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Controlled Brewster Effect in the Scattering of Electromagnetic Waves on Pseudo-Rotating Lattices of Dielectric Resonators

Trubin O. O.

National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine

E-mail: *atrubin9@gmail.com*

Various conditions for the occurrence of anomalous scattering, in which most of the power is emitted by the lattice only in the direction of propagation of the incident wave, are analyzed. An analytical model for the scattering of plane electromagnetic waves on arrays of pseudo-rotating dielectric resonators (DRs) of cylindrical and rectangular shape is developed. New analytical relations are derived for the functions that determine the coupling between the field of a plane wave and the main types of magnetic oscillations of the rotated DR. The angles between the axis of the DR and the directions of propagation of plane waves, at which the coupling between the DR and the incident wave reaches extreme values, are studied. The conditions of non-resonant scattering and scattering with the absence of a reflected petal, known as the Brewster effect, were established. The general relations between the angles of inclination of DRs, polarization and the angles of incidence of waves on lattices, which lead to special cases of scattering, were found. There is a similarity between non-resonant scattering and the known Malyuzhynets effect, which describes the passage of waves through lattices of other types. Scattering models for lattices of rotated cylindrical and rectangular DRs were built. The difference between the classical Brewster effect and the petal-free cases of scattering on lattices built on the basis of the use of pseudo-rotating DRs was noted. In particular, it's shown that, unlike other methods of realizing metasurfaces of this class, cases of scattering without petals on lattices of pseudo-rotating resonators are possible when the angles of incidence are changed in a wider band. The obtained theoretical results allow us to propose a new class of devices built on the basis of the use of pseudo-rotating DRs, to significantly reduce the calculation time and to optimize complex multi-resonator structures. New types of lattices built on pseudo-rotating DRs can be used to design a wide class of antennas, as well as multiplexing devices in terahertz, infrared, and optical wavelength range communication systems.

Keywords: lattice; dielectric resonator; scattering; Brewster effect; rotation; coupled oscillations

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Introduction

The effects of anomalous scattering of electromagnetic waves on various types of metasurfaces have recently attracted the attention of researchers [1–24]. One such phenomenon is the absence of reflection of an incident polarized wave from a metasurfaces, known in optics as the Brewster effect. The Brewster effect, discovered in the 19th century during the scattering of polarized waves at a flat interface between two dielectrics, was discovered during scattering on dielectric metasurfaces [1–6, 8, 9, 14, 15, 17, 18, 20]; on metasurfaces made of various metal planar resonators [11,16,19,21], surfaces of more complex structure [7,12], as well as on surfaces with a random distribution of inhomogeneities [22]. If previously the Brewster effect was used mainly to obtain polarized waves [25], today it is used in various measuring devices for detecting and studying the properties of various micro-objects in the optical range, for example, graphene [10,13] and also for the determination of optical constants [23, 24].

In metamaterials, the Brewster effect usually occurs under the condition of simultaneous excitation of frequency-degenerate magnetic and electrical oscillations of different types in lattice elements (see for example [15]). Simultaneous excitation of several types of oscillations leads to a specific interaction of grating elements with the incident field and, in some cases, can cause re-emission in a direction that does not coincide with the direction of the reflected wave. Meanwhile, it is known that degenerate oscillations exist only in a limited range of resonator sizes, and in addition, they are not fundamental - lower in frequency, so their practical use is difficult due to the negative influence of other types of oscillations adjacent in frequency.

In this work, we consider lattices of dielectric resonators (DRs) with non-degenerate types of natural oscillations, including fundamental type oscillations, with the possibility of rotating their axes. This allows us to observe various scattering anomalies in a purer form. The conditions, when all the lattice resonators are oriented in such a way that their field in the wave zone becomes zero in the direction of the reflected lobe, are analyzed. In this case, a phenomenon similar to the Brewster effect occurs. In contrast to the classical Brewster effect, these conditions can be implemented in DR lattices for both cases of scattering of waves of two types: s- or p-polarization, and in addition, at any given angles of incidence, by selecting the types of oscillations and the spatial orientation of the resonator axes.

Along with the Brewster effect, during resonant scattering on periodic DR structures, the work analyzes the conditions under which all lattice resonators are not coupled with the incident wave. In this case, the dielectric structure interacts with the incident field only in a non-resonant manner, which leads to the effect of quasi-complete transmission of waves through the grating.

1 Statement of the problem

Let us consider the problem of scattering a plane electromagnetic wave $(\mathbf{E}^+, \mathbf{H}^+)$ on a lattice of dielectric resonators. In the general case, it is convenient to represent the total scattering field $(\mathbf{E}, \mathbf{H})$ in the form of a superposition:

$$\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^s; \quad \mathbf{H} = \mathbf{H}^+ + \mathbf{H}^s, \tag{1}$$

where $(\mathbf{E}^s, \mathbf{H}^s)$ is the field scattered by the lattice. In return, the scattered field can be decomposed into two terms, the first of which is due to the contribution of non-resonant components $(\mathbf{E}^{nr}, \mathbf{H}^{nr})$, and the second is the sum of the fields of coupled oscillations of the lattice resonators $\left(\mathbf{E}^L, \mathbf{H}^L\right)$:

$$\mathbf{E}^s = \mathbf{E}^{nr} + \mathbf{E}^L; \quad \mathbf{H}^s = \mathbf{H}^{nr} + \mathbf{H}^L. \tag{2}$$

The non-resonant components of the lattice field are represented as an integral over a continuous spectrum [26], and the resonant components for the most part can be expressed as a finite sum over the coupled oscillations of the resonator system $(\mathbf{e}^t, \mathbf{h}^t)$ $(t = 1, 2, \ldots, N)$ [29]:

$$\mathbf{E}^L = \sum_{t=1}^{N} a^t \mathbf{e}^t; \ \ \mathbf{H}^L = \sum_{t=1}^{N} a^t \mathbf{h}^t. \tag{3}$$

In turn, the field of coupled lattice oscillations can be represented as a superposition of the fields of partial resonators: $(\mathbf{e}_n, \mathbf{h}_n)$ $(n = 1, 2, \ldots, N)$:

$$\mathbf{e}^t = \sum_{n=1}^{N} b_n^t \mathbf{e}_n; \quad \mathbf{h}^t = \sum_{n=1}^{N} b_n^t \mathbf{h}_n. \tag{4}$$

The field amplitudes a^t and b_n^t are determined from the system of equations [28], previously found on the basis of perturbation theory.

The scattered field of the lattice in the wave zone in the case of identical DR can be represented in the form:

$$\begin{aligned} \mathbf{e}^\infty &= \sum_{t=1}^{N} A_t \mathbf{e}_t^\infty \approx \mathbf{e}_1^\infty \sum_{t=1}^{N} A_t e^{ik_0 d_t}; \\ \mathbf{h}^\infty &= \sum_{t=1}^{N} A_t \mathbf{h}_t^\infty \approx \mathbf{h}_1^\infty \sum_{t=1}^{N} A_t e^{ik_0 d_t}, \end{aligned} \tag{5}$$

where $(\mathbf{e}_1^\infty, \mathbf{h}_1^\infty)$ – field of an isolated DR in the wave zone; A_t – amplitude of the forced oscillation of the t-th partial DR:

$$A_t = \sum_{s=1}^{N} a^s b_t^s; \tag{6}$$

d_t – projection of the coordinate vector of the t-th DR onto the direction of measurement of the scattered field $\mathbf{n}_0$.

From (5) the scattering field of the lattice in the wave zone can also be written in the form:

$$\mathbf{e}^\infty = \mathbf{e}^\infty(\theta, \varphi) = \mathbf{e}_0 f \left\langle \theta_k, \varphi_k | \theta, \varphi \right\rangle \frac{e^{-ik_0 r}}{k_0 r}, \tag{7}$$

where $\mathbf{e}_0$ is the unit vector, defining the polarization of the scattered electric field in the wave-zone. The function $f \left\langle \theta_k, \varphi_k | \theta, \varphi \right\rangle$ is named scattering amplitude.

In most cases of resonant scattering, at frequencies near coupled lattice oscillations $\left(\mathbf{E}^L, \mathbf{H}^L\right)$, the conditions are satisfied:

$$\left|\mathbf{E}^{nr}\right| \ll \left|\mathbf{E}^L\right|; \quad \left|\mathbf{H}^{nr}\right| \ll \left|\mathbf{H}^L\right|, \tag{8}$$

therefore, non-resonant terms $(\mathbf{E}^{nr}, \mathbf{H}^{nr})$ in (2) can be neglected.

However, in some cases, special conditions may arise that cause a non-trivial distribution of the scattered field.

1) As follows from (5), the scattered field of the lattice, in the case of identical DR, is determined by the field distribution of a single resonator in the wave zone $(\mathbf{e}_1^\infty, \mathbf{h}_1^\infty)$. It is obvious that if in any direction the power flux density of the resonator is equal to zero, then the power flux density dissipated by the entire lattice as a whole will be close to zero. This important remark allows us to determine the possible conditions for "screening" the scatered field in a given direction. In particular, we can choose the orientation of the axes of all lattice resonators in such a way as to satisfy a given condition.

2) At certain values of the angles of incidence, as well as at certain spatial orientations of the DR axes, conditions may arise under which the incident wave $(\mathbf{E}^+, \mathbf{H}^+)$ may not be coupled with the resonators of the lattice. In this case, all amplitudes (3) $a^s = 0$ and as follows from (5, 6) the contribution of non-resonant scattering become decisive. Conditions (8) are

not met. In this case, the lattice will interact with the incident field only in a non-resonant manner (2). The coupling between the field of the incident wave and the grating resonators is noticeably reduced, which leads to quasi-complete passage of waves through the lattice.

The implementation of the noted conditions is most easily ensured by the same orientation of the axes of all partial resonators of the lattice, which becomes possible only with their pseudo-rotations. By pseudo-rotation we will understand a non-trivial rotation of the DR axes in which the relative spatial position of the resonator axes remains constant. In the case of pseudo-rotations, the directions of the resonator axes change only relative to the lattice plane; in this case, the relative location of the axes remains unchanged, for example, parallel (Fig. 3, a, c).

This paper examines special cases of 1), 2) scattering of plane electromagnetic waves on lattices of pseudo-rotating dielectric resonators.

2 Scattering of electromagnetic waves by Plain Lattice pseudo-rotatable DRs

To solve the problem of scattering of electromagnetic waves on a lattice, we use a theory based on the expansion of the field into coupled oscillations of the DR (3). In this case, the amplitudes a^t, according to [29], are presented as follows:

$$a^t = -Q^D/(\omega_0 Q_t(\omega)) \, \frac{\det C_t}{\det B}, \quad (t = 1,\, 2, \dots,\, N), \quad (9)$$

where

$$B = \begin{bmatrix} b_1^1 & b_1^2 & \dots & b_1^N \\ b_2^1 & b_2^2 & \dots & b_2^N \\ . & . & \dots & . \\ b_N^1 & b_N^2 & \dots & b_N^N \end{bmatrix};$$

$$C_t = \begin{bmatrix} b_1^1 & b_1^2 & \dots & c_1^{+*}/w_1 & \dots & b_1^N \\ b_2^1 & b_2^2 & \dots & c_2^{+*}/w_2 & \dots & b_2^N \\ . & . & \dots & . & \dots & . \\ b_N^1 & b_N^2 & \dots & c_N^{+*}/w_N & \dots & b_N^N \end{bmatrix}.$$

Coefficients $(c_1^+, c_2^+, \dots, c_N^+)$, taking into account the coordinates of the DR centers, determine the degree of coupling of each lattice resonator with the incident wave $(\mathbf{E}^+,\ \mathbf{H}^+)$:

$$c_t^+ = -\tfrac{1}{2} \oint\limits_{s_t} \left\{ [\mathbf{e}_t^1,\, \mathbf{n}] \left(\mathbf{H}^+\right)^* + [\mathbf{n},\, \mathbf{h}_t^1] \left(\mathbf{E}^+\right)^* \right\} \mathrm{ds}. \quad (10)$$

Here $(\mathbf{e}_t^1,\, \mathbf{h}_t^1)$ is the field of natural oscillations in the material of the t-th DR $(t = 1, 2, \dots, N)$; $Q_t(\omega) = \omega/\omega_0 + 2iQ^D\left(\omega/\omega_0 - 1 - \lambda_t/2\right)$; Q^D – dielectric quality factor of the resonator; ω – circular frequency; ω_0 - circular natural frequency of isolated resonators; $\lambda_t = 2(\tilde{\omega}^t - \omega_0)/\omega_0$; $\tilde{\omega}^t$ is the t-th complex natural frequency of the lattice [28, 29]; $\mathbf{n}$ – normal to surface s_t of the t-th DR of the lattice.

Field of main natural oscillations of the DR $H_{10\delta}$ in the “single-wave” approximation in the dielectric region; $\rho \le r_0$; $|z'| \le L/2$ (Fig. 1, a) is represented in the form:

$$\begin{aligned} &e_\rho = e_{z'} = 0; \quad e_\alpha = -h_1 \frac{i\omega\mu_0}{\beta} \cdot J_1(\beta\rho) \cos \beta_z z'; \\ &h_\rho = h_1 \frac{\beta_z}{\beta} \cdot J_1(\beta\rho) \sin \beta_z z'; \\ &h_\alpha = 0; \quad h_z = h_1 J_0(\beta\rho) \cos \beta_z z', \end{aligned} \quad (11)$$

where L – height, r_0 – radius of the dielectric cylinder; $J_m(x)$ – Bessel function of the first kind, of the m-th order; h_1 – amplitude of the magnetic field of natural oscillations; β, β_z – wave numbers.

Let us assume that a plane electromagnetic wave $(\mathbf{E}^+,\ \mathbf{H}^+)$ is incident on a cylindrical dielectric resonator at the frequency of the fundamental natural oscillations $H_{01\delta}$:

$$\mathbf{E}^+ = \mathbf{E}_0 e^{-i\mathbf{k}_0\mathbf{r}}; \quad \mathbf{H}^+ = \mathbf{H}_0 e^{-i\mathbf{k}_0\mathbf{r}}, \quad (12)$$

where $\mathbf{E}_0 = E_0\mathbf{n}_E$; $\mathbf{H}_0 = E_0/w_0\mathbf{n}_H$; $\mathbf{k}_0 = k_0\mathbf{n}_k$; $\mathbf{n}_E$; $\mathbf{n}_H$; $\mathbf{n}_k$ – unit vectors defining the polarization of a plane wave: $(\mathbf{n}_k = [\mathbf{n}_E, \mathbf{n}_H])$; $\mathbf{r} = (x, y, z)$ – radius vector, specified in a rectangular coordinate system, at the center of the DR (Fig. 1, b).

Substituting (11), (12) into (10), we find a general analytical expression for the c-function that defines the relationship between the field of a plane wave and $H_{10\delta}$ oscillations for arbitrary orientations of the cylindrical DR axis as well as arbitrary angles of incidence in the coordinate system (Fig. 1, b):

$$\begin{aligned} c_1^+ = &-2\pi i E_0^* \cdot \frac{h_1}{\beta} \cdot \left(r_0^2 \frac{L}{2}\right) \cdot \left(k_1^2 - k_0^2\right) \frac{(\mathbf{n}_H, \mathbf{n}_R)}{\sqrt{1 - (\mathbf{n}_k, \mathbf{n}_R)^2}} \times \\ &\times \frac{\left[q_\perp \sqrt{1 - (\mathbf{n}_k, \mathbf{n}_R)^2} J_0\left(q_\perp \sqrt{1 - (\mathbf{n}_k, \mathbf{n}_R)^2}\right) J_1(p_\perp) - p_\perp J_0(p_\perp) J_1\left(q_\perp \sqrt{1 - (\mathbf{n}_k, \mathbf{n}_R)^2}\right)\right]}{\left(q_\perp \sqrt{1 - (\mathbf{n}_k, \mathbf{n}_R)^2}\right)^2 - p_\perp^2} \times \\ &\times \frac{[p_z \sin p_z \cos(q_z(\mathbf{n}_k, \mathbf{n}_R)) - q_z(\mathbf{n}_k, \mathbf{n}_R) \cos p_z \sin(q_z(\mathbf{n}_k, \mathbf{n}_R))]}{(q_z(\mathbf{n}_k, \mathbf{n}_R))^2 - p_z^2}. \end{aligned} \quad (13)$$

Here $\mathbf{n}_R = (\sin\theta_R \cos\varphi_R,\ \sin\theta_R \sin\varphi_R,\ \cos\theta_R)$ is a unit vector that determines the direction of the DR axis in a spherical coordinate system (θ_R, φ_R); $p_\perp = \beta r_0$; $p_z = \beta_z L/2$; $q_\perp = k_0 r_0$; $q_z = k_0 L/2$; $k_0 = \omega/c$; $k_1 = k_0\sqrt{\varepsilon_{1r}}$; ε_{1r} – relative dielectric constant of the resonator.

From relation (13) it follows that the degree of interaction of the lattice resonators with a plane wave is determined depending on the directions of their axes $\mathbf{n}_R$; direction of incidence $\mathbf{n}_k$ and, in addition, the type of polarization of the incident wave, specified in this case by the vector $\mathbf{n}_H$.

From (13) it also follows that the DR is not coupled with the incident wave if:

for p-scattering ($\mathbf{E}^+$ vector lies in the plane of incidence):

$$\varphi_k - \varphi_R = 0, \pi; \tag{14}$$

for s-scattering (the $\mathbf{E}^+$ vector is directed orthogonal to the plane of incidence):

$$\mathrm{ctg}(\theta_R)\mathrm{tg}(\theta_k) = \cos(\varphi_R + \varphi_k); \tag{15}$$

in a special case, if

$$\varphi_k = -\varphi_R \quad \text{and} \quad \theta_k = \theta_R. \tag{16}$$

The numerical solution of equation (15) is shown in Fig. 1, f, for different directions of the DR axis. The dependences $\left|c_t^+\right|^2$ for p- and s-scattering on the (θ_k, φ_k) angles of incidence for $(\theta_R, \varphi_R) = (\pi/4, \pi/4)$ are shown in Fig. 1, d, e, respectively.

The DR coupling reaches a maximum if $\mathbf{n}_R = \pm\mathbf{n}_H$, as well as for

p-scattering, if

$$\theta_R = \pi/2 \quad \text{and} \quad \varphi_k - \varphi_R = \pm\pi/2; \tag{17}$$

for s-scattering:

$$\varphi_k + \varphi_R = 0; 2\pi \quad \text{and} \quad \theta_k - \theta_R = \pm\pi/2, \tag{18}$$

or

$$\varphi_k + \varphi_R = \pi \quad \text{and} \quad \theta_R + \theta_k = \pi/2. \tag{19}$$

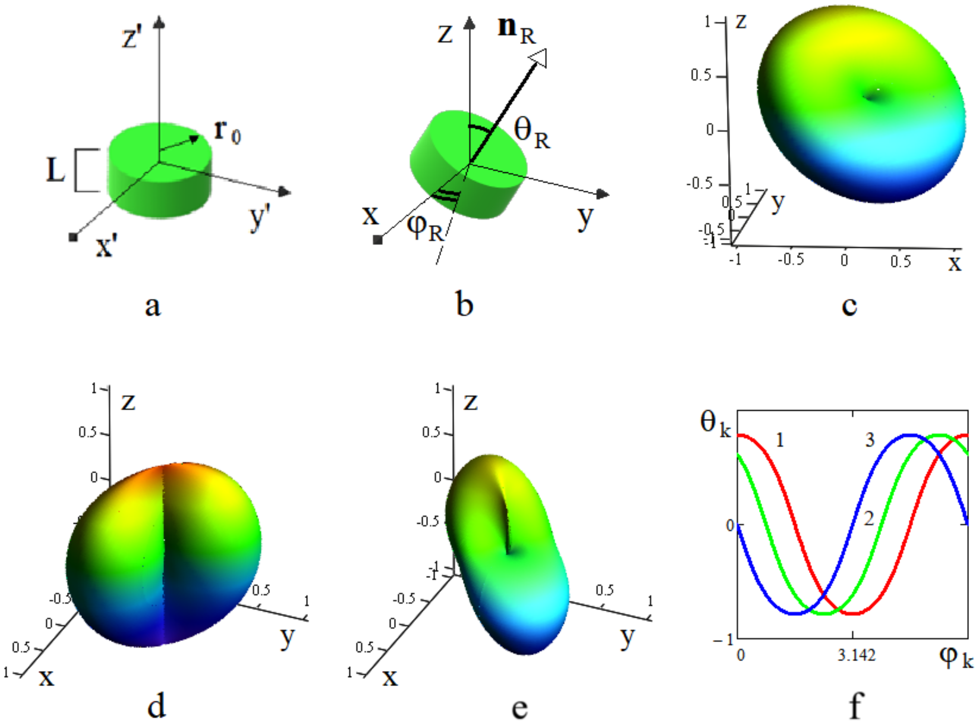

Fig. 1. Functions reflecting the degree of coupling of plane waves with the field of the main types of natural oscillations of a cylindrical DR. a) Cylindrical DR in a local coordinate system (x', y', z'); b) Rotated DR relative to the external coordinate system (x, y, z); c) Distribution of the wave zone field of a rotating DR for $\theta_R = 0,25\pi$; $\varphi_R = 0,25\pi$; d) Spatial distribution of the modulus of the c-function for p-polarization; e) for s-polarization ($\varepsilon_{1r} = 36$; $\theta_R = 0,25\pi$; $\varphi_R = 0,25\pi$); f) Dependences between the angles of the $\mathbf{k}_0$ vector, determining the direction of incidence at which the plane wave is not coupled to the resonator for s-scattering. Curve 1 – $\varphi_R = 0$; 2 – $\varphi_R = 0,25\pi$; 3 – $\varphi_R = 0,5\pi$ ($\theta_R = 0,25\pi$)

The field of natural oscillations of the magnetic type H_{nml} of a rectangular DR with dimensions (a_0, b_0, L), in the dielectric region in the local coordinate system (Fig. 2, a) are represented in the form:

$$\begin{aligned}
e_{x'} &= -h_1 \frac{i\omega\mu_0\beta_y}{(\beta_x^2+\beta_y^2)} \cdot \begin{pmatrix}\sin\beta_x x' \\ \cos\beta_x x'\end{pmatrix} \begin{bmatrix}\cos\beta_y y' \\ -\sin\beta_y y'\end{bmatrix} \begin{Bmatrix}\sin\beta_z z' \\ \cos\beta_z z'\end{Bmatrix};\\
e_{y'} &= h_1 \frac{i\omega\mu_0\beta_x}{(\beta_x^2+\beta_y^2)} \cdot \begin{pmatrix}\cos\beta_x x' \\ -\sin\beta_x x'\end{pmatrix} \begin{bmatrix}\sin\beta_y y' \\ \cos\beta_y y'\end{bmatrix} \begin{Bmatrix}\sin\beta_z z' \\ \cos\beta_z z'\end{Bmatrix};\\
e_{z'} &= 0;\\
h_{x'} &= h_1 \frac{\beta_x\beta_z}{(\beta_x^2+\beta_y^2)} \cdot \begin{pmatrix}\cos\beta_x x' \\ -\sin\beta_x x'\end{pmatrix} \begin{bmatrix}\sin\beta_y y' \\ \cos\beta_y y'\end{bmatrix} \begin{Bmatrix}\cos\beta_z z' \\ -\sin\beta_z z'\end{Bmatrix};\\
h_{y'} &= h_1 \frac{\beta_y\beta_z}{(\beta_x^2+\beta_y^2)} \cdot \begin{pmatrix}\sin\beta_x x' \\ \cos\beta_x x'\end{pmatrix} \begin{bmatrix}\cos\beta_y y' \\ -\sin\beta_y y'\end{bmatrix} \begin{Bmatrix}\cos\beta_z z' \\ -\sin\beta_z z'\end{Bmatrix};\\
h_{z'} &= h_1 \cdot \begin{pmatrix}\sin\beta_x x' \\ \cos\beta_x x'\end{pmatrix} \begin{bmatrix}\sin\beta_y y' \\ \cos\beta_y y'\end{bmatrix} \begin{Bmatrix}\sin\beta_z z' \\ \cos\beta_z z'\end{Bmatrix}.
\end{aligned} \tag{20}$$

In a dielectric, the $(\beta_x, \beta_y, \beta_z)$ constants satisfy the equation:

$$\beta_x^2 + \beta_y^2 + \beta_z^2 = k_1^2.$$

Here $k_1 = \varepsilon_{1r}^{1/2} k_0$; $k_0 = \omega/c$; ω – circular frequency; c – speed of light.

For convenience, oscillations of different types of symmetry are designated by the indices s or c. The s index corresponds to the distribution of the $h_{z'}$ magnetic field z components (20), proportional to the sin function, and the c index corresponds to the distribution of the corresponding z field components, proportional to the cos function. For example, we will denote the magnetic type H_{csc} corresponding to an even $h_{z'}$ field distribution in the direction of the x' and z' axes, as well as an odd one along the y' axis, as (20).

In the case of scattering of a plane wave on a rotating rectangular DR for any magnetic oscillation, the results of calculating are presented in the form:

$$c_1^+ = c_0 \cdot \left[\beta_y \omega_x((\mathbf{n}_x^R, \mathbf{n}_k))\varpi_y((\mathbf{n}_y^R, \mathbf{n}_k)) \cdot (\mathbf{n}_E, \mathbf{n}_x^R) - \beta_x \varpi_x((\mathbf{n}_x^R, \mathbf{n}_k))\omega_y((\mathbf{n}_y^R, \mathbf{n}_k)) \cdot (\mathbf{n}_E, \mathbf{n}_y^R)\right] \cdot \omega_z((\mathbf{n}_z^R, \mathbf{n}_k)), \tag{21}$$

where

$$\begin{aligned}
c_0 &= \frac{1}{2}(\varepsilon_{1r}-1)h_1 E_0^* \frac{k_0^2}{(\beta_x^2+\beta_y^2)} a_0 b_0 L;\\
\omega_v(n) &= \frac{1}{p_v^2-(q_v n)^2} \begin{pmatrix} -i[p_v\cos p_v \sin(q_v n) - q_v n \sin p_v \cos(q_v n)] \\ [p_v \sin p_v \cos(q_v n) - q_v n \cos p_v \sin(q_v n)] \end{pmatrix};\\
\varpi_v(n) &= \frac{1}{p_v^2-(q_v n)^2} \begin{pmatrix} [p_v \sin p_v \cos(q_v n) - q_v n \cos p_v \sin(q_v n)] \\ i[p_v\cos p_v \sin(q_v n) - q_v n \sin p_v \cos(q_v n)] \end{pmatrix},
\end{aligned} \tag{22}$$

where $(v = x, y, z)$, $p_x = \beta_x a_0/2$; $p_y = \beta_y b_0/2$; $p_z = \beta_z L/2$; $q_x = k_0 a_0/2$; $q_y = k_0 b_0/2$; $q_z = k_0 L/2$; $\mathbf{n}_x^R$, $\mathbf{n}_y^R$, $\mathbf{n}_z^R$ – orts of the local coordinate system (x', y', z') in the “external” coordinate (x, y, z) system of the DR (Fig. 2, a); $\mathbf{n}_E$ – unit vector of the electric field of the incident wave $(\mathbf{E}^+, \mathbf{H}^+)$; $\mathbf{n}_k$ – unit vector that determines the direction of propagation of the incident wave. The directions of the orts $\mathbf{n}_v^R = \mathbf{n}_v^R(\theta_R, \varphi_R)$ are indicated in spherical coordinates. The expressions in brackets (22) correspond to the field distribution in (20).

As follows from (21), (22), a rectangular DR is not coupled with an incident wave on the main magnetic type of oscillation H_{ccc} the direction of the vectors $\mathbf{n}_k$ and $\mathbf{n}_z^R$ coincides, i.e. $\left|(\mathbf{n}_k, \mathbf{n}_z^R)\right| = 1$. The second possible condition is $\left|(\mathbf{n}_E, \mathbf{n}_z^R)\right| = 1$. That is, if the z' resonator axis is directed along the $\mathbf{k}_0$ vector, or in the $\mathbf{n}_E$ direction of polarization of the electric field of the incident wave.

Fig. 2, b–e shows the dependences of the modulus c which is the function of the rectangular DR, calculated according to formula (21) for three main types of magnetic-type oscillations (20). As follows from the data presented, the rotation of a rectangular DR has a particularly noticeable effect on oscillations of higher types (Fig. 2, d, e), which have a more complex spatial field distribution.

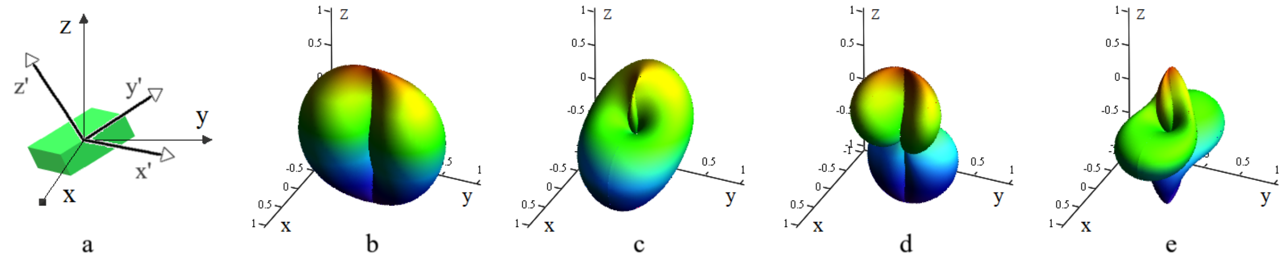

Fig. 2. Spatial distributions of functions reflecting the magnitude of the coupling between the main magnetic oscillations of a rectangular DR and a plane wave in open space. a) Rotated Rectangular DR in the external (x, y, z) and local (x', y', z') coordinate system; b, c)$|c^{+}(\theta, \varphi)|$ dependences on the $\mathbf{n}_k$ directions of incidence of a plane wave of rotated DR for $\mathbf{n}_z^R = (0.25\pi, 0)$; $\mathbf{n}_x^R = (0.75\pi, 0)$ for H_{ccc} oscillation; d, e) for H_{ccs} oscillation; b, d) for p-scattering; c, e) for s-scattering

Thus, the found general expressions of c-functions make it possible to establish conditions 2), under which the scattering of plane waves on lattices of DRs with magnetic oscillations of the main type is determined only by non-resonant interaction. In this case, the incident wave passes almost completely through the lattice without noticeable reflections. The phenomenon considered is reminiscent of the Malyuzhinets effect, also studied in [20] when waves are scattered on gratings of dielectric bars.

3 Brewster effect at the scattering of electromagnetic waves by Plain Lattices of pseudo-rotatable DRs

Let us consider anomalous scattering on different square lattices of pseudo-rotated DR (Fig. 3, a, c, g). In the general case, if condition (8) is satisfied, the dependence of the scattering amplitude qualitatively coincides with those considered in [27]: radiation lobes reflected and transmitted through the lattice are observed in the wave zone, lying in the plane of incidence.

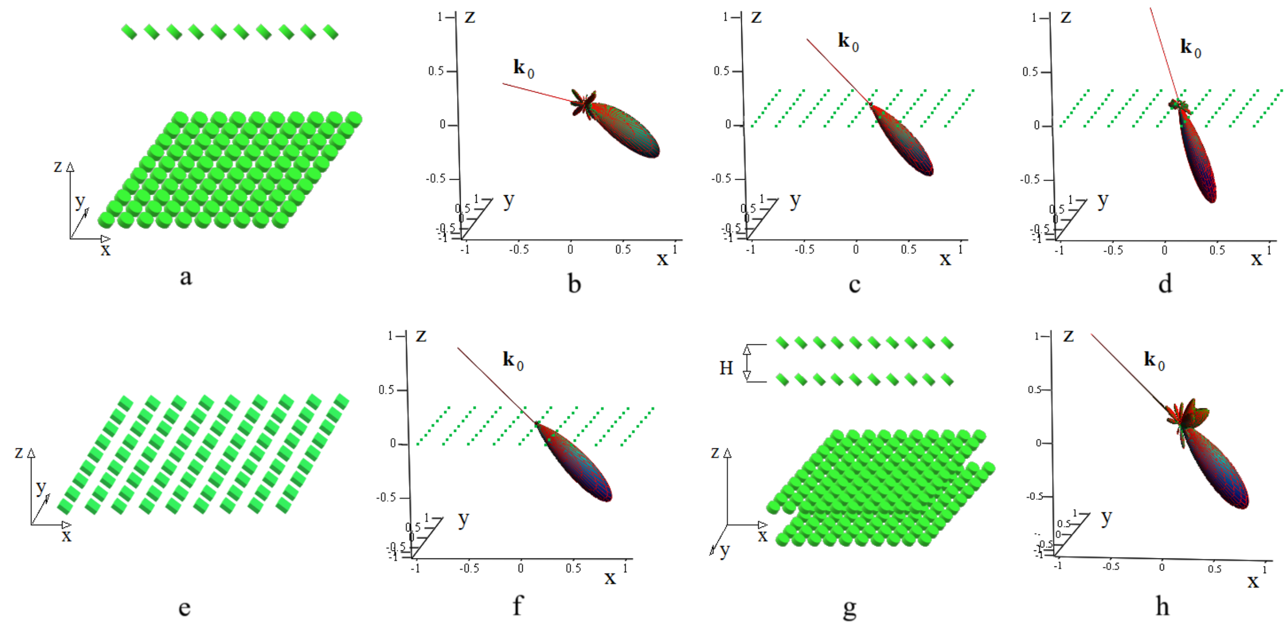

Fig. 3. Results of scattering of plane waves on lattices of pseudo-rotating DR in open space under the condition of minimizing the reflected lobe (a, e). Square lattices of various types of pseudo-rotating DR of cylindrical and rectangular shape; b-d) Angular dependences $|f\langle\theta_k, \varphi_k|\theta, \varphi\rangle|^2$ for s-scattering at angles of incidence $\theta_k = 0,6\pi; 0,75\pi; 0,9\pi$; $\varphi_k = 0,25\pi$ for the lattice a); f, h) $\theta_k = 0,75\pi$; $\varphi_k = 0$; when turning the DR axes $\theta_R = \pi - \theta_k$; $\varphi_R = \varphi_k$. The straight line shows the $\mathbf{k}_0$ direction of fall

However, if the direction of the minimum radiation of the main magnetic oscillations of each partial resonator coincides with the direction of propagation of the reflected wave (Fig. 1, c), there is an almost complete absence of the reflection lobe (Fig. 3, b-d) when scattering not only on a plane, but also double-layer grid (Fig. 3, h), called the Brewster effect. It should be noted that the general patterns of reflection from DR lattices have a number of fundamental differences from the classical Brewster effect.

Firstly, "zero reflection" from the DR lattice is possible for both s- and p-scattering, while the Brewster effect occurs only for p-scattering. In the second case, the lattice of cylindrical DR should be excited, for example, by $E_{10\delta}$ azimuthally homogeneous oscillations of the electric type.

Secondly, in the general case, when a wave is incident at the Brewster angle, the reflected and refracted rays are perpendicular to each other [25]. In the case of scattering on the DR lattice, this condition is not satisfied. As noted above, the power flux density of the reflected waves can be reduced over a wide range of incidence angles by pseudo-rotating the lattice of resonators (see Fig. 3, b-d). For the main types of oscillations, this is achieved by orienting the resonator axes in the direction of the reflected lobe.

Conclusion

The work determines the conditions for quasi-complete passage of waves through lattices of cylindrical and rectangular DR with the main types of oscillations. In this case, the scattered field is determined only by the non-resonant interaction of the lattice elements with the field of the incident wave.

The conditions for the occurrence of the Brewster effect for DR lattices of different types have been established. It is shown that lattices of pseudo-rotating DRs have greater capabilities for controlling the reflected power flow compared to the classical case of scattering at a flat surface between media of different dielectric constants.

The use of pseudo-rotations of dielectric resonators makes it possible in practice to achieve an effective reduction in the power of reflected waves in a given scattering direction; separating scattering channels of resonant and non-resonant types [26].

The results of this study can be used to ensure electromagnetic compatibility of a wide class of antennas in the microwave and terahertz ranges, built using DR lattices, as well as in the development of optical devices in the infrared and optical wavelength ranges in modern communication systems.

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Керований ефект Брюстера при розсіюванні електромагнітних хвиль на псевдообертових решітках діелектричних резонаторів

Трубін О. О.

Досліджуються особливі випадки розсіювання плоских електромагнітних хвиль решітками діелектричних резонаторів (ДР) з невиродженими типами коливань, які обертаються навколо заданої вісі. Знайдено аналітичні вирази функцій, які в загальному вигляді відображають ступінь взаємодії діелектричних резонаторів прямокутної та циліндричної форми з плоскими хвилями різних видів поляризації. Розраховано залежності від кутів падіння плоскої хвилі для декількох основних типів коливань, які забезпечують зв'язок ДР з плоскою хвилею. На основі отриманих формул встановлено умови нерезонансного розсіювання, а також розсіювання з відсутністю відбитої пелюстки, відомої як ефект Брюстера. Знайдені загальні співвідношення між кутами нахилу ДР, поляризацією та кутами падіння хвиль на решітки, які призводять до особливих випадків розсіювання. Відмічається схожість між нерезонансним розсіюванням та відомим ефектом Малюжинця, який описує проходження хвиль крізь решітки інших типів. Побудовано моделі розсіювання для обертових решіток циліндричних та прямокутних ДР. Розраховано залежності амплітуд розсіювання від кутів падіння хвиль, які відповідають попереднім теоретичним висновкам. Відмічається різниця між класичним ефектом Брюстера та безпелюстковими випадками розсіювання на решітках, побудованих на основі використання псевдообертових ДР. Показано, що на відміну від інших способів реалізації метаповерхонь цього класу, безпелюсткові випадки розсіювання на решітках псевдообертових резонаторів можливі при зміні кутів падіння в більш широкій смузі. Отримані теоретичні результати моделювання дозволяють запропонувати новий клас пристроїв, побудованих на основі використання псевдообертових ДР, суттєво скоротити час розрахунків та оптимізувати багато складних резонаторних структур, побудованих на їх основі. Нові типи решіток, побудованих на псевдообертових ДР, можна використовувати для побудови широкого класу антен, пристроїв мультиплексування в системах зв'язку терагерцового, інфрачервоного та оптичного діапазонів довжин хвиль.

Ключові слова: решітка; діелектричний резонатор; розсіювання; ефект Брюстера; обертання; зв'язані коливання