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Towards a New Implementation of Bandpass Filters on Basis of Band-Stop Structures on Dielectric WGM Microresonators

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A new method of constructing band-stop filters based on a system of optical microresonators with whispering gallery oscillations of ultra-high Quality factor (Q) is proposed, which are widely used in various integral filters of the optical wavelength range. To reduce the mutual coupling, the system of microresonators is located on different sides of the regular transmission line. With the help of perturbation theory, an electrodynamic model of filters was developed, which describes a complex system of interconnected microresonators with doubly degenerate types of natural oscillations. The obtained general analytical expressions are used to describe the characteristics of the scattering of natural waves of the line on the system of optical microresonators, which form a band-stop filter or an alternative band-pass filter. The frequency dependencies of the filter scattering matrix were calculated and analyzed. On the basis of the built analytical model, and using the periodicity of the microresonators' own oscillations, with a change in the free spectral range (FSR) value, the possibility of constructing a new class of band-pass filters has been proven. The new filters differ from the known ones in that they simultaneously use two types of natural oscillations of adjacent microresonators. Amplitude-frequency characteristics of filters with different sizes of operating frequency bands are calculated. Based on the comparison of the obtained data, a conclusion about the reduced dependence of the losses of the proposed filters on the bandwidth is made. It is noted that the amplitude-frequency characteristics of the new class of filters are close to linear ones. The obtained practical simulation results allow significantly reducing the calculation time and optimizing complex multi-resonator structures for optical communication systems.

Keywords: dielectric microresonator; whispering gallery oscillations; band stop filter; FSR – free spectral range; bandpass filter

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Introduction

Microresonators with whispering gallery oscillations have extremely high radiation quality factors and are convenient for implementation in various integrated circuits in the infrared and optical ranges [1–15]. Along with these advantages, it is well known that the whispering gallery oscillations are quite close to each other, generating a quasi-one-dimensional spectrum of natural frequencies close to periodic. The presence of additional natural oscillations leads to the appearance of spurious bands, which noticeably worsens the characteristics of devices built on the basis of such resonators. To compare the spectral parameters of the resonators, the concept of the frequency distance between neighboring resonant peaks was introduced as free spectral range (FSR). The frequency distance between adjacent resonant peaks for ring microresonators is inversely proportional to their radius, therefore, by changing the shape and relative dimensions of the microresonators, it is possible to change the distance between the frequencies of neighboring oscillations within small limits, and, consequently, the location of parasitic bands.

The periodicity of whispering gallery oscillations, together with a possible change in the FSR value, can be made a positive feature and used, for example, to implement alternative bandpass filters based on band-stop structures. Such filters should have properties that are, in a sense, the opposite of those built on currently traditional coupled resonator structures.

1 Statement of the problem

The purpose of the article is to study the possibility of constructing a new class of bandpass optical filters based on the use of band-stop structures on a system of coupled microcavities with whispering gallery oscillations, using the periodicity of their natural oscillations. Investigation of the scattering characteristics of a new type of filters was carried out.

2 Calculation of the scattering parameters of the Band-stop filters

To calculate the characteristics of the wave scattering matrix of the transmission line, the theory considered in [16] was used. Suppose we have a system of the N microresonators, which is located in open space and at the same time coupled with transmission line. Suppose that each of the microresonators has a doubly degenerate type of natural oscillations on the frequency $\omega_0 = 2\pi f_0$, each of which is characterized by a given symmetry with respect to the selected plane: even $(\vec{e}_n^{\,e}, \vec{h}_n^{\,e})$, or odd $(\vec{e}_n^{\,o}, \vec{h}_n^{\,o})$ [17]. Let a wave $\left(\vec{E}^+, \vec{H}^+\right)$ falls on 1 port on N microresonator system via regular waveguide (Fig. 1, a).

The eigenoscillation field of the system we represented as a superposition of fields of isolated microresonators in the frequency band near ω_0 :

$$\begin{aligned} \vec{e}^{\,s} &= \sum_{n=1}^{N} b_n^{es} \vec{e}_n^{\,e} + \sum_{n=1}^{N} b_n^{os} \vec{e}_n^{\,o}; \\ \vec{h}^{s} &= \sum_{n=1}^{N} b_n^{es} \vec{h}_n^{e} + \sum_{n=1}^{N} b_n^{os} \vec{h}_n^{o}. \end{aligned} \tag{1}$$

The matrix B

$$B = \begin{bmatrix} b_1^{e1} & b_1^{e2} & \dots & b_1^{e2N} \\ b_1^{o1} & b_2^{o2} & \dots & b_2^{o2N} \\ & . & . & . \\ b_N^{o1} & b_N^{o2} & \dots & b_N^{o2N} \end{bmatrix} \tag{2}$$

defining the amplitudes of coupled microresonators oscillations $b_t^{e,o,s}$ should satisfy the equation system as an eigenvector of coupling operator K [16].

For band-drop filter shown in Fig. 1, a, the coupling matrix of the microresonators has the form:

$$\begin{aligned} K = \Big\| & i(\tilde{k}_1^e \delta_{s1} + \tilde{k}_1^o \delta_{s2} + \ldots + \tilde{k}_N^e \delta_{s(2N-1)} + \\ & + \tilde{k}_N^o \delta_{s(2N)} + \tilde{k}_{OS})\delta_{sn} + \kappa_{sn}(1 - \delta_{sn}) \Big\| , \end{aligned} \tag{3}$$

where $\tilde{k}_u^{e,o}$ is the coupling coefficient of the u-th microresonator with the transmission line 1-2 (Fig. 1, a) on an even (or odd) mode oscillation; $\tilde{k}_{OS}$ is the coupling coefficient of the microresonator with open space; $\kappa_{sn} = k_{sn}^{e,o} + i\tilde{k}_{sn}^{e,o}$ is the mutual coupling coefficient between microresonators for even (odd) mode (for simplicity we proposed that if $|s - n| > 1$ then $k_{sn}^{e,o} = 0$); δ_{st} is the Kronecker δ-function.

The found eigenfunctions of coupling oscillations, defining by matrix K (2) we used for solving the scattering problem of the wave $(\vec{E}^+, \vec{H}^+)$ on a system of microresonators. For the solution of the problem we represented decomposition [17]:

$$\vec{E} \approx \vec{E}^+ + \sum_{s=1}^{2N} a^s \vec{e}^{\,s}; \quad \vec{H} \approx \vec{H}^+ + \sum_{s=1}^{2N} a^s \vec{h}^{s}, \tag{4}$$

where a^s are the unknown amplitudes ($s = 1, 2, \ldots, 2N$) and $(\vec{e}^{\,s}, \vec{h}^{s})$ are the s-th eigenoscillation field of the coupled microresonator system (1) with complex frequency $\tilde{\omega}^s$.

As a result, the transfer coefficient between the 1-2 ports is determined using the expression [16]:

$$\begin{aligned} T_{11}(\omega) &= -\frac{Q^D}{\det B} \cdot \sum_{s=1}^{2N} \frac{\det B_1^s}{Q_s(\omega)}, \\ T_{12}(\omega) &= 1 - \frac{Q^D}{\det B} \cdot \sum_{s=1}^{2N} \frac{\det B_2^s}{Q_s(\omega)}, \end{aligned} \tag{5}$$

where $Q_s(\omega) = \omega/\omega_0 + 2iQ^D(\omega/\omega_0 - 1 - \lambda_s/2)$; Q^D is the dielectric Q-factor of microresonators ($s = 1, \ldots, 2N$); $\lambda_s = 2(\tilde{\omega}^s - \omega_0)/\omega_0$.

The matrix, defining a scattering on the band-stop filter:

$$\begin{aligned} B_1^s &= \begin{bmatrix} b_1^{e1} & \dots & \sum\limits_{u=1}^{2N} b_u^s \tilde{k}_{u1}^{ae-+} & \dots & b_1^{e2N} \\ b_1^{o1} & \dots & \sum\limits_{u=1}^{2N} b_u^s \tilde{k}_{u1}^{ao-+} & \dots & b_1^{o2N} \\ . & \dots & . & \dots & . \\ b_N^{o1} & \dots & \sum\limits_{u=1}^{2N} b_u^s \tilde{k}_{uN}^{ao-+} & \dots & b_N^{o2N} \end{bmatrix}; \\ B_2^s &= \begin{bmatrix} b_1^{e1} & \dots & \sum\limits_{u=1}^{2N} b_u^s \tilde{k}_{u1}^{ae++} & \dots & b_1^{e2N} \\ b_1^{o1} & \dots & \sum\limits_{u=1}^{2N} b_u^s \tilde{k}_{u1}^{ao++} & \dots & b_1^{o2N} \\ . & \dots & . & \dots & . \\ b_N^{o1} & \dots & \sum\limits_{u=1}^{2N} b_u^s \tilde{k}_{uN}^{ao++} & \dots & b_N^{o2N} \end{bmatrix}. \end{aligned} \tag{6}$$

Here $\tilde{k}_{(v)s}^{a\pm} = \tilde{k}_{(v)s}^{e,o\pm}$ is the coupling coefficient of the s-th microresonator with the v-th transmission line. Here a takes values even or odd depending on the type of u-th microresonator oscillations.

In this case, index s is determined by the column number in matrix B_s^v . Here

$$\tilde{k}_{mn}^{ab-+} = \left(c_m^{a-} c_n^{b+*}\right)/(\omega_0 w_n) = (\tilde{k}_{mn}^{ab})_0 e^{-i\Gamma(z_m + z_n)};$$

$$\tilde{k}_{mn}^{ab++} = \left(c_m^{a+} c_n^{b+*}\right)/(\omega_0 w_n) = (\tilde{k}_{mn}^{ab})_0 e^{-i\Gamma(z_m - z_n)};$$

$c_n^{a\pm}$ is the expansion coefficient of the n-th microresonator field with a – even or odd mode on the propagating wave of the transmission line; w_n is the energy stored in the dielectric of microresonator;

Γ is the longitudinal wave number of the transmission line; z_n is the longitudinal coordinate of the n-th microresonator center.

Relations (2), (4)–(6) allow one to calculate the characteristics of wave scattering of a regular transmission line on an array of resonators with whispering gallery oscillations.

3 Scattering parameters of alternative bandpass filters based on notch filter resonator structures

Let us first consider band stop filters based on microresonators with whispering gallery oscillations (Fig. 1, a). Reducing the fluctuations of the module of the coefficients of the scattering matrix can be achieved by decreasing the coupling between the microresonators; for this, it is proposed to place each of them on the opposite side of the line (Fig. 1, a). To calculate and analyze the frequency dependences of the characteristics of the S-matrix, we will use relations (1)-(6).

Figure 1, b–e shows the results, based on (2)-(6), of the dependences on the frequency of the S-matrix ($S_{v1} = 20\lg|T_{1v}|$; $\Psi_{v1} = \arg(T_{1v})$) of a 2-, 4-section band-stop filters. Where T_{1v} is the transfer coefficient between the 1 and v ports (5).

As it can be seen from the above data, the filters shown in Fig. 1, a, are characterized by smoother frequency characteristics of attenuation outside the stop bands (Fig. 1, b-e).

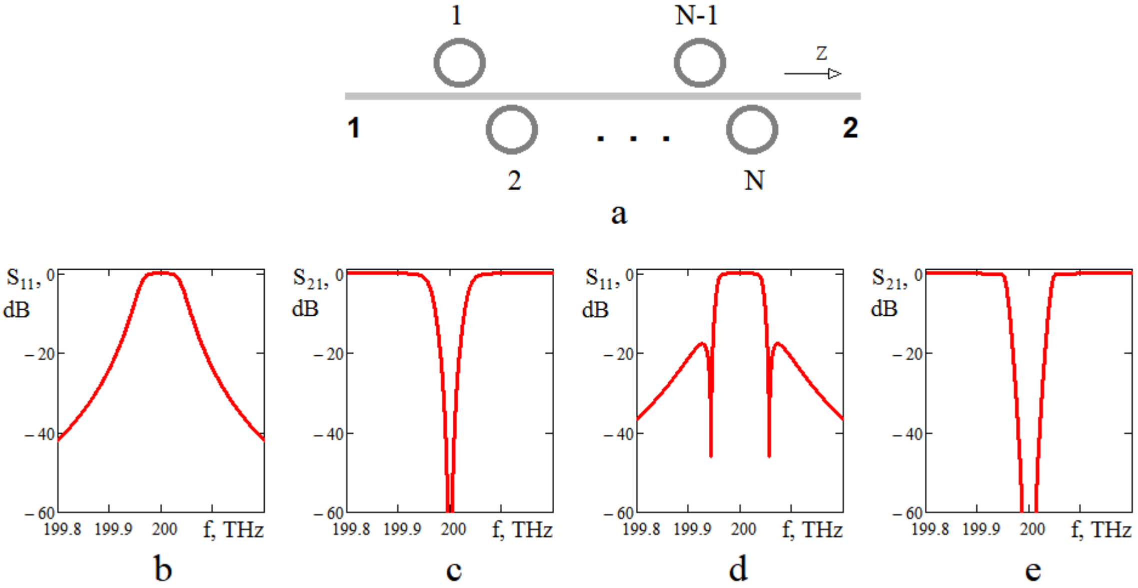

Fig. 1. Laterally coupled microresonators band-stop filter structure – a. Module S-matrix responses of the 2-section (b-c), 4-section (d-e) band-stop filter as functions of frequency. Coupling coefficients of microresonators with transmission lines: $\tilde{k}_s^e = \tilde{k}_s^o = 2 \cdot 10^{-4}$ for even, odd oscillations. Microresonator coupling coefficients with open space: $\tilde{k}_{OS} = 1 \cdot 10^{-7}$. Mutual coupling coefficients of the microresonators for even, odd oscillations $k_{st}^o = k_{st}^e \approx 0$. Frequency of free microresonators oscillations $f_0 = 200$ THz; $Q^D = 10^6$; $\Gamma\Delta z_{s,s+1} = 31\pi/2$

We use the periodicity of the frequency spectrum of the resonators to optimize their frequency characteristics, located between adjacent rejection bands for use as bandpass filters. In this case, the fundamental difference between the new structures is that the different slopes of their frequency dependence in the considered bandwidth are now determined by the different types of natural oscillations of the microresonators. Using (1)-(6), we calculated several adjacent stop bands implemented by the same microresonator system shown in Fig. 1, a. After that, we optimized their S-parameters to obtain the given scattering characteristics located between two fixed rejection bands. Such filters based on whispering gallery oscillations are called alternative with respect to band-stop filters built on the basis of a known structure microresonators in a regular line.

Figure 2 shows the results of calculating the scattering parameters of an alternative narrow-band (a–c), medium (d–f) and wider (g, h) passband filter implemented on 8 microcavities. Let us pay attention to the linearity of the phase-frequency dependences in the filter passbands (Fig. 2, b, e, h).

As a result, we obtained the following data.

The frequency response of the first bandpass filter (Fig. 2, a-c) has such parameters: center frequency $f_0 =$

199, 880 THz; minimum loss in the passband -1,4 dB; −3 dB bandwidth: 280 GHz; squareness frequency response (ratio of -30 dB bandwidth to -3 dB bandwidth) 5,17.

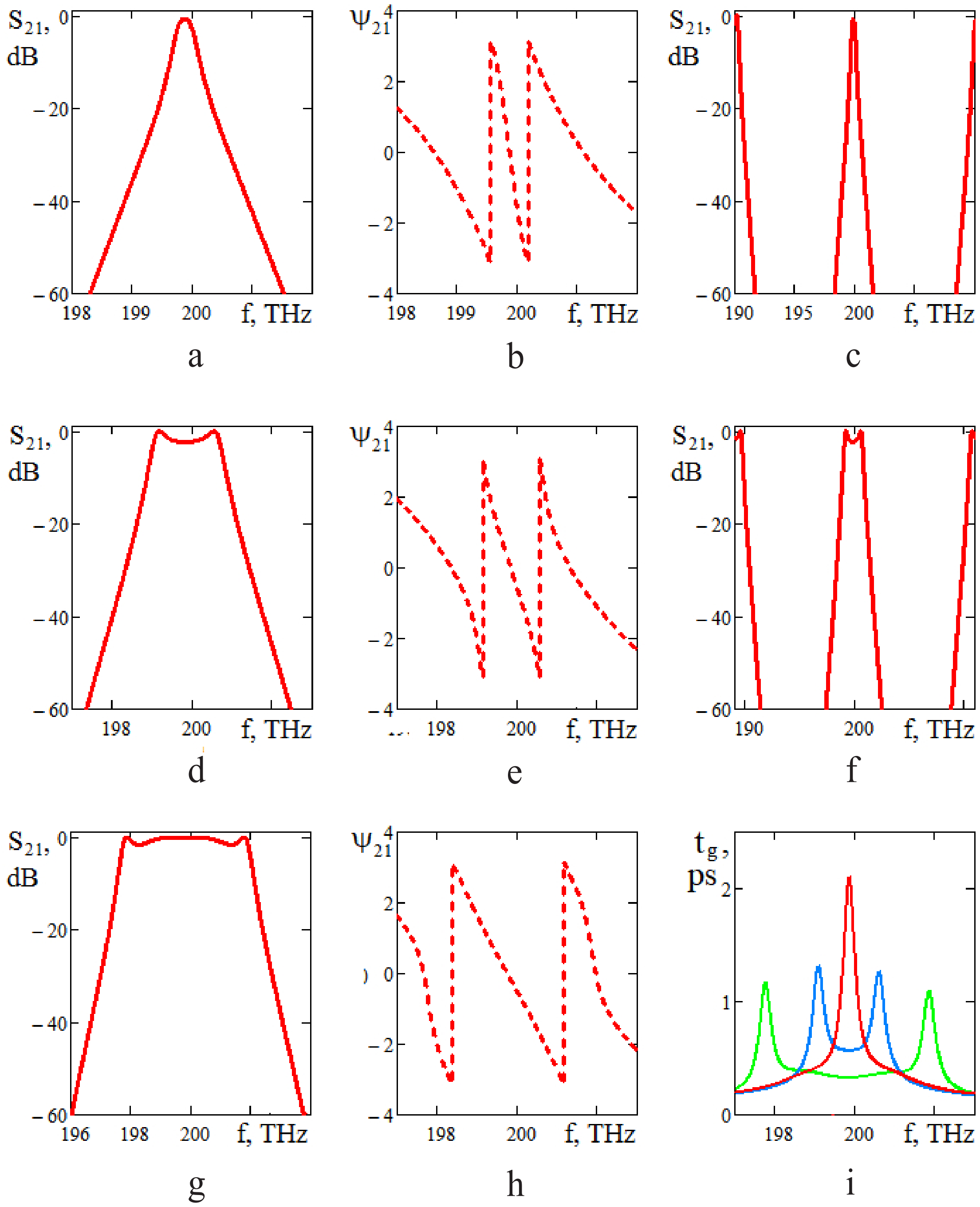

Fig. 2. S-matrix responses of different 8-section (a-i) alternative filters as functions of frequency. Coupling coefficients of the microresonators with transmission lines: $\tilde{k}^e_{nm} = 2,45\cdot10^{-2}$; $\tilde{k}^o_{nm} = 2,05\cdot10^{-2}$ (a); $\tilde{k}^e_{nm} = 2,49\cdot10^{-2}$; $\tilde{k}^o_{nm} = 2,1\cdot10^{-2}$ (d); $\tilde{k}^e_{nm} = 2,35\cdot10^{-2}$; $\tilde{k}^o_{nm} = 2,30\cdot10^{-2}$ (g). Open Space microresonator coupling coefficients: $\tilde{k}_{OS} = 1\cdot10^{-7}$. Mutual coupling coefficients of the microresonators for even oscillations: $k^e_{12} = k^e_{21} = 2\cdot10^{-5}$; for odd oscillations $k^o_{12} = k^o_{21} = -2\cdot10^{-5}$ (a); $k^e_{12} = k^e_{21} = 2\cdot10^{-5}$, $k^o_{12} = k^o_{21} = -2\cdot10^{-5}$ (d); $k^e_{12} = k^e_{21} = 1,5\cdot10^{-5}$, $k^o_{12} = k^o_{21} = -1,5\cdot10^{-5}$ (g). Phase-frequency dependences of filters (b, e, h). Group delay time of filters: a – red; d - blue; g is the green curve (i)

For filter shown in Fig. 2, d–f, center frequency $f_0 = 199,87$ THz; minimum loss -0,42 dB; -3 dB bandwidth: 1,67 THz; squareness frequency response 1,86.

For filter shown in Fig. 2, g–h, center frequency $f_0 = 199,83$ THz; minimum loss -0,094 dB; -3 dB bandwidth: 4,26 THz; squareness frequency response 1,35.

Thus, as the bandwidth of the filters decreases, the squareness of their frequency response deteriorates. From the data shown in Fig. 2i, we see that, in this case, the decrease in the bandwidth is accompanied by increase in the group delay.

Discussion and Conclusion

Summarized, the obtained electromagnetic simulation results demonstrate the possibility of realizing a wide class bandpass filters based on bandstop structures, using microresonators with whispering gallery mod. The proposed filters use the periodicity properties of the frequency spectrum of natural oscillations of microresonators, as well as non-degenerate oscillations of different types adjacent in frequencies. It is predicted that with the same number of resonators, narrower filters have a larger group delay of signals compared to wideband ones. Since the losses in the passband of alternative filters are determined by the contribution of mainly non-resonant scattering, their value should be relatively small for both broadband and narrowband structures.

The proposed new type of filters can find practical application in multiplexers, amplifiers, lasers and other devices of infrared and optical communication systems.

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До нової реалізації смугових фільтрів на основі загороджуючих структур на діелектричних WGM мікрорезонаторах

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Запропоновано новий спосіб побудови смугово-загороджуючих фільтрів на основі системи оптичних мікрорезонаторів з коливаннями шепочучої галереї надвисокої добротності, які широко використовуються в різноманітних інтегральних фільтрах оптичного діапазону довжин хвиль. Для зменшення взаємного зв'язку, система мікрорезонаторів розташована з різних сторін регулярної лінії передачи. За допомогою теорії збурень, розроблена електродинамічна модель фільтрів, яка описує складну систему зв'язаниих між собою мікрорезонаторів з двократно виродженими типами власних коливань. Отримані загальні аналітичні вирази використовуються для опису характеристик розсіювання власних хвиль лінії на системі оптичних мікрорезонаторів, які утворюють смугово-загороджуючий або альтернативний смугово-пропускаючий фільтри. Розраховані та проаналізовані частотні залежності матриці розсіювання фільтрів. На основі побудованої аналітичної моделі та, використовуючи періодичність власних коливань мікрорезонаторів зі зміною величини FSR, доведена можливість побудови нового класу смугово-пропускаючих фільтрів. Нові фільтри відрізняються від відомих тим, що використовують одночасно два типи власних коливань мікрорезонаторів, суміжних за частотами. Розраховано амплітудно-частотні характеристики фільтрів з різними величинами смуг робочих частот. На основі порівняння отриманих даних, зроблено висновок про зменшену залежність втрат запропонованих фільтрів від ширини смуги пропускання. Відмічається, що амплітудно-частотні характеристики нового класу фільтрів близькі до лінійних. Отримані практичні результати моделювання дозволяють суттєво скорочувати час розрахунків та оптимізувати складні багаторезонаторні структури для оптичних систем зв'язку.

Ключові слова: діелектричний мікрорезонатор; коливання шепочучої галереї; смуго-загороджуючий фільтр; FSR – free spectral range; смуговий фільтр