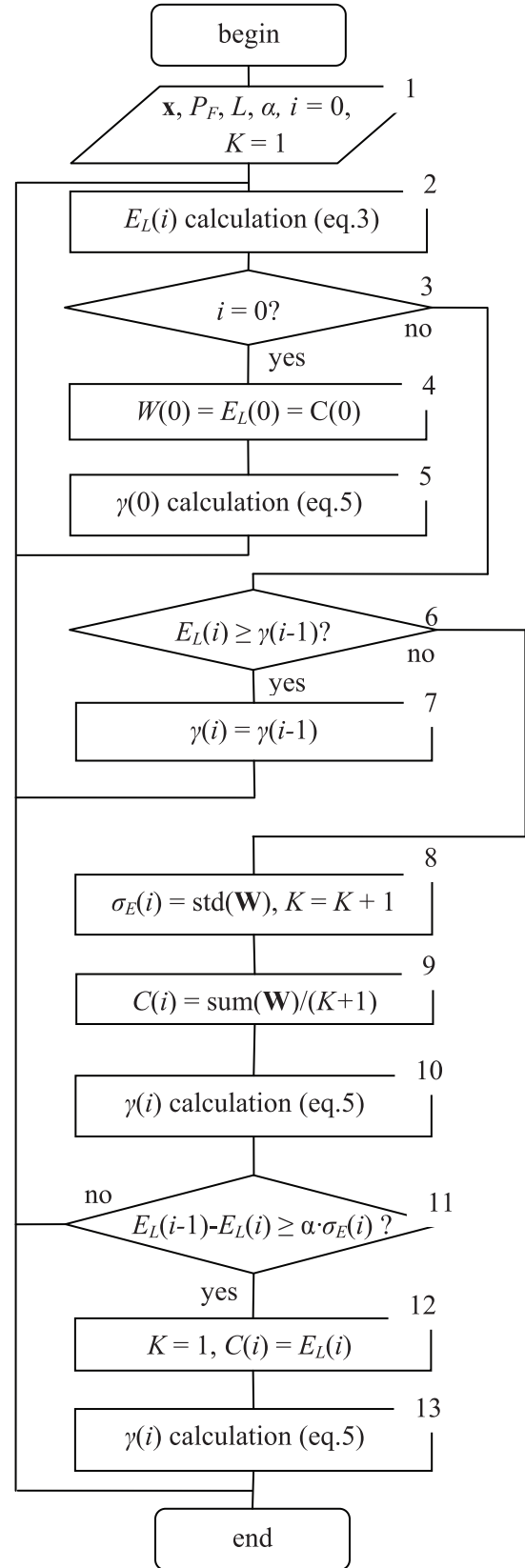


Fig. 1. Block diagram of stochastic signal detection algorithm

If it is not the first processed signal fragment ( $i > 0$ ), value of average signal energy  $E_L(i)$  calculated at this iteration is compared with threshold value  $\gamma(i-1)$  calculated at the previous iteration (block 6). If this condition is fulfilled, a decision of signal start detection is made and threshold value at current iteration is equal to corresponding value for previous iteration (block 7). If threshold is not exceeded, then SD  $\sigma_E(i)$  of vector  $\mathbf{W}$  is calculated and value of  $K$  is increased by 1 (block 8). Vector  $\mathbf{W}$  contains values of  $E_L(i)$  for those integration intervals where threshold is not exceeded. In block 9, refined value of energy  $C$  is calculated, which is used to calculate threshold in block 10.

In block 11, a sharp decrease of signal energy is detected. If such decrease is detected, a decision is made about signal end. Then value of auxiliary variable  $K$  is equal to 1 and value of coefficient  $C$  is equal to value of average energy over a given integration interval (block 12). Blocks 11-13 are mainly intended to detect signal end in case when it was present at initial moment of time, as well as to process false alarms.

## 5 Results and discussion

At a small value of parameter  $\alpha$ , an additional mechanism of false signal samples detection due to sharp random changes in noise power appears. Figure 2 shows results of threshold processing of noise samples for  $P_F = 10^{-3}$  and  $\alpha = 4$  (a) and  $\alpha = 6$  (b) at  $L = 20$ . Figures 3a, b show similar graphs for  $L = 100$ . In these figures  $E_L(i)$  and  $\gamma(i)$  denote average energy and threshold for the  $i$ -th integration interval, respectively. Value of  $n$ -th sample of received signal is denoted as  $x(n)$ . For both cases,  $10^5$  samples of noise were analyzed.

Comparative analysis of Fig. 2 and Fig. 3 allows us to conclude that by increasing length of integration interval  $L$ , we can reduce frequency of false threshold exceedances for fixed values of noise sample length and  $P_F$ . The same effect can be achieved by increasing coefficient  $\alpha$ .

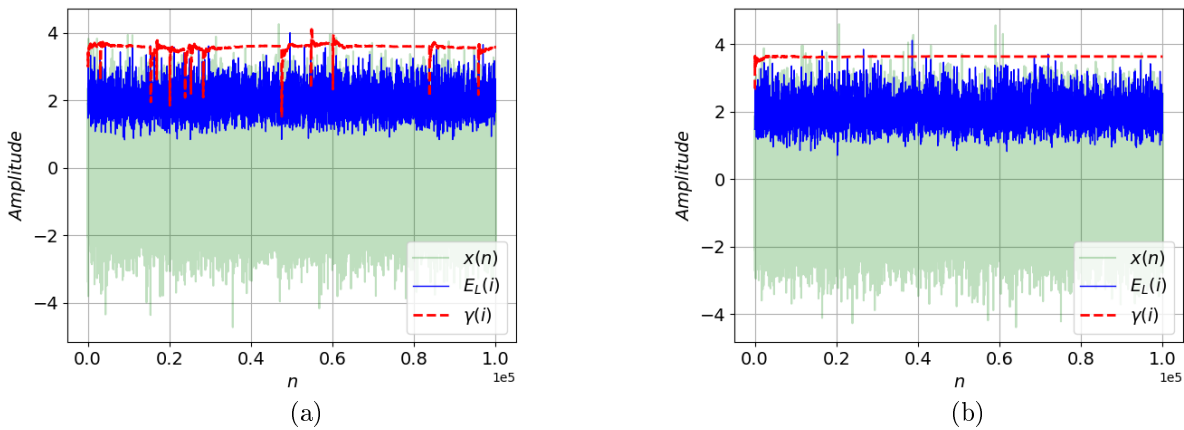


Fig. 2. Thresholding of noise samples for  $\alpha = 4$  (a) and  $\alpha = 6$  (b) at  $L = 20$

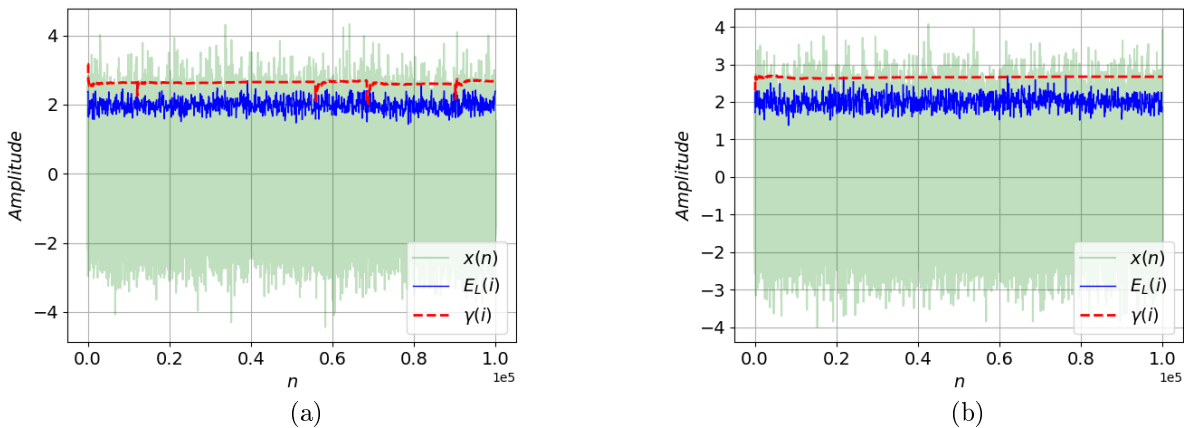


Fig. 3. Thresholding of noise samples for  $\alpha = 4$  (a) and  $\alpha = 6$  (b) at  $L = 100$

In the study pulses with a rectangular envelope filled with white Gaussian noise were used as a signal. Signals using OFDM technology (LTE, Wi-Fi) have a similar structure.

Figure 4 shows results of received samples thresholding in frequency channel at SNR of 0 dB for the case when there is no signal at the start of observation. Length of integration window was  $L = 20$  (Fig. 4a) and  $L = 100$  (Fig. 4b). Total number of analyzed samples was  $10^4$ . Probability of false alarm was set at  $10^{-4}$ , and value of the parameter  $\alpha = 6$ .

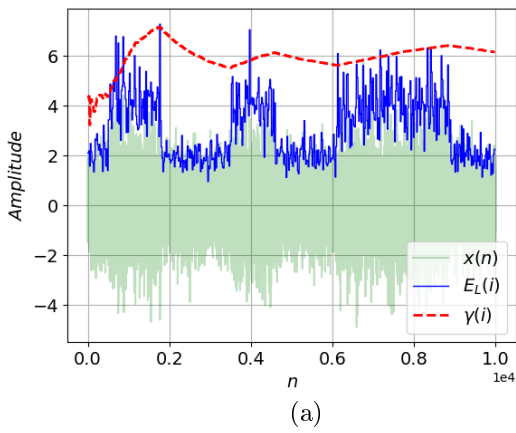


Figure 5 shows similar graphs for SNR value of 5 dB.

Figure 4 and Figure 5 shows that at the initial stage of detector processing, threshold value fluctuates near its true level, stabilizing with an increase in number of integration interval  $i$ . Moreover, amplitude of such oscillations decreases when  $L$  increases. Also, when  $L$  increases SD of signal envelope decreases and signal can be detected at lower SNR.

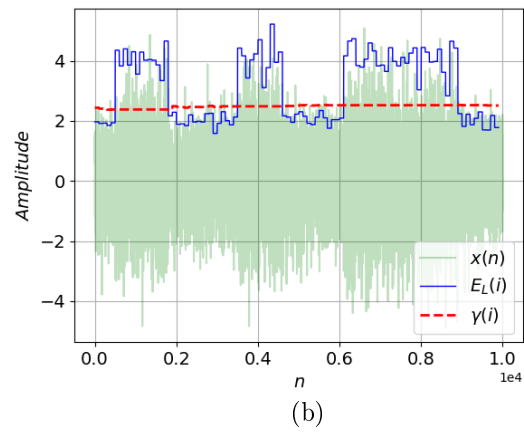


Fig. 4. Thresholding for the case of no signal at observation start for  $L=20$  (a) and  $L=100$  (b) at SNR = 0 dB

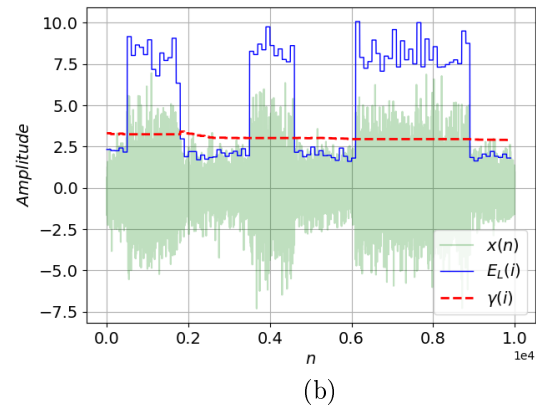
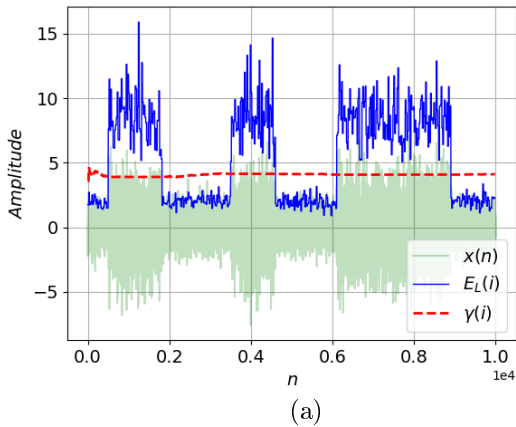


Fig. 5. Thresholding for the case of no signal at observation start for  $L=20$  (a) and  $L=100$  (b) at SNR = 5 dB

Figure 6 shows results of received samples thresholding in frequency channel at SNR of 0 dB for the case when signal is present at the initial time of observation. Length of the integration window was  $L = 20$  (Fig. 6a) and  $L = 100$  (Fig. 6b).

Figure 7 shows similar graphs for SNR of 5 dB.

Figure 6 and Figure 7 shows that the presence of a signal at initial moment of observation makes it difficult to detect its end and further estimate the noise level, especially at small values of  $L$ .

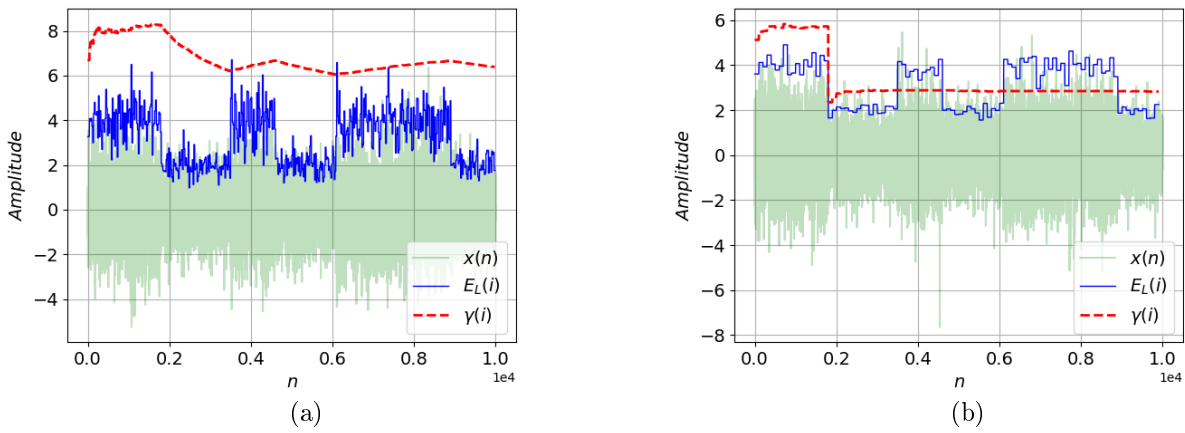


Fig. 6. Thresholding for the case of a signal present at observation start for  $L = 20$  (a) and  $L = 100$  (b) at  $\text{SNR} = 0$  dB

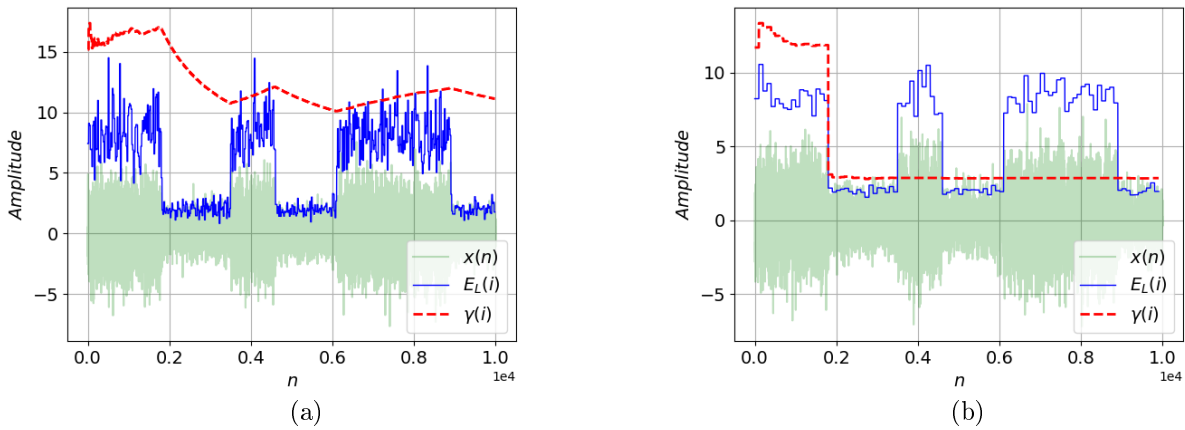


Fig. 7. Thresholding for the case when signal is present at initial time of observation for  $L = 20$  (a) and  $L = 100$  (b) at  $\text{SNR} = 5$  dB

Figure 8 shows dependence of correct detection probability via SNR for different  $k$  (ratio of integration interval and signal duration) and pulse duration of 100 samples (a) and 1000 samples (b) at  $P_F = 10^{-4}$ . Time mismatch  $\Delta t$  between signal begin and integration interval start was chosen randomly for each value of  $k$ . Criteria for signal detection was at least one value of  $E_L$  exceeds threshold within the true pulse duration.

From depicted above graphs following conclusions can be made:

1. For the same  $L$  at a fixed SNR, probability of correct detection will be maximized when signal begin and integration interval coincide.
2. For a fixed  $k$ , when  $L$  is increased by an order SNR gain for signal detection is about 4 dB.

3. With the same SNR and  $\Delta t$ , maximal probability of correct detection is achieved at  $k = 1$ .

Experiment also has shown that detection curves are the same for both coherent signal structure and noise signal.

Figure 9 shows dependence of correct detection probability via SNR for different  $P_F$  at  $k = 1$  and pulse durations of 100 samples (a) and 1000 samples (b).

It is established that to ensure probability of correct detection  $P_D = 0.9$  when probability of false alarm decreases from  $10^{-4}$  to  $10^{-10}$ , it is necessary to have an additional SNR of about 2.5 dB. When  $P_F$  decreases from  $10^{-10}$  to  $10^{-16}$ , only about 1.5 dB is required.

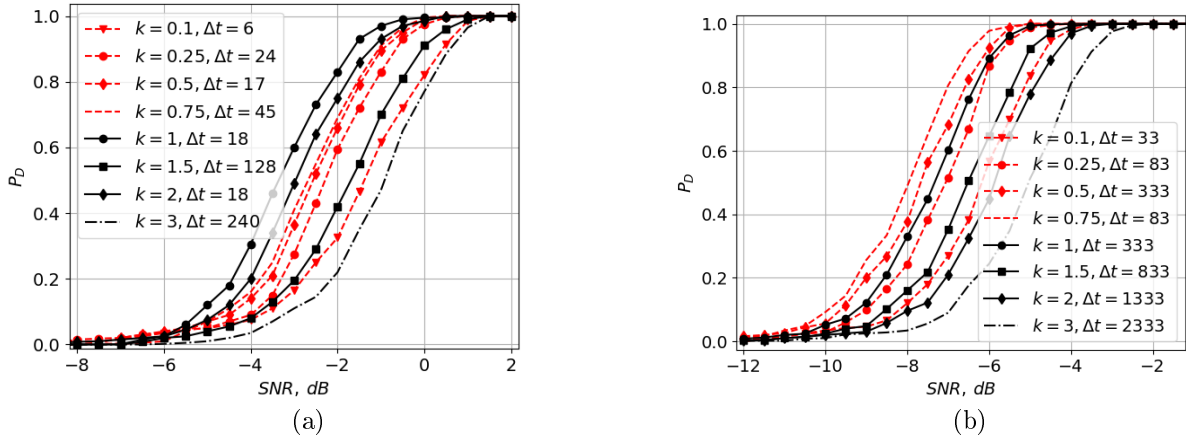


Fig. 8. Dependences of correct detection probability via SNR for different  $k$  and pulse durations of 100 samples (a) and 1000 samples (b)

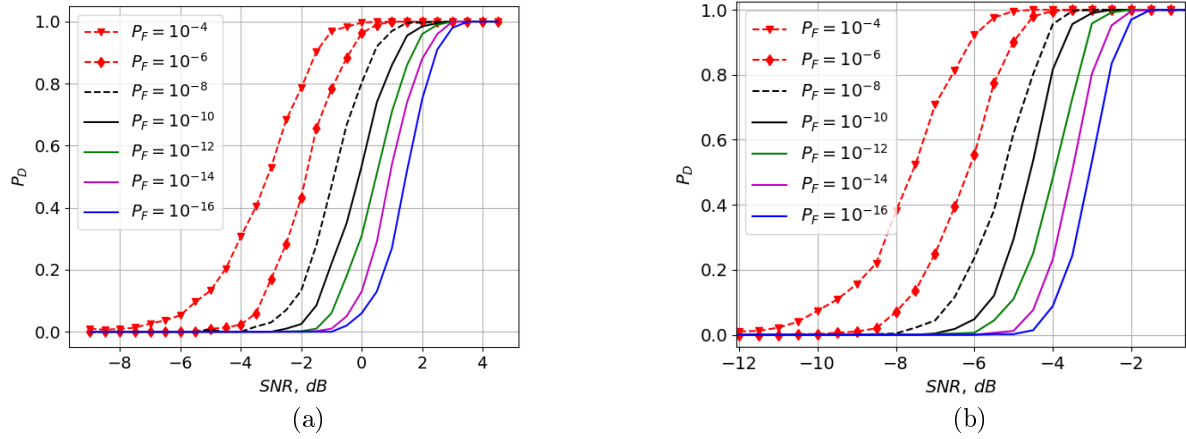


Fig. 9. Dependences of correct detection probability via SNR for different  $P_F$  and pulse durations of 100 samples (a) and 1000 samples (b)

Figure 10 shows detection curves for developed detector using statistics  $E_L$  and statistics  $E$  which is instantaneous smoothed signal energy using a moving average window [14]. Length of integration window  $L$  was 100, and probability false alarm for both detectors was set at  $10^{-4}$ . As we can see, developed detector requires about 5 dB less SNR to provide the same probability of correct detection  $P_D$  as the detector based on  $E$  statistic. Moreover, for the latter algorithm, noise power must be known.

Compared to the method of signal smoothing using a moving average window of  $L$  samples, proposed method requires  $L$  times fewer operations of squares sums calculation of  $L$  numbers and  $L$  times fewer operations of numbers dividing (when normalized). Also, this approach greatly simplifies algorithm for finding signals time boundaries after thresholding, since signal envelope calculated in this way will less fluctuate. For short pulsed signals at low SNR, proposed approach, in comparison with moving

average, gives smaller errors of signal duration estimates.

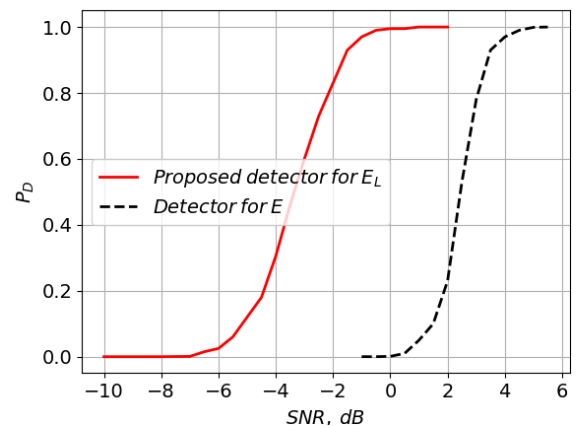


Fig. 10. Detection curves for  $P_F = 10^{-4}$  and  $L = 100$

## 6 Recommendations

1. At low SNR (less than 2-3 dB) and a small value of  $P_F$ , false detections of signal begin and end may occur. In this case,  $L$  should be chosen as large as possible.

2. At large  $L$ , error in signals time boundaries estimates, especially for short pulses, can exceed duration of pulses themselves. Therefore, it is advisable to use some detection strobe with possible signal boundaries taking into account the predicted errors of time parameters estimates.

3. Threshold adjustment in case of signal absence recommended to perform not continuously, but within a certain time interval. After that, reset accumulated value and refine noise level again.

4. For pulsed signals detection, it is recommended to choose  $L$  that fulfill such condition:  $0.5 \leq k \leq 2$ .

5. To detect continuous signals, it is recommended to choose  $L$  as large as possible. The limit in this case will be maximum permissible error in determining signals begin.

6. Obtained noise power estimates can be used to calculate (using Parseval theorem) noise level in frequency domain. This will allow you to calculate threshold for detecting signals in frequency domain.

## Conclusions

Proposed detector allows detecting and estimating stochastic signals time parameters at the background of an unknown and variable noise power. Developed algorithm is insensitive to parasitic signal amplitude modulation associated with frequency-selective fading in propagation channel and other effects. Increasing length of integration interval by an order provides SNR gain in signal detection for about 4 dB. Maximum value of pulsed signal correct detection probability is achieved when pulse duration and integration interval have the same length. Compared with a method of smoothing signal using moving average, proposed method has less computational complexity, simplifies search for signal time boundaries, and gives smaller errors of signal duration estimates. Detector can be implemented in cognitive radio systems, automatic radio monitoring complexes, and other radio systems where it is necessary to fast estimate occupancy of frequency channels. Prospects for further research in this area should be focused on development algorithms to estimate more precisely time parameters of detected signals.

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## Детектор енергії стохастичних сигналів в шумовій невизначеності

Бугайов М. В.

Метою даної статті є автоматизація процесів детектування стохастичних сигналів та оцінки часових параметрів за умов невідомої частотно-часової структури сигналів та потужності шуму. Суть запропонованого методу полягає у виявленні та відстеженні усереднених за  $L$  відліками сигналу часових змін енергії прийнятого сигналу в обраному частотному каналі. Порогове значення для заданої ймовірності помилкової тривоги



розраховується з використанням поточних оцінок потужності сигналу. Цей поріг є динамічним і уточнюється лише у вільних від сигналів інтервалах часу. У тих часових вікнах, де енергія перевищує поріг, приймається рішення про наявність сигналу. Запропоновано алгоритм детектування стохастичних сигналів. Якщо сигнал присутній у початковий момент часу, запропонований алгоритм може виявити лише його кінець за різким зменшенням енергії сигналу. Після цього оцінюється новий рівень шуму та уточнюється порогове значення. Отримано криві виявлення стохастичного сигналу з використанням запропонованого алгоритму. Показано, що при збільшенні кількості відліків  $L$  на порядок вираш у відношенні сигнал/шум при виявленні си-

гналу становить близько 4 дБ. Максимальне значення вірогідності правильного виявлення імпульсного сигналу досягається при однаковій тривалості імпульсу та довжині інтервалу інтегрування. Запропонований метод має меншу обчислювальну складність порівняно з методом згладжування сигналу з ковзним середнім вікном, спрощує пошук часових границь сигналу та дає менші похибки в оцінках тривалості сигналу. Сформульовано рекомендації щодо впровадження розробленого алгоритму.

*Ключові слова:* стохастичний сигнал; детектор енергії; часові параметри; частотний канал; рівень шуму; поріг; інтервал інтегрування