*Visnyk NTUU KPI Seriia – Radiotekhnika Radioaparatobuduvannia*, 2024, Iss. 97, pp. 5–11

UDC 621.396.962

# Mathematical Model of Shifted Time of Combined Signal as Part of Fragments with Linear and Quadratic Frequency Modulation

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The issues of synthesis of nonlinear-frequency modulated probing signals, which, in comparison with well-known linear-frequency modulated signals, have a lower maximum level of side lobes of the autocorrelation function, have a practical orientation, relate to actual problems of theory and practice of forming signals with intra-pulse modulation for radio electronic means for various purposes. The paper considers a combined nonlinear-frequency modulated signal consisting of linearly and quadratically-frequency modulated fragments. The peculiarity of the proposed approach to the description of its mathematical model is the introduction of frequency-phase compensation components, which reduces the maximum level of the side lobes of the autocorrelation function of the signal. Calculation of values of compensation components is based on consideration of the effect of derivatives of function of instantaneous phase of fragments up to the most significant order inclusive. The limitation of the method should include the requirement for the existence of their final quantity. In the first section of the article, an analysis of known studies and publications is carried out, which states that for the mathematical model of shifted time, considered in the work, the proposed method of compensating for frequency-phase distortions has not been previously considered. Therefore, in the second section of the work, the corresponding task of the study is formulated. In order to achieve the formulated task of research, the third section of the work develops a mathematical model of the shifted time of the combined signal, which contains the compensatory components of the specified distortions. The importance of their consideration in the resulting signal is theoretically substantiated and clearly demonstrated. As a result of the research, the theory of synthesis of combined signals has been developed, the composition and determination of the magnitude of frequency-phase distortions, which are caused by the appearance of the third derivative of the instantaneous phase function of the quadratically-frequency modulated fragment, have been established. As a direction for further research, it is planned to develop and study a mathematical model of shifted time of a three-fragment combined signal with a quadratically-frequency modulated fragment.

*Keywords:* nonlinear frequency modulation; quadratic frequency modulation; autocorrelation function; maximum level of side lobes

DOI: 10.20535/RADAP.2024.97.5-11

## Statement of the research task

Issues of further improvement of the theory and practice of synthesis of signals with intra-pulse modulation (IPM) for radio-electronic means (REM) for various purposes are devoted to a large number of fundamental works, for example, [1–4], but some issues remain unresolved.

One of such issues should include the formation of non-linear frequency modulated (NLFM) probing signals, which, in comparison with well-known linear frequency modulated (LFM, FM) signals, have a lower maximum level of side lobes (MPSLL, SLL) autocorrelation functions (ACF).

Known sources provide a large number of variants of multi-fragment signals and sets of their parameters, but the issues of frequency-phase distortion at the joints of their fragments were not deeply considered. These distortions make it difficult to select time-frequency parameters that minimize MPSLL. In works [5–7], for some variants of NLFM signals, methods are proposed that provide compensation for frequency and phase jumps at the junction of fragments. For different combinations of fragments and methods of their mathematical representation (in the current or shifted time), the compensation components have a different analytical expression.

It is advisable to expand the nomenclature of signals by considering other combinations of laws of IPM fragments, in this case the use of the term “combined signals” (CS) is justified. It is expected that by compensating for frequency-phase distortions occurring in signals of this type, it is possible to achieve a wider range of input time-frequency parameter values and a lower MPSLL.

# 1 Analysis of studies and publications

Determining factors for realizing the possibility of reducing MPSLL due to the use of NLFM signals are the choice of the type and structure of such a signal. In view of this, the vast majority of publications devoted to such issues [8–24].

During the review of well-known scientific sources, the main attention was focused on the selection of time-frequency parameters of the CS, which ensure the minimization of MPSLL when applying a particular mathematical model (MM) of the signal. For example, researchers [8, 9] have developed an optimization algorithm for this purpose, the essence of which is to select the duration and deviation of the frequency of CS fragments. In [10], a similar approach to parameter optimization is used for the two-segment LFM-LFM signal. In the works of other authors [11, 12], in order to avoid a complete search of the input parameters of MM during optimization, it is proposed to use genetic algorithms. Using similar approaches, a large number of studies have been performed [13–24], but the essence of the proposed measures is reduced to the selection of time-frequency parameters of CS fragments so that the resulting phase structure provides ACF with a minimum MPSLL value.

The authors of publications [5–7] applied a different approach, proved that at the junction of fragments of the CS there are frequency-phase distortions that can be calculated in advance and compensated in MM, which in the vast majority of cases provides a decrease in MPSLL. It should be noted that the analytical expressions of the compensating components for different IPM laws and their combinations, as well as their dependence on the time representation of MM, are significantly different, which requires the development of a new MM for each type of CS. In this paper, it is proposed to develop MM of two-fragment CS, which has fragments with different types of IPM.

# 2 Formulation of the study task

The purpose of the work is to develop a two-fragment CS shifted time MM, the first fragment of which has LFM, and the second – quadratic FM (QFM), with frequency-phase distortion compensation, which should reduce the MPSLL ACF of the resulting signal.

# 3 Presentation of the study material

## 3.1 Development of MM shifted time NLFM signal with LFM and QFM fragments

Development of MM of shifted time is performed, considering that the CS consists of two fragments, the first of which is LFM and has the law of frequency change $f(t) = F(t)$ and the second – QFM $f(t) = F(t^2)$.

As an initial model, we use MM with a time shift of a two-fragment NLFM signal, which includes LFM fragments with an increasing FM law [24]:

$$\varphi(t) = \begin{cases} 2\pi\left[f_0 t + \dfrac{\beta_1 t^2}{2}\right], \quad 0 \le t \le T_1; \\ 2\pi\left[(f_0+\beta_1 T_1)(t-T_1)+\beta_2\left(\dfrac{t^2}{2}-T_1 t\right)\right], \\ \quad T_1 \le t \le T_{12}; \end{cases} \tag{1}$$

where $f_0$ is an initial signal frequency NLFM;
$T_{12} = T_1 + T_2$ is a total duration of the first and second fragments;
$\Delta f_1, \Delta f_2$ is a frequency deviation of the first and second fragments;
$\beta_1 = \frac{\Delta f_1}{T_1}$ is a FM rate (FMR) of the first LFM fragment;
$\beta_2 = \frac{\Delta f_2}{T_2}$ is a FMR of the second LFM fragment.

During the development of the new MM, the main attention is paid to the second CS fragment, the instantaneous phase of which has three derivatives, the distortion of the phase structure of the resulting signal is determined relative to the higher derivative – third derivative. In [6] it is defined as an FM acceleration, and has a constant value, that is, it does not depend on time:

$$\alpha_2 = \frac{d\varphi_2{}'''\left(\frac{t^3}{6} - \frac{t^2}{2}T_1\right)}{dt^3} = Const.$$

For further consideration, the expression for $\alpha_2$ must be written in an analytical form with respect to the parameters of the second QFM fragment of the signal. To do this, we use the definition of frequency deviation for this fragment:

$$\Delta f_2 = \alpha_2 \iint\limits_{T_2} dt,$$

where:

$$\alpha_2 = \frac{2\Delta f_2}{T_2^2}. \tag{2}$$

The expression for the instantaneous frequency of the QFM fragment is obtained by double integration with respect to FM accelerating $\alpha_2$:

$$f_2\left(\frac{t^2}{2}-T_1t\right)=f_0+\beta_1T_1+\iint\limits_{t-T_1}\alpha dt= \\ =f_0+\beta_1T_1+\alpha_2\left(\frac{t^2}{2}-T_1t\right)+C_1, \quad (3)$$

where $C_1$ is integration constant, which is determined based on the initial condition $t=T_1$:

$$C_1=\frac{1}{2}\alpha_2T_1^2. \quad (4)$$

Accordingly, the expression for the instantaneous phase is obtained by integrating (3):

$$\varphi_2(t-T_1)=2\pi\left[(f_0+\beta_1T_1)(t-T_1)+ +\alpha_2\left(\frac{t^3}{6}-T_1\frac{t^2}{2}\right)+\frac{\alpha_2T_1^2}{2}(t-T_1)+C_2\right]. \quad (5)$$

Let's determine the integration constant of $C_2$ by analogy with (4):

$$C_2=\alpha_2\frac{T_1^3}{6}. \quad (6)$$

It is necessary to focus on the fact that the finding of $C_1, C_2$ is of particular importance, since they make a decisive contribution to the frequency-phase distortion and allow taking into account the values of instantaneous frequencies and phases at the time of the actual beginning of the second fragment, that is, when $t=T_1$.

Taking into account (1) and as a result of substitution (4) in (3), we obtain an instant frequency MM with a compensation component for the resulting CS:

$$f(t)=\begin{cases} f_0+\beta_1t, \quad 0\le t\le T_1; \\ f_0+\beta_1T_1+\alpha_2\left[\left(\frac{t^2}{2}-T_1t\right)+\frac{T_1^2}{2}\right], \\ \quad T_1\le t\le T_{12}, \end{cases} \quad (7)$$

and on the basis of (1), (5) and (6) we write the MM of the instantaneous phase of this signal:

$$\varphi(t)=\begin{cases} 2\pi\left[f_0t+\frac{\beta_1t^2}{2}\right], \quad 0\le t\le T_1; \\ 2\pi\left[\left(f_0+\beta_1T_1+\frac{\alpha_2T_1^2}{2}\right)(t-T_1)+\right. \\ \left.+\alpha_2\left\{\left(\frac{t^3}{6}-T_1\frac{t^2}{2}\right)+\frac{T_1^3}{6}\right\}\right], \\ \quad T_1\le t\le T_{12}. \end{cases} \quad (8)$$

In (7) and (8), FM acceleration field is calculated according to (2).

For the descending FM law, the signs of the components in (7) and (8), in addition to those containing $f_0$, are reversed.

Analysis of (7) and (8) shows that the third derivative of the instantaneous phase of the QFM fragment causes the appearance of additional components, the compensation of which must be carried out in MM. Despite the fact that the compensation of the frequency jump at the junction of fragments occurs automatically due to the mathematical reception of the shift of the initial time of the second fragment by the zero mark, the frequency-phase jumps at the junction of fragments caused by the third derivative of the instantaneous phase do not disappear. Let's analyse the changes in MM (7), (8) caused by the presence of this derivative.

When moving from LFM to QFM fragment FM acceleration takes a non-zero value, which causes a jump in the instantaneous frequency of the signal, which is equal to $C_1$. This frequency jump causes the instantaneous phase to jump by the magnitude of the $C_2$ and causes its additional linear increment to be taken into account:

$$C_2=\alpha_2\frac{T_1^2}{2}(t-T_1). \quad (9)$$

Ignoring component (9) leads to a significant frequency deviation in QFM fragment, which is not taken into account in known MMs.

In the next section, we will check the obtained theoretical results using mathematical modeling.

### 3.2 Results of mathematical modeling

Mathematical modeling of the proposed signal was carried out using the Matlab application package.

The feasibility of using CS of the type LFM-QFM (7), (8) is determined by comparing it with the classic LFM and two-fragment LFM-LFM signal similar in frequency-time parameters. These signals have equivalent time-frequency parameters. To ensure the visualization of the simulation, we assume $f_0=0$, the remaining parameters are determined to ensure the possibility of observing the detailed structure of the synthesized signals, namely, the form of oscillograms, ripples of the apex and side slopes of the spectra, changes in the amplitude and frequency of beating of the side lobes of the ACF. Parameters of investigated signals and obtained results are given in Table 1.

Tabl. 1 Parameters of investigated signals and obtained results

| Kind pf sign. | $T_1$, $\mu$s | $T_2$, $\mu$s | $\Delta f_1$, kHz | $\Delta f_2$, kHz | MPSL, dB | Rapid decline SLL, dB/dec | Width MLACF (-3 dB), $\mu$s |
|---|---|---|---|---|---|---|---|
| LFM | 120 | | 450 | | -13,5 | 19,5 | 1,97 |
| LFM-LFM | 20 | 100 | 150 | 300 | -17,69 | 21,7 | 2,23 |
| LFM-QFM | 60 | 60 | 100 | 350 | -19,12 | 25,0 | 2,41 |

Analysis of Table 1 shows that for CS of the LFM-QFM type in relation to the classic LFM signal, the MPSLL decrease is 5.62 dB (42%), the SLL decrease rate is 5.5 dB/dec (28%), while the ML ACF expansion is 22%.

For predetermined time-frequency parameters, the LFM-QFM signal relative to the LFM-LFM signal provides an 8% decrease in MPSLL and a 15% increase in the SLL decay rate. ML width at -3 dB increased by approximately 8%, which is the best result among the presented indicators.

To confirm the necessity and importance of taking into account the compensation components in the MM CS shifted time, we compare the simulation results when using MM with and without compensation for frequency-phase distortion in (7), (8). The influence of the compensating component can be demonstrated by putting $C_1 = 0$ in (7), (8), that is, excluding it from the calculation, which causes the manifestation of the effect of FM acceleration $\alpha_2$ on the structure of the resulting signal.

Fig. 1 and Fig. 2 show the results of modeling LFM-QFM signals with parameters $T_1 = 60\mu$s, $T_2 = 60\mu$s, $\Delta f_1 = 100$ kHz, $\Delta f_2 = 500$ kHz. The absence of the $C_1$ component in (7) and (8) leads to a loss of signal physicality (Fig. 2), namely, it partially passes into the region of negative frequencies, the QFM component of the signal spectrum undergoes mirror distortion. The dependence of CS frequency on time, oscillogram, amplitude-frequency spectrum and ACF, shown in Fig. 2, indicate the complete unsuitability of such a signal for practical use in real REM.

In order to check operability of the developed MM (8) and determine possible values of input parameters ensuring its stable operation, a group of six LFM-QFM signals was studied. Initial parameters and results are shown in Table 2.

During the simulation, it was found that the stable operation of MM (8) is ensured by the ratio of the duration of CS fragments as 1:1. A significant decrease in MPSLL occurs due to the ratio of frequency deviations of LFM and QFM fragments 1:3 or more. With an increase in the proportion of QFM fragment frequency deviation in the total CS frequency deviation, along with a decrease in MPSLL, an increase in the SLL ACF decay rate is observed. As the fragment duration increases, that is, as the FM acceleration decreases, the MPSLL increases and the SLL ACF decrease rate decreases.

Tabl. 2 Results of experimental studies of two-fragment LFM-QFM signals

| No. | $T_1$, $\mu$s | $T_2$, $\mu$s | $\Delta f_1$, kHz | $\Delta f_2$, kHz | MPSLL, dB | Rapid decline SLL, dB/dec |
|---|---|---|---|---|---|---|
| 1. | 60 | 60 | 200 | 200 | -14,01 | 24,0 |
| 2. | 60 | 60 | 100 | 300 | -18,28 | 25,5 |
| 3. | 60 | 60 | 100 | 600 | -20,46 | 31,0 |
| 4. | 60 | 60 | 100 | 900 | -21,59 | 33,0 |
| 5. | 80 | 80 | 100 | 600 | -20,0 | 25,0 |
| 6. | 80 | 80 | 100 | 900 | -20,6 | 30,0 |

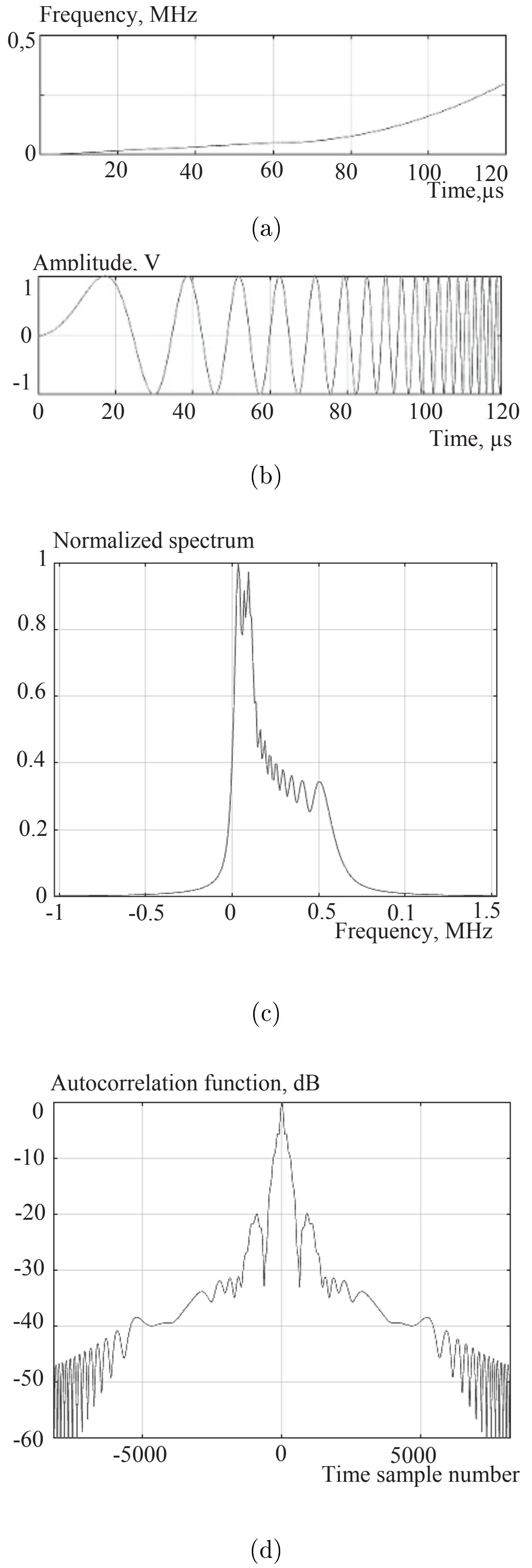


Fig. 1. Frequency variation plot (a), oscillogram (b), spectrum (c), ACF (d) LFM-QFM signal with compensating components

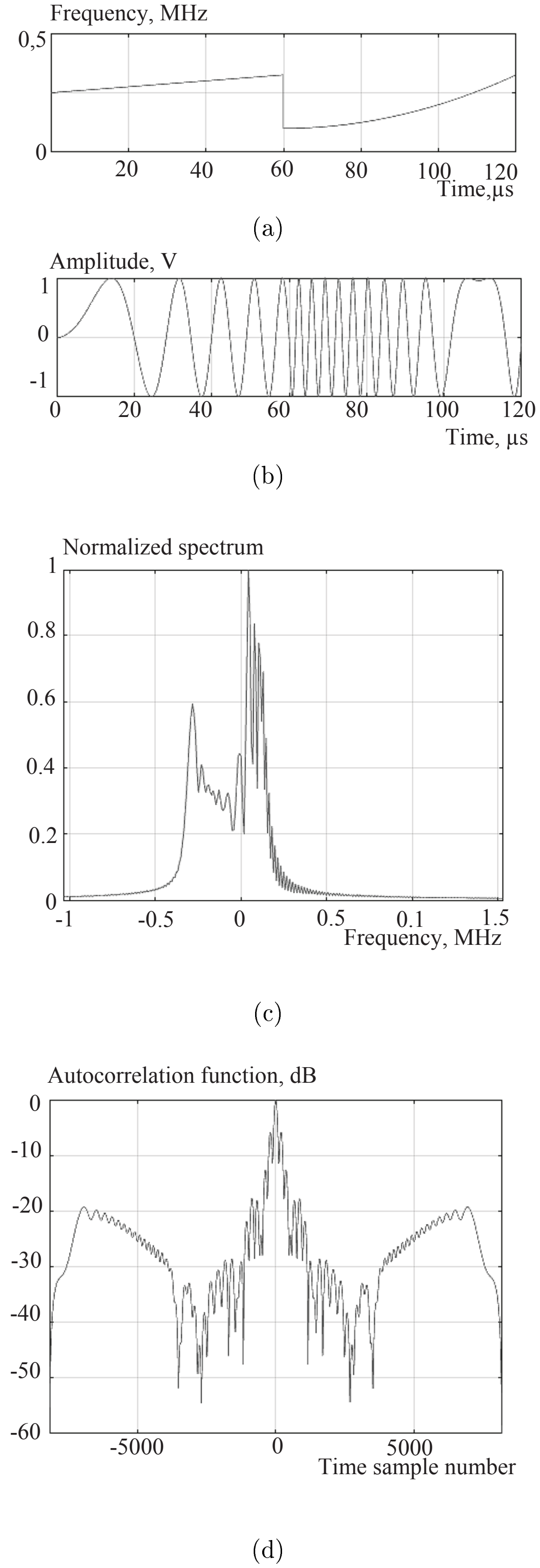


Fig. 2. Plot of frequency change (a), oscillogram (b), spectrum (c), ACF (d) LFM-QFM signal without compensating component $C_1$

# Conclusions

The result of the work is the development of a new MM of shifted time for CS as part of the first LFM and the second QFM fragments. Unlike those known in the proposed MM, integration constants are taken into account when calculating the instantaneous frequency and phase of the signal, which makes it possible to determine the components for compensating for the resulting distortions. As a result of the development of this MM, the theory of synthesis of these CS is developed, which consists in establishing the composition and determining the magnitude of frequency-phase distortions that are caused by the appearance of the third derivative of the QFM fragment, for the case of using MM shifted time.

During the development of this MM, a method based on the use of an older derivative of the instantaneous phase of the QFM fragment was used. A limitation of the proposed method is the requirement of existence of a finite number of derivatives of the function of instantaneous phase of fragments.

If this condition is not met, then the method [5–7] should be used to calculate the value of frequency-phase jumps, which is based on finding the difference between the final values of the previous frequency and phase and their initial values for the next CS fragment.

The results of the comparative analysis indicate that for the parameters of the proposed signal of the LFM-QFM type, the lowest MPSLL value is -21.59 dB, while the SLL decay rate is 33.0 dB/dec, which is also the best value among those considered.

**The practical significance** of these results is to expand the range of NLFM signals used, which contributes to the improvement of electromagnetic compatibility and noise immunity of REM for various purposes. It should be noted that the proposed approach to compensating for the time-frequency distortions of the CS is potentially capable of expanding the range of changes in the values of the input parameters of the MM compared to the known ones.

**Prospects for further research**. In the future, it is planned to develop and study MM shifted time of three-fragment CS, which will include a fragment from QFM.

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## Математична модель зсунутого часу комбінованого сигналу у складі фрагментів з лінійною та квадратичною частотною модуляцією

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Питання синтезу нелінійно-частотно модульованих зондувальних сигналів, які у порівнянні з широковідомими лінійно-частотно модульованими сигналами мають нижчий максимальний рівень бічних пелюсток автокореляційної функції, мають практичну спрямованість, відносяться до актуальних проблем теорії та практики формування сигналів з внутрішньо-імпульсною модуляцією для радіоелектронних засобів різного призначення.

У роботі розглядається комбінований нелінійно-частотно модульований сигнал, що складається з лінійно- та квадратично-частотно модульованих фрагментів. Особливістю запропонованого підходу до опису його математичної моделі є введення частотно-фазових компенсаційних складових, що забезпечує зниження максимального рівня бічних пелюсток автокореляційної функції сигналу. Обчислення значень компенсаційних складових засновано на врахуванні впливу похідних функції миттєвої фази фрагментів до найвищого порядку включно. До обмеження способу слід віднести вимогу існування їх кінцевої кількості.

У першому розділі статті проведено аналіз відомих досліджень та публікацій, який свідчить, що для математичної моделі зсунутого часу, яка розглядається у роботі, запропонований спосіб компенсації частотно-фазових спотворень раніше не розглядався. Тому у другому розділі роботи сформульовано відповідне завдання дослідження. Задля досягнення сформульованого завдання дослідження у третьому розділі роботи розроблено математичну модель зсунутого часу комбінованого сигналу, яка містить компенсаційні складові вказаних спотворень. Теоретично обґрунтовано та наочно продемонстровано важливість їх врахування у результуючому сигналі.

У результаті проведених досліджень розвинуто теорію синтезу комбінованих сигналів, встановлено склад та визначено величини частотно-фазових спотворень, які спричинені появою третьої похідної функції миттєвої фази квадратично-частотно модульованого фрагменту.

У якості напряму подальших досліджень планується розробка та дослідження математичної моделі зсунутого часу трифрагментного комбінованого сигналу з квадратично-частотно модульованим фрагментом.

*Ключові слова:* нелінійна частотна модуляція; квадратична частотна модуляція; автокореляційна функція; максимальний рівень бічних пелюсток