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Method for Selecting Pulsed Signals by Their Duration in Fading Channels

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The duration of pulsed signals is one of the main parameters to be estimated in radio monitoring systems. When signals propagate in channels with deep fading, even at high signal-to-noise ratios, the pulse shape will be distorted. In sophisticated electronic environment, it is also may be random interference in signal processing channel, which leads to the occurrence of false pulses with random durations. Therefore, the values of the signal pulses durations will be concentrated near their true value, and the rest of the detected pulses will have a significantly random duration. That's why, the development and study of methods for selecting pulse signals by their durations in sophisticated signal environment is actual scientific problem. The aim of the work is improving pulsed signals processing methods in fading channels by selecting its' durations. The study found that the estimates of signal pulse durations are normally distributed. Pulse durations that are not related to signals are subjected to an exponential distribution. The input data for the proposed method is only a sample of measured pulse durations. The values of the parameters of both the exponential and normal distributions are unknown. In this case, the problem of selecting pulses by their durations is formalized to the estimation of the mean values of normal distributions. To do this, it is proposed to search for the maxima of the smoothed estimate of the probability density function. The scientific novelty of the obtained results is that a method for estimating the mean value of a normal distribution at the background of exponentially distributed values was proposed. An example of this approach is the estimation of pulsed signal durations in channels with deep fading and impulse interference. Based on the developed method, algorithms for automatic pulse selection for radio monitoring systems can be implemented.

Keywords: pulsed signal; fading; signal selection; normal distribution; exponential distribution; sample

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Introduction

One of the main tasks of radio monitoring systems is to estimate the time and frequency parameters of radio signals. One of these parameters is the duration of pulse signals for radar stations, as well as the duration of packets, symbols, frequency elements for communication, control, and telemetry systems. Estimation of signals time parameters is usually performed by thresholding a complex envelope. Due to the non-stationarity of the radio signal propagation channel, some of the pulses will be affected by fading, which will distort their shape. Therefore, in channels with deep fading, even at high signal-to-noise ratios, only some of the detected pulses will have a close to true duration. The rest of the detected pulses will have random durations. In a sophisticated electronic environment, it is also possible for random interference to occur in the frequency channel, which will lead to false pulses detection. These factors make it difficult to select signals by their duration. That's why, the development and study of methods for selecting pulsed signals by their duration in a sophisticated signal environment is current scientific task.

1 Related works

The problems of detecting and estimating the time parameters of pulse signals in various domains of science have been considered in numerous papers. In particular, an approach to detecting weak pulse signals against chaotic noise is proposed in [1]. To detect and determine the beginning and end of pulse signals, a generalized method using machine learning is proposed in [2]. A method for detecting non-repeating irregular pulse signals is presented in [3]. In [4], it is proposed to use wavelet transforms and deep learning to classify pulse signals in a noisy channel. A brief overview of methods for estimating pulse durations and repetition periods is given in [5]. A method for estimating pulse durations using auto-compression is proposed in [6, 7]. In [8], an algorithm for estimating the time parameters of pulses without thresholding is considered.

The reviewed works do not address the issue of selecting pulses by their duration when operating in channels with deep fading and in sophisticated electronic environment.

2 Problem statement

The aim of the work is improving pulsed signals processing methods in fading channels by selecting its' durations.

3 Method for pulse duration selection

Let's consider a typical case where a frequency channel with deep fading contains only white Gaussian noise ξ with a known standard deviation (SD) σ_ξ and signals with unknown durations may appear at unknown times. In this case, the threshold value can be calculated for a given probability of false alarm. The threshold processing is performed on the samples of the complex envelope signal smoothed with a moving average window of length L. For noise, the probability density function (PDF) of the smoothed samples has a chi-square distribution with $2(L+1)$ degrees of freedom.

For a given threshold value, the times at which it is crossed by the complex noise envelope will be random variables. Moreover, the threshold crossings will appear with some fixed average intensity λ for a particular threshold value. Such events will create a stream of random variables τ which are time intervals between two consecutive threshold crossings, which are exponentially distributed [9] according to the following expression:

$$p(\tau) = \lambda e^{-\lambda\tau}. \quad (1)$$

It is worth noting that the values of τ follow an exponential distribution regardless of the type of distribution of complex envelope samples. Therefore, the duration of random interference will also follow an exponential distribution.

For an exponential distribution, the value of the parameter λ is related to the values of the mathematical expectation $m_{\tau e}$ and the SD $\sigma_{\tau e}$ according to the following expression [9]:

$$\lambda = \frac{1}{m_{\tau e}} = \frac{1}{\sigma_{\tau e}}. \quad (2)$$

Thus, it is possible to implement the signals selection by their durations, assuming that the number of pulse duration values is limited and each duration occurs a significant number of times in the observation interval.

Errors of pulse duration estimates are normally distributed. Pulse durations that are not related to signals will have an exponential distribution. In other words, the exponentiality of the PDF of τ values in the channel indicates the randomness of the pulse origin caused, among other things, by deep amplitude fading.

The initial data for proposed method is a sample of measured pulse durations of size M. In this sample, an unknown number of values N_e follows an exponential distribution with an unknown parameter λ. The remaining values follow K normal distributions. Then the sample size can be written as follows:

$$M = N_e + \sum_{i=1}^{K} N_{ni}. \quad (3)$$

The number of values for the i-th normal distribution N_{ni}, their mean values $m_{\tau ni}$ and the SD $\sigma_{\tau ni}$ are also unknown parameters. Then the PDF of the mixture of exponential and normal values can be written in the following form:

$$p(\tau) = \frac{1}{K+1}\left(\frac{1}{m_{\tau e}} e^{-\frac{\tau}{m_{\tau e}}} + \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{K} \frac{1}{\sigma_{\tau ni}} e^{-\frac{(\tau - m_{\tau ni})^2}{2\sigma_{\tau ni}^2}}\right). \quad (4)$$

In practice, the sample size M is finite, so the shape of the histogram based on it, which is an empirical analog of the PDF, will depend on the values of N_e, N_{ni} and K. The ratio N_e/M determines the normalizing factor for the exponential distribution, and N_{ni}/M – for the i-th normal distribution. Then expression (4) can be rewritten as follows:

$$p_H(\tau) = \frac{N_e}{M m_{\tau e}} e^{-\frac{\tau}{m_{\tau e}}} + \frac{1}{M\sqrt{2\pi}} \sum_{i=1}^{K} \frac{N_{ni}}{\sigma_{\tau ni}} e^{-\frac{(\tau - m_{\tau ni})^2}{2\sigma_{\tau ni}^2}}. \quad (5)$$

Figure 1 shows the graphs of the resulting PDF constructed according to expression (4), as well as the exponential and three normal distributions. The parameters of the individual distributions are given in the legend of the graph. The addition of individual distributions leads to the appearance of local maxima and minima.

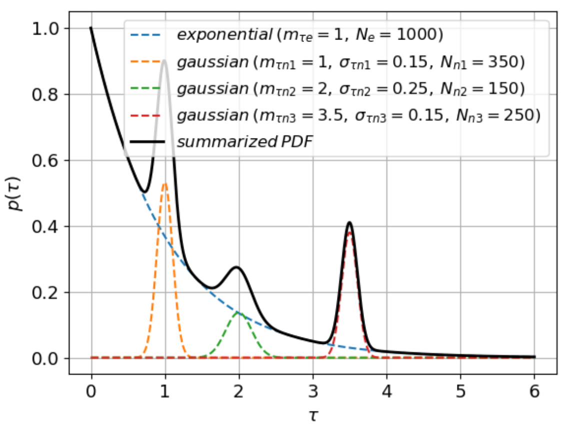


Fig. 1. Resulting PDF, exponential and normal distributions

Then the problem of pulse selection is formalized to the estimation of the mean values of the normal distributions $m_{\tau ni}$. In Fig. 1, the values of $m_{\tau ni}$ correspond to the maxima of the sum PDF, so their estimation will allow us to obtain estimates of pulse durations. Also, to find the maxima, it is advisable to use the property of the decreasing exponential distribution with increasing argument.

To solve this problem, it is necessary to obtain an analytical expression for estimating the PDF for a given M using a sample of τ values.

Kernel probability density estimation is most often used to construct the PDF [10]. This approach is non-parametric and based on local smoothing of sample data. The parameters for implementing this method are the type of kernel (window) and the width of the smoothing band h. Most often, the Gaussian kernel is used to estimate the PDF [11] because it provides the greatest smoothing. The choice of h is a compromise because it determines the degree of smoothing. A value of h that is too small will result in the display of unimportant sample details. Choosing a large value of h will result in the loss of some information due to excessive smoothing. In [12], for non-normal distributions, it is proposed to calculate the width of the smoothing interval using the following expression:

$$h = 0.9 \cdot \min\left(\hat{\sigma}, \frac{IQR}{1.34}\right) M^{-0.2}, \tag{6}$$

where IQR – interquartile range, which is more robust to outliers than the SD; $\hat{\sigma}$ – estimate of sample SD.

Then we estimate the PDF of a sample of values of τ by the following expression [11]:

$$\begin{aligned} P(t) &= \frac{1}{M}\sum_{i=1}^{M} K(\tau_i, t), \\ K(\tau_i, t) &= \frac{1}{h\sqrt{2\pi}} e^{-\frac{(t-\tau_i)^2}{2}}, \end{aligned} \tag{7}$$

where t is an argument used to build the kernel.

The maxima of the PDF estimate are found as the value of $P(t)$ for those t for which the value of the derivative of $P(t) = 0$ and changes its sign from positive to negative. The value of the derivative of the exponential component of the sum distribution

$$p_e{}'(\tau) = -\frac{N_e}{Mm_{\tau e}^2} e^{-\frac{\tau}{m_{\tau e}}} \tag{8}$$

is a decreasing function for all values of τ.

For the i-th normal distribution, the value of the derivative will be as follows:

$$p_{ni}{}'(\tau) = \frac{(m_{\tau ni} - \tau)\, N_{ni}}{\sqrt{2\pi} M \sigma_{\tau ni}^3} e^{-\frac{(\tau - m_{\tau ni})^2}{2\sigma_{\tau ni}^2}} \tag{9}$$

and reaches on zero values at the point $\tau = m_{\tau ni}$.

However, for relatively small M (several hundred), for large values of τ, due to their small number, the derivative of $P(t)$ will have zero values for those values of the argument that do not correspond to pulse durations. These values are associated with the noise of the sample due to its limited size. To filter out these random maxima, it is necessary to take into account the sample size M and the rate of decay of the exponential component. That is, among all the detected maxima of $P(t)$, it is necessary to select those that correspond to pulse durations, and discard the rest as noise. To do this, it is necessary to perform threshold processing of the detected maxima. The threshold function should have a form close to the exponent. We write it in the following form:

$$\gamma(\tau) = E(\tau) + \varphi(\tau), \tag{10}$$

where $E(\tau)$ – exponent; $\varphi(\tau)$ – some descending function.

The value of the function $\varphi(\tau)$ reflects the fluctuations of the function $P(t)$ associated with a limited sample size M. It is also necessary to take into account the fact that with an increase in τ and a fixed M, the number of values that fall within the histogram interval of width h will decrease. Therefore, for larger τ, the sampling noise will be more pronounced than for smaller ones. This is due to the fact that for a sample of exponentially distributed values of size N_e, the number of small values will be greater than the number of large values, since the probability of occurrence of its value decreases with increasing τ. Therefore, the function $\varphi(\tau)$ should decay more slowly than the exponent. The SD of the sampling function $P(t)$ for each τ will be proportional to the number of τ values falling into the corresponding histogram interval. Based on the above considerations, the value of the threshold function is written in the following form:

$$\gamma(\tau) = \frac{N_e}{Mm_{\tau e}} e^{-\frac{\tau}{m_{\tau e}}} + \sigma_0 q e^{-\left(\frac{\tau}{m_{\tau e}}\right)^d}, \tag{11}$$

where σ_0 – SD of the number of exponential values falling in the first interval of the histogram; q is a positive number that determines the probability of exceeding the threshold by a noise emission; d – parameter responsible for the function's $\varphi(\tau)$ decay rate.

When constructing a histogram, the number of times τ values fall into the i-th interval n_i is a random χ^2 variable with n_i degrees of freedom. The dependence of σ_0 on the sample size of the exponential distribution N_e and its mean value $m_{\tau e}$ was obtained by statistical modeling:

$$\sigma_0 \approx \frac{1}{m_{\tau e}} \left(0,017 + \frac{e^{-N_e^{0,145}+0,9}}{3}\right). \tag{12}$$

The estimate of σ_0 was obtained from 10^4 sample realizations, so the error is about 0.31%. Expressions (11-12) should be used for normalized values of the PDF estimates.

In Fig. 2 is shown the dependence of the sample mean m_0 and the SD σ_0 of the number of values of an exponential random variable with parameter $\lambda = 1$ falling into the first interval of the histogram n_0 on the sample size N_e. As you can see, the value of the mean practically does not change and is approximately equal to 1, so its influence on the value of the function $\varphi(\tau)$ can be neglected. The value of n_0/N_e is subject to the χ^2 distribution. If the number of hits in the first interval of the normalized histogram is more than 30 PDF, n_0/N_e can be approximately considered normal and the value of q in expression (10) can be chosen equal to 4-5.

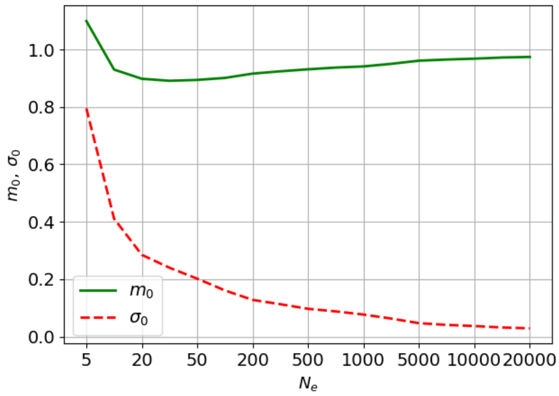


Fig. 2. Dependence m_0 and σ_0 of sample size N_e

Thus, to obtain the function $\gamma(\tau)$, it is necessary to estimate the mean value of the exponential distribution $m_{\tau e}$ and the number of values of τ from the sample of size M distributed exponentially N_e. The values of $m_{\tau e}$ and N_e we will search basing on the assumption that the first value of the normalized histogram n_0/M, as well as all local minima of the PDF estimation function $P(t)$ belong to the exponential distribution with parameters $m_{\tau e}$ and N_e. To do this, it is enough to solve the system of two equations:

$$\begin{cases} E_1 = \dfrac{N_e}{M m_{\tau e}} e^{-\frac{\tau_1}{m_{\tau e}}} \\ E_2 = \dfrac{N_e}{M m_{\tau e}} e^{-\frac{\tau_2}{m_{\tau e}}} \end{cases}, \qquad (13)$$

where τ_1, τ_2 – arguments for which the function $P(t)$ has local minima.

After solving this system, we get the following values:

$$m_{\tau e} = \frac{\tau_2 - \tau_1}{\ln(E_1) - \ln(E_2)}, \quad N_e = M m_{\tau e} E_1 e^{\frac{\tau_1}{m_{\tau e}}}. \qquad (14)$$

The minimum values of $P(t)$ are also random variables. To obtain reliable estimates of them, it is necessary to calculate the values of $m_{\tau e}$ and N_e for several different values of τ at which $P(t)$ reaches its minimum and to average the results. Then, to estimate the values of $m_{\tau e}$ and N_e, the following sequence of operations should be performed:

1. Find all the minima of the function $P(t)$ and append the value of the normalized histogram for the first interval n_0/M to created array.
2. Sort the resulting array in descending order. This is necessary because, due to sampling noise, the minimum $P(t)$ for a larger τ may exceed the minimum $P(t)$ for a smaller τ, which does not correspond to an exponential distribution and will lead to additional errors in parameter estimates.
3. For the first equation in system (13), substitute n_0/M for E_1 and 0 for τ_1. For the second equation, use the minima of the function $P(t)$ with the corresponding arguments τ.
4. Average obtained values of $m_{\tau e}$ and N_e.

Then the values of the estimated pulse durations will be determined according to the following expression:

$$m_{\tau n} = \arg_{\tau} \{\max(P(\tau)) \geq \gamma(\tau)\}. \qquad (15)$$

In this expression, τ is used as an argument to the function $P(t)$. These parameters are related to each other by the following relationship:

$$\tau = \tau_{\max} t / M , \qquad (16)$$

where τ_{max} is the maximum value of τ for each sample.

If the values in the sample follow only one normal distribution, then there may be no minima of the smoothed PDF. In this case, there should be only one maximum of the PDF estimate, and the pulse duration can be calculated as the arithmetic mean of τ.

4 Simulations and numerical results

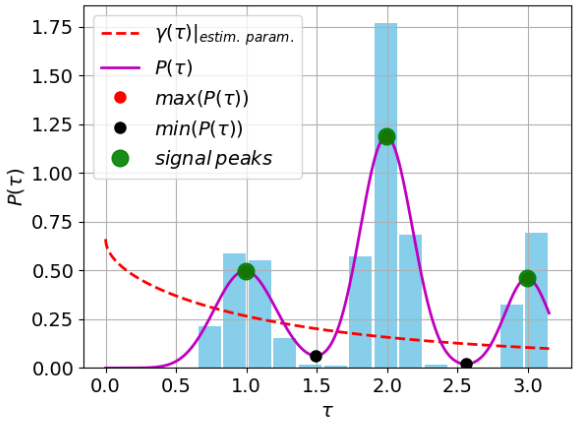
The effectiveness of the developed method will be determined by the probability of detecting the values of pulse durations and the relative error by estimating the values of their durations. Obviously, these characteristics will depend on the parameters included in expression (5), namely: $N_e, m_{\tau e}, N_{ni}, m_{\tau ni}$ and $\sigma_{\tau ni}$.

When working in real conditions, there may be situations when a sample of size M contains only exponentially distributed values, one or more only normally distributed values, and a mixture of values according to expression (5). Figure 3a shows the results of pulse duration detection and estimation for the case when the sample of size M contains only three normally distributed random variables with the parameters given in Table 1. The threshold value is calculated according to expression (11), and the parameter values are estimated according to expression (14). The following parameter values were chosen: $d = 0,5$ and $q = 5$. As we can see, two minima and three maxima were obtained for the estimation of the PDF. Moreover, all the maxima exceeded the calculated threshold.

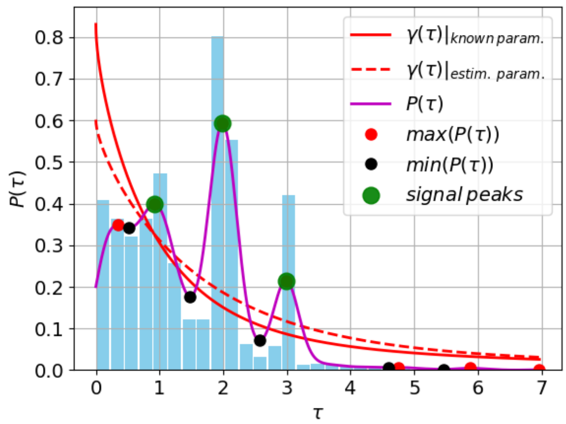
Figure 3b shows the corresponding results for the case when the sample contains 1000 exponentially distributed values with parameter 1 in addition to the normal values given in Table 1. This figure also shows the threshold for the case when the parameters of the exponential distribution N_e and $m_{\tau e}$ are known.

Table 1 — Parameters of normal values

Pulse number	N_n	$m_{\tau n}$	$\sigma_{\tau n}$
1	300	1	0,15
2	600	2	0,1
3	200	3	0,05



(a)



(b)

Fig. 3. Results of pulse detection for case of only normal values (a) and mixture of normal and exponential values (b)

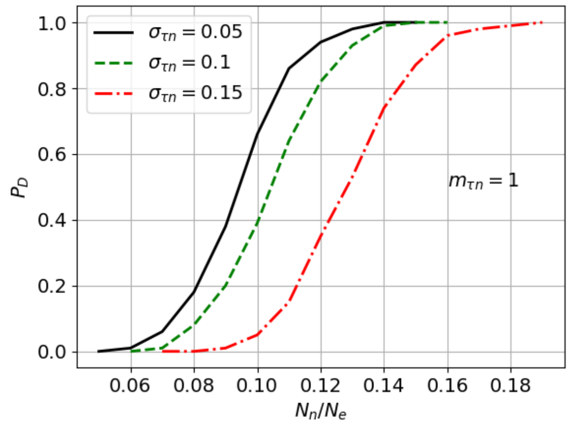
In Fig. 4a is shown the probability of pulse detection as a function of the relative number of normally distributed values at the background of exponential values at $m_{\tau n} = 1$ for different values of the SD. Fig. 4b shows similar detection curves at $\sigma_{\tau n} = 1$ for different normal distribution means. The following conclusions can be drawn from these dependencies:

as N_n/N_e increases, the probability of pulse detection also increases;

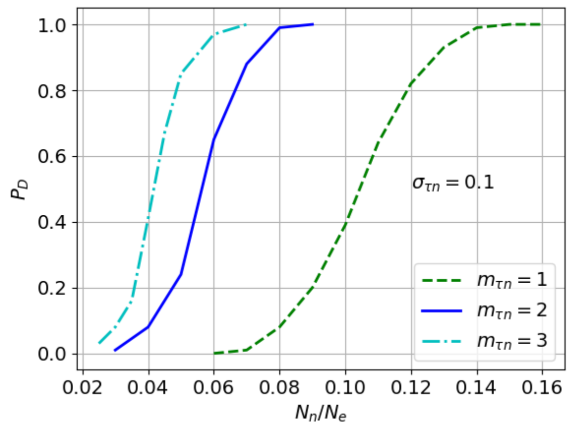
as $m_{\tau n}$ increases and other parameters are fixed, the detection characteristics improve;

as $\sigma_{\tau n}$ increases and other parameters are fixed, the detection characteristics deteriorate;

for $N_n/N_e > 0,2$, $\sigma_{\tau n}/\sigma_{\tau e} < 0,15$ and $m_{\tau n}/m_{\tau e} > 1$, guaranteed pulse detection is provided.



(a)



(b)

Fig. 4. Dependence of pulse detection probability via relative number of normal values for different SD (a) and means (b)

Figure 5 shows the dependence of the relative error ε_τ of the pulse duration estimation on the relative number of normally distributed values for different means and standard deviations of the normal distribution. These graphs show that for any values of the distribution parameters, the value of ε_τ decreases with increasing N_n/N_e according to an approximately exponential dependence. With a decrease in $\sigma_{\tau n}$, the value of ε_τ also decreases. However, at large $N_n/N_e > 0,5$, this difference becomes insignificant. For $N_n/N_e = 0,5$ and $\sigma_{\tau n} = 0,1$, the error in estimating the pulse duration does not exceed 1.5%.

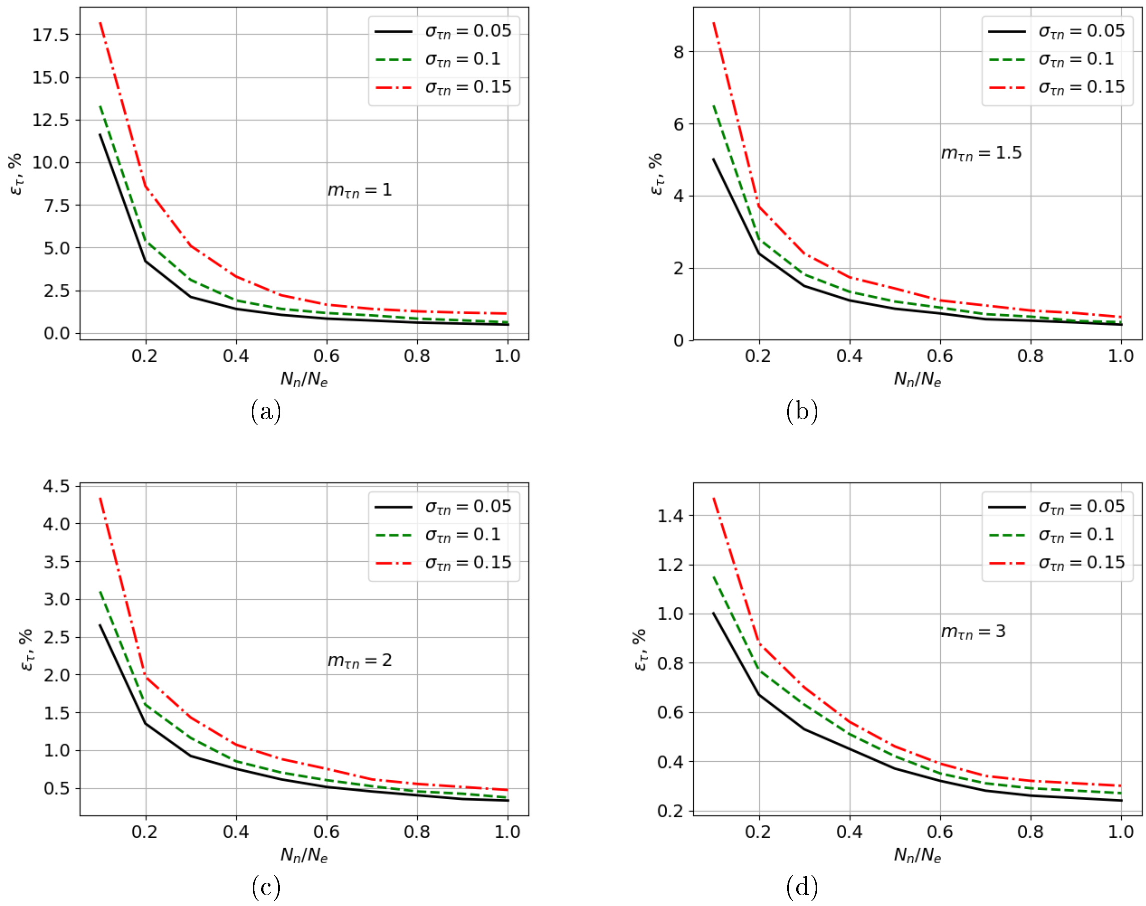


Fig. 5. Dependence of pulse duration estimate error via relative number of normal values for different mean: (a) $m_{\tau n} = 1$; (b) $m_{\tau n} = 1,5$; (c) $m_{\tau n} = 2$; (d) $m_{\tau n} = 3$

In case of absence of exponential values in the sample, the error estimates of pulse durations will be normal random variables, and their values will be determined by the sample size and the variability of the data itself.

Conclusions

The scientific novelty of the obtained results is that a method for estimating the mean value of a normal distribution at the background of exponentially distributed values is proposed. An example of this approach is the estimation of pulse signal durations in channels with deep fading and impulse interference. The reliability of the proposed method is confirmed by the results of simulation modeling. Based on the developed method, algorithms for automatic pulse selection for radio monitoring systems can be implemented. Prospects for further research in this area should be focused on generalizing of the obtained results for other types of distributions of signal and interfering variables.

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Метод селекції імпульсних сигналів за їх тривалостями в каналах із завмираннями

Бугайов М. В., Закіров С. В.

Тривалість імпульсних сигналів є одним із основних параметрів, що підлягають оцінюванню у системах радіомоніторингу. При поширенні сигналів у каналах із глибокими завмираннями навіть при високих значеннях відношення сигнал-шум форма імпульсів буде спотвореною. У складній радіоелектронній обстановці також можливе попадання в канал оброблення випадкових завад, що призведе до появи хибних імпульсів. Тому значення тривалостей корисних імпульсів будуть зосереджені біля їх істинного значення, а решта виявлених імпульсів матимуть суттєво випадкову тривалість. У зв'язку з цим розроблення та дослідження методів селекції імпульсних сигналів за їх тривалостями у складній сигнальній обстановці є актуальним науковим завданням.

Метою роботи є удосконалення методів оброблення імпульсних сигналів у каналах із завмираннями шляхом селекції їх тривалостей.

У ході досліджень встановлено, що оцінки значень тривалостей сигнальних імпульсів розподілені за нормальним законом. Тривалості імпульсів, що не пов'язані із сигналами, підпорядковані експоненціальному розподілу. Вихідними даними для роботи запропонованого методу є лише вибірка виміряних тривалостей значень імпульсів. Значення параметрів як експоненціального, так і нормальних розподілів є невідомими. В такому разі задача селекції імпульсів за їх тривалостями формалізується до оцінювання середніх значень нормальних розподілів. Для цього запропоновано проводити пошук максимумів згладженої оцінки розподілу щільності ймовірностей.

Наукова новизна отриманих результатів полягає в тому, що запропоновано метод оцінювання середнього значення нормального розподілу на фоні експоненціально розподілених величин. Прикладним застосуванням даного підходу є оцінювання тривалостей імпульсних сигналів у каналах із глибокими завмираннями та імпульсними перешкодами. На основі розробленого методу можуть бути реалізовані алгоритми автоматичної селекції імпульсів для систем радіомоніторингу.

Ключові слова: імпульсний сигнал; завмирання; селекція сигналів; нормальний розподіл; експоненціальний розподіл; вибірка