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Scattering of Electromagnetic Waves by Loss and Gain Systems of Dielectric Resonators

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The problem of wave scattering on a system of coupled Dielectric Resonators (DR) made of an active or absorbing dielectric is considered. The solution of the scattering problem is decomposed over the field of natural oscillations of the DR system. The field describing the natural oscillations of the DR system is decomposed by the field of partial resonators, which are made of a dielectric with a complex permittivity. A system of equations is given, the solution of which allows to determine the frequencies and amplitudes of the natural oscillations of the system of active or absorbing resonators. In the work, a new system of linear equations for amplitudes of forced oscillations of resonators was obtained. General solutions for the scattering field on resonators located in a regular transmission line or in a break of a regular line have been found. Several examples of calculation of the frequency dependences of the scattering matrix for different bands to pand band-pass filters, consisting of coupled active or absorbing dielectric resonators are given. The possibilities of the proposed method are demonstrated on the example of optimization of scattering characteristics on band-stop and band-pass filters made of an active dielectric. It is shown that the use of resonators made of an active dielectric will make it possible to build and optimize the frequency characteristics of a new class of devices that simultaneously perform the functions of filters and amplifiers. The conditions under which it is possible to build filters with the functions of amplifiers are defined. In the future, the proposed devices may find application in optical communication systems.

Keywords: scattering; dielectric resonator; scattering matrix; active dielectric; band-pass filter; band-stop filter; add-drop filter; Double-channel SCISSOR

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Introduction

Dielectric resonators (DRs) made from an active dielectric becomes new promising elements of integrated circuits in the optical and infrared ranges [2, 8, 12, 16–19]. The electromagnetic properties of resonators made of an active dielectric attract the attention of researchers [1–21], but have not been fully studied at present. An important component of our knowledge of DRs is the analytical theory of scattering, built on the basis of well-established physical assumptions [23, 24], using integrals of bilinear relations between electromagnetic field vectors, directly following from Maxwell's equations [23]. The construction of an analytical theory of scattering makes it possible to effectively design a wide class of various elements of communication systems, optical processors, different quantum devices, etc.

1 Statement of the problem

The purpose of this article is to construct new analytical electromagnetic theory of scattering on systems of coupled dielectric resonators, consisting of an active dielectric. The proposed solution is based on the application of perturbation theory for describing the coupled oscillations of active DRs and subsequent use of this eigenfunctions to solving the scattering problem.

2 Coupled oscillations of DRs with loss and gain media

For theory constructing, we use electromagnetic field scattering expansions in terms of coupled oscillations of the DR system [24], so we will first consider the natural oscillations of a system consisting of different DRs (Fig. 1) with loss and gain. Let us assume that we know electric and magnetic field of each of isolated resonator $(\mathbf{e}_s, \mathbf{h}_s)$ $(s = 1, 2, \ldots, N)$, as well as complex frequencies $\tilde{\omega}_s = \omega_0 + i\omega_s''$ of its natural oscillations, where ω_0 is the real part and ω_s'' is the imaginary part of complex frequency $\tilde{\omega}_s$. We also assume that each of the resonators is made of a loss or gain dielectric: $\tilde{\varepsilon}_s = \varepsilon_s' - i\varepsilon_s''$ $(s = 1, 2, \ldots, N)$. Here ε_s' is the real part; ε_s'' is the imaginary part of complex dielectric constant $\tilde{\varepsilon}_s$ of the s-th DR (Fig. 1). Below (Fig. 1-6) we will denote in yellow resonators consisting of an active dielectric ($\varepsilon_s'' < 0$) with gain, and in gray resonators made from a dielectric with losses ($\varepsilon_s'' > 0$).

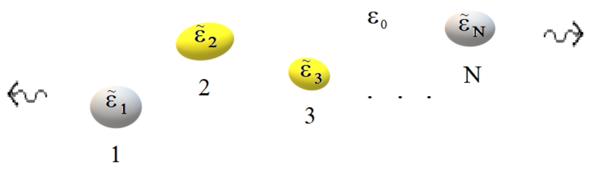

Fig. 1. Loss and gain coupling DR system

We will seek a solution to the problem of natural oscillations of the field of a system of N coupled DRs $(\mathbf{e}, \mathbf{h})$ in the form of an expansion in terms of natural oscillations of isolated partial resonators $(\mathbf{e}_s, \mathbf{h}_s)$ located in a given structure:

$$\mathbf{e} = \sum_{s=1}^{N} b_s \mathbf{e}_s; \quad \mathbf{h} = \sum_{s=1}^{N} b_s \mathbf{h}_s. \tag{1}$$

Here b_s there are unknown expansion coefficients.

Using Maxwell's equations for the field of coupled oscillations (1) and the field of partial resonators, after integration over the volume V_t of the t-th partial resonator, taking into account:

$$\int_{V_t} \mathbf{e}_s \cdot \mathbf{e}_t^* dv \approx \delta_{st} \int_{V_t} |\mathbf{e}_t|^2 dv;$$

$$\int_{V_t} \mathbf{h}_s \cdot \mathbf{h}_t^* dv \approx \delta_{st} \int_{V_t} |\mathbf{h}_t|^2 dv,$$

obtain:

$$\sum_{s \neq t}^{N} \kappa_{st} b_s + [i(\tilde{k}_t + \tilde{k}_t^{\varepsilon}) - \lambda] b_t = 0; \quad (t = 1, 2, \ldots, N), \tag{2}$$

where $\lambda = 2(\tilde{\omega} - \omega_0)/\omega_0$; $\tilde{\omega}$ is an unknown frequency of coupled oscillations. Parameter

$$\tilde{k}_t^{\varepsilon} = P_t^{\varepsilon} / \omega_0 w_t, \tag{3}$$

where $w_t = 1/4 \int_{V_t} (\varepsilon_t' |\mathbf{e}_t|^2 + \mu_0 |\mathbf{h}_t|^2) dv$ is the energy stored in the resonator material, $P_t^{\varepsilon} = \frac{\omega_0}{2} \varepsilon_t'' \int_{V_t} |\mathbf{e}_t|^2 dv$ determines the average power loss ($\varepsilon_t'' > 0$) (or released power (if $\varepsilon_t'' < 0$)) of the resonator; $\tilde{k}_t$ – coupling coefficient of the t-th DR with open space [24]; $\kappa_{st} = k_{st} + i\tilde{k}_{st}$ – coupling coefficient between s–th and t–th resonator in the transmission line [24]. It is easy to verify that for high-Q oscillations $\tilde{k}_n^{\varepsilon} \approx \varepsilon_n''/\varepsilon_n'$.

System of equations (2) allows us to calculate the frequencies and fields of coupled loss and gain DRs.

Functions λ are eigenvalues of new coupling operator: $K = \|\kappa_{st}\|$, where

$$\kappa_{st} = \begin{cases} (k_{st} + i\tilde{k}_{st}), & \text{if } \ s \neq t \\ i(\tilde{k}_t + \tilde{k}_t^{\varepsilon}), & \text{for } \ s = t \end{cases}; \qquad \lambda = 2 \cdot \left(\frac{\tilde{\omega} - \omega_0}{\omega_0} \right). \tag{4}$$

Here also ω_0 is a real part of the frequency of resonators without other DRs.

When $\varepsilon_n'' = 0$ system of equations (2) coincides with that found earlier [24]. As follows from (2), taking into account the complex parameters of dielectrics $\tilde{\varepsilon}_s = \varepsilon_s' - i\varepsilon_s''$, the values of the coupling coefficients of the resonators take on a more complex form. In this case, an increase in dissipative losses, in the perturbation theory approximation, is equivalent to an increase in radiation (3), (4), and the presence of an active dielectric, releasing energy into the system from an external source, is equivalent to a decrease in radiation from the resonator.

Provided: $\kappa_{st} = 0$, from (2) we obtain that for coupled oscillations ($b_t \neq 0$) the quality factor of the resonators is determined only by radiation losses $\tilde{k}_t$, and also, taking into account the sign of $\tilde{k}_t^{\varepsilon}$, by losses or gain in the dielectric:

$$Q_t = \mathrm{Re}(\tilde{\omega}_t)/2\,\mathrm{Im}(\tilde{\omega}_t) = 1/(\tilde{k}_t + \tilde{k}_t^{\varepsilon}).$$

In the general case, by equating the determinants of the system (2) to zero, we find the complex frequencies $\tilde{\omega}$ of coupled oscillations of the DR system. In this case, each t-th value $\lambda = \lambda^t$ define complex $\tilde{\omega}^t$ ($t = 1, 2, \ldots, N$) corresponding to a characteristic distribution of the amplitudes of coupled oscillations of the resonators, determined by the vector: $\|b_s^t\| = (b_1^t \;\; b_2^t \;\; \ldots \;\; b_N^t)$. At that, the field of each of t-th coupled oscillations takes the simple form:

$$\mathbf{e}^t = \sum_{s=1}^{N} b_s^t \mathbf{e}_s; \quad \mathbf{h}^t = \sum_{s=1}^{N} b_s^t \mathbf{h}_s. \tag{5}$$

The dependences of the amplitudes and frequencies of coupled oscillations on $\tilde{k}_n^{\varepsilon} \approx \varepsilon_n''/\varepsilon_n'$ are shown in Fig. 2 for 2 DR located in a regular line; as well as in Fig. 3 for 2 DR placed in a transmission line break. The curve numbers indicate the number of the type of coupled oscillations of the resonator system (5). The data presented demonstrate the dependence of the relative amplitudes (b) and complex frequencies (c, d) of coupled oscillations on the imaginary part of the dielectric constant of the material. We also note the existence of a region of parameters $\tilde{k}_t^{\varepsilon}$ (Fig. 3, b), in which the oscillations of coupled resonators become quasi-separate: the amplitude of one of the resonators significantly exceeds the amplitude of the other.

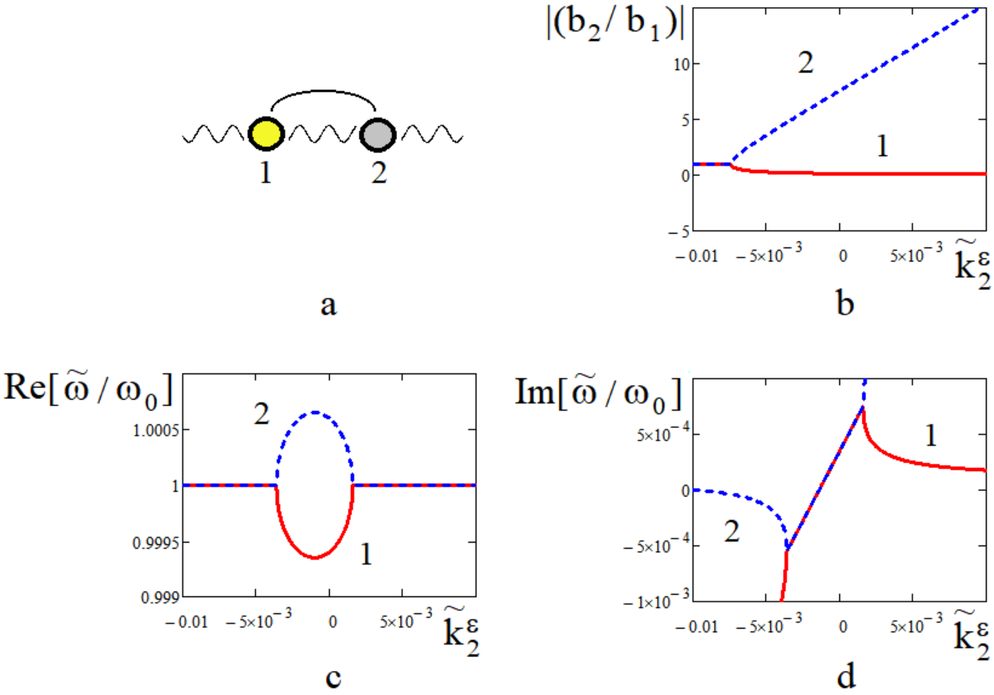

Fig. 2. Amplitude ratio of two DRs (b) in a regular transmission line (a); the real (c) and imaginary (d) parts of the relative frequency of coupled oscillations (denoted by 1, 2) depending on the parameter $\tilde{k}_n^{\varepsilon} \approx \varepsilon_n''/\varepsilon_n'$ ($\Gamma\Delta z_{12} = 31\pi/2$; $k_{12} = -0,0001$; $\tilde{k}_1 = \tilde{k}_2 = 0,0012$). Resonators are shown as circles; the field of propagating waves is shown by wavy curves, and the field of non-propagating (reactive) waves is shown by solid smooth lines (a)

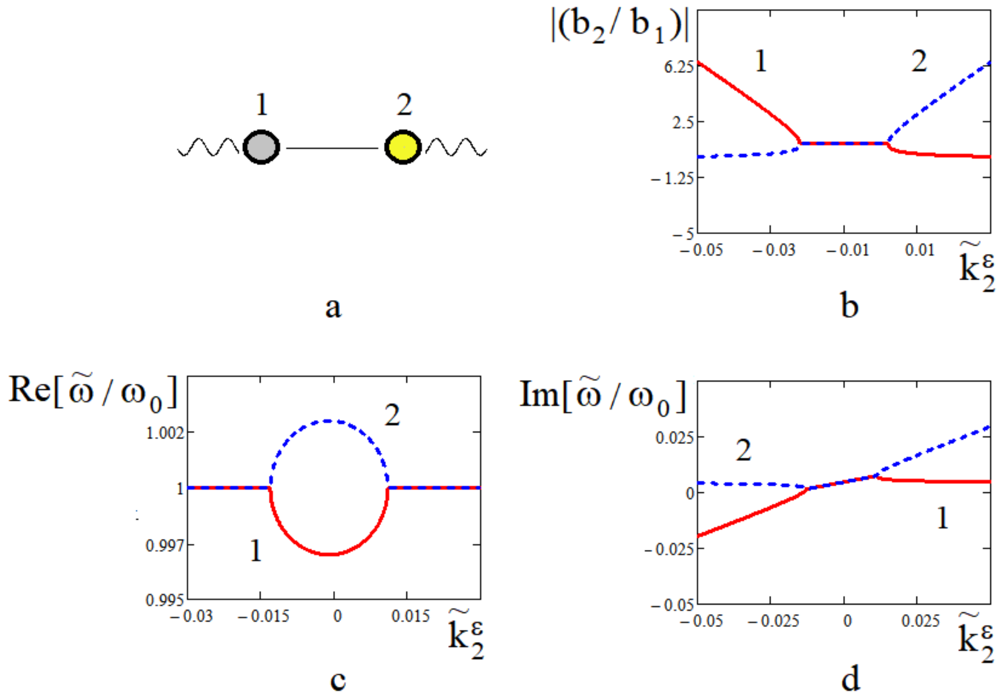

Fig. 3. Amplitude ratio of two DRs in a transmission line break (a); real (c) and imaginary (d) parts of the relative frequency of coupled oscillations (1, 2) depending on the parameter $\tilde{k}_n^{\varepsilon} \approx \varepsilon_n''/\varepsilon_n'$ ($k_{12} = 0,006$; $\tilde{k}_1 = \tilde{k}_2 = 0,01$)

3 Equation system for the amplitudes of forced oscillations of the DRs with loss and gain dielectric

Let us now construct a solution to the problem of the wave scattering on a system of coupled DRs with loss and gain dielectric (Fig. 1), based on the perturbation theory of Maxwell's equations, using expansions (5), and the conclusions of the previous section. Let assume that a wave $\left(\mathbf{E}_l^+, \mathbf{H}_l^+\right)$ falls on a system of resonators located in a transmission line. We will assume that each of the resonators is also made of a material with complex dielectric constant: $\tilde{\varepsilon}_s = \varepsilon_s{}' - i\varepsilon_s{}''$ $(s = 1, 2, \ldots, N)$, wherein $|\varepsilon_n{}''| << \varepsilon_n{}'$.

We present the solution to the scattering problem in the form [23]:

$$\begin{aligned} \mathbf{E}(\omega) &\approx \mathbf{E}_l^+ + \sum_{s=1}^{N} a^s(\omega)\mathbf{e}^s; \\ \mathbf{H}(\omega) &\approx \mathbf{H}_l^+ + \sum_{s=1}^{N} a^s(\omega)\mathbf{h}^s, \end{aligned} \qquad (6)$$

where $(\mathbf{e}^s, \mathbf{h}^s)$ – electromagnetic field of coupled oscillations of a resonator system (5); $a^s(\omega)$ – unknown expansion coefficients; ω – circular frequency.

Using expressions similar to (5), (6) written for the scattered field $(\mathbf{E}(\omega),\ \mathbf{H}(\omega))$, transmission line field $\left(\mathbf{E}_l^+,\ \mathbf{H}_l^+\right)$, and also the fields of partial resonators $(\mathbf{e}_s, \mathbf{h}_s)$ (5), we arrive at the equation system for amplitudes:

$$\sum_{s=1}^{N} \left(\frac{\omega - \tilde{\omega}^s}{\omega_0}\right) \cdot b_t^s a^s = i\frac{\omega}{\omega_0}\left(\frac{1 - i\frac{\varepsilon_t{}''}{\varepsilon_t{}' - \varepsilon_0}}{2 - i\frac{\varepsilon_t{}''}{\varepsilon_t{}'}}\right)\left(c_t^+\right)^*/\omega_0 w_t,$$

$$(t = 1, 2, \ldots, N),$$

where $c_t^\pm$ is the coefficient of expansion of the partial t-th DR field $(\mathbf{e}_t, \mathbf{h}_t)$ over the transmission line field $\left(\mathbf{E}_l^+,\ \mathbf{H}_l^+\right)$ [24]; $*$ - complex conjugate symbol.

For a not too wide frequency band $|\omega - \omega_0|/\omega_0 <<$ 1; $\omega \approx \omega_0$ and for $\varepsilon_t{}' >> \varepsilon_0$:

$$\sum_{s=1}^{N} \left(\frac{\omega - \tilde{\omega}^s}{\omega_0}\right) \cdot b_t^s a^s = i\left(\frac{1 - i\frac{\varepsilon_t{}''}{\varepsilon_t{}'}}{2 - i\frac{\varepsilon_t{}''}{\varepsilon_t{}'}}\right)\left(c_t^+\right)^*/\omega_0 w_t. \qquad (7)$$

Denoting by

$$\begin{aligned} D_s(\omega) &= \frac{\omega - \tilde{\omega}^s}{\omega_0}; \\ G_t &= -i\left(\frac{1 - i\frac{\varepsilon_t{}''}{\varepsilon_t{}'}}{2 - i\frac{\varepsilon_t{}''}{\varepsilon_t{}'}}\right), \end{aligned} \qquad (8)$$

rewrite (7)

$$\sum_{s=1}^{N} D_s(\omega) \cdot b_t^s a^s = -G_t \cdot \left(c_t^+\right)^*/\omega_0 w_t. \qquad (9)$$

From (6), (9) the transmission T and the reflection coefficient R of the DR system in the line can be obtained in the form:

$$\begin{aligned} T &= T_0 + \sum_{u=1}^{N}\left(\sum_{s=1}^{N} b_s^u c_s^+\right) a_u = T_0 - \frac{1}{A(\omega)}\sum_{s=1}^{N} A_s^+(\omega); \\ R &= R_0 + \sum_{u=1}^{N}\left(\sum_{s=1}^{N} b_s^u c_s^-\right) a_u = R_0 - \frac{1}{A(\omega)}\sum_{s=1}^{N} A_s^-(\omega). \end{aligned} \qquad (10)$$

Here T_0, R_0 are the transmission and reflection coefficients of the line without DRs;

$$\begin{aligned} A_s^\pm(\omega) &= \det\begin{bmatrix} b_1^1 D_1(\omega) & b_1^2 D_2(\omega) & \ldots & G_1 \sum_{u=1}^{N} b_u^s \tilde{k}_{u1}^{\pm+} & \ldots & b_1^N D_N(\omega) \\ b_2^1 D_1(\omega) & b_2^2 D_2(\omega) & \ldots & G_2 \sum_{u=1}^{N} b_u^s \tilde{k}_{u2}^{\pm+} & \ldots & b_2^N D_N(\omega) \\ \cdot & \cdot & \ldots & \cdot & \ldots & \cdot \\ b_N^1 D_1(\omega) & b_N^2 D_2(\omega) & \ldots & G_N \sum_{u=1}^{N} b_u^s \tilde{k}_{uN}^{\pm+} & \ldots & b_N^N D_N(\omega) \end{bmatrix}; \\ A(\omega) &= \det\begin{bmatrix} b_1^1 D_1(\omega) & b_1^2 D_2(\omega) & \ldots & b_1^N D_N(\omega) \\ b_2^1 D_1(\omega) & b_2^2 D_2(\omega) & \ldots & b_2^N D_N(\omega) \\ \cdot & \cdot & \ldots & \cdot \\ b_N^1 D_1(\omega) & b_N^2 D_2(\omega) & \ldots & b_N^N D_N(\omega) \end{bmatrix} \end{aligned} \qquad (11)$$

for band-stop filters;

$$A_s^+(\omega) = \det\begin{bmatrix} b_1^1 D_1(\omega) & b_1^2 D_2(\omega) & \dots & G_1 b_N^s \tilde{k}_{N1}^{++} & \dots & b_1^N D_N(\omega) \\ b_2^1 D_1(\omega) & b_2^2 D_2(\omega) & \dots & 0 & \dots & b_2^N D_N(\omega) \\ . & . & \dots & . & \dots & . \\ b_N^1 D_1(\omega) & b_N^2 D_2(\omega) & \dots & 0 & \dots & b_N^N D_N(\omega) \end{bmatrix};$$

$$A_s^-(\omega) = \det\begin{bmatrix} b_1^1 D_1(\omega) & b_1^2 D_2(\omega) & \dots & G_1 b_1^s \tilde{k}_{11}^{-+} & \dots & b_1^N D_N(\omega) \\ b_2^1 D_1(\omega) & b_2^2 D_2(\omega) & \dots & 0 & \dots & b_2^N D_N(\omega) \\ . & . & \dots & . & \dots & . \\ b_N^1 D_1(\omega) & b_N^2 D_2(\omega) & \dots & 0 & \dots & b_N^N D_N(\omega) \end{bmatrix} \qquad (12)$$

for band-pass filters.

From (8), (11)-(12) obtain:

$$T = T_0 + \sum_{u=1}^{N}\left(\sum_{s=1}^{N} b_s^u c_s^+\right) a_u = T_0 - \frac{\omega_0}{B}\sum_{s=1}^{N}\frac{B_s^+}{\omega - \tilde{\omega}^s};$$

$$R = R_0 + \sum_{u=1}^{N}\left(\sum_{s=1}^{N} b_s^u c_s^-\right) a_u = R_0 - \frac{\omega_0}{B}\sum_{s=1}^{N}\frac{B_s^-}{\omega - \tilde{\omega}^s}, \qquad (13)$$

$$B_s^{\pm} = \det\begin{bmatrix} b_1^1 & b_1^2 & \dots & G_1 \sum_{u=1}^{N} b_u^s \tilde{k}_{u1}^{\pm+} & \dots & b_1^N \\ b_2^1 & b_2^2 & \dots & G_2 \sum_{u=1}^{N} b_u^s \tilde{k}_{u2}^{\pm+} & \dots & b_2^N \\ . & . & \dots & & \dots & . \\ b_N^1 & b_N^2 & \dots & G_N \sum_{u=1}^{N} b_u^s \tilde{k}_{uN}^{\pm+} & \dots & b_N^N \end{bmatrix};$$

$$B = \det\begin{bmatrix} b_1^1 & b_1^2 & \dots & b_1^N \\ b_2^1 & b_2^2 & \dots & b_2^N \\ . & . & \dots & . \\ b_N^1 & b_N^2 & \dots & b_N^N \end{bmatrix} \qquad (14)$$

also for band-stop filters and

$$B_s^+ = \det\begin{bmatrix} b_1^1 & b_1^2 & \dots & G_1 b_N^s \tilde{k}_{N1}^{++} & \dots & b_1^N \\ b_2^1 & b_2^2 & \dots & 0 & \dots & b_2^N \\ . & . & \dots & . & \dots & . \\ b_N^1 & b_N^2 & \dots & 0 & \dots & b_N^N \end{bmatrix};$$

$$B_s^- = \det\begin{bmatrix} b_1^1 & b_1^2 & \dots & G_1 b_1^s \tilde{k}_{11}^{-+} & \dots & b_1^N \\ b_2^1 & b_2^2 & \dots & 0 & \dots & b_2^N \\ . & . & \dots & . & \dots & . \\ b_N^1 & b_N^2 & \dots & 0 & \dots & b_N^N \end{bmatrix} \qquad (15)$$

for the band-pass filters. In this case, frequencies $\tilde{\omega}^t$ as well as amplitudes $\left\|\mathrm{b}_s^t\right\|$ of coupled oscillations we found from (2). Here $\tilde{k}_{sn}^{++} = (c_s^+ c_n^{+*})/(\omega_0 w_n) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s - z_n)}$; $\tilde{k}_{sn}^{-+} = (c_s^- c_n^{+*})/(\omega_0 w_n) = (\tilde{k}_{sn})_0 e^{-i\Gamma(z_s + z_n)}$; Γ – transmission line wave number; z_s - longitudinal coordinate of the center of the s-th DR in the transmission line; $(\tilde{k}_{sn})_0$ coupling coefficient of the s-th and t-th resonators without taking into account the dependence of their longitudinal coordinates in the line.

As follows from the found expressions (13), the frequency dependences of the reflection and transmission coefficients, in approximation (6) are determined only by the complex frequencies of coupled oscillations of the DR system $\tilde{\omega}^s$ $(s = 1, 2, \dots, N)$.

4 Solution of the problem of scattering by active DR system in a transmission line

Let us first consider band-stop filters made on two loss and gain DRs (Fig. 4, a, d, g). We assume that the natural frequencies of the partial isolated resonators are $f_0 = 200$ THz. The coupling coefficients of the resonators with the transmission line are the same: $\tilde{k}_1 = \tilde{k}_2 = 0,01$. The distance between resonators $\Delta z_{12} = |z_1 - z_2|$ in a regular line: $\Gamma\Delta z_{12} = 31\pi/2$, where Γ is a wave number of line.

On Fig. 4 show the frequency characteristics of the scattering matrix of the band-stop filter made of two resonators with loss (b, c); band-stop filter made of two resonators with gain and loss (e, f) and filter made of two resonators with gain (h, i), obtained from (13), (14). Where $S_{21} = 20\lg|T|$; $S_{11} = 20\lg|R|$. As can be seen from the calculations, the frequency dependences of the S-matrix in the case of resonators with losses correspond to the characteristics of a conventional notch filter made of a “bad” dielectric (b, c). For two resonators with gain and loss (e, f), the release of energy by one resonator is partially compensated by absorption in the resonator with losses (e). If both resonators consist of an active dielectric, the transmitted and reflected signal is increased (h, i) and the filter, with appropriate settings, can be used as an amplifier.

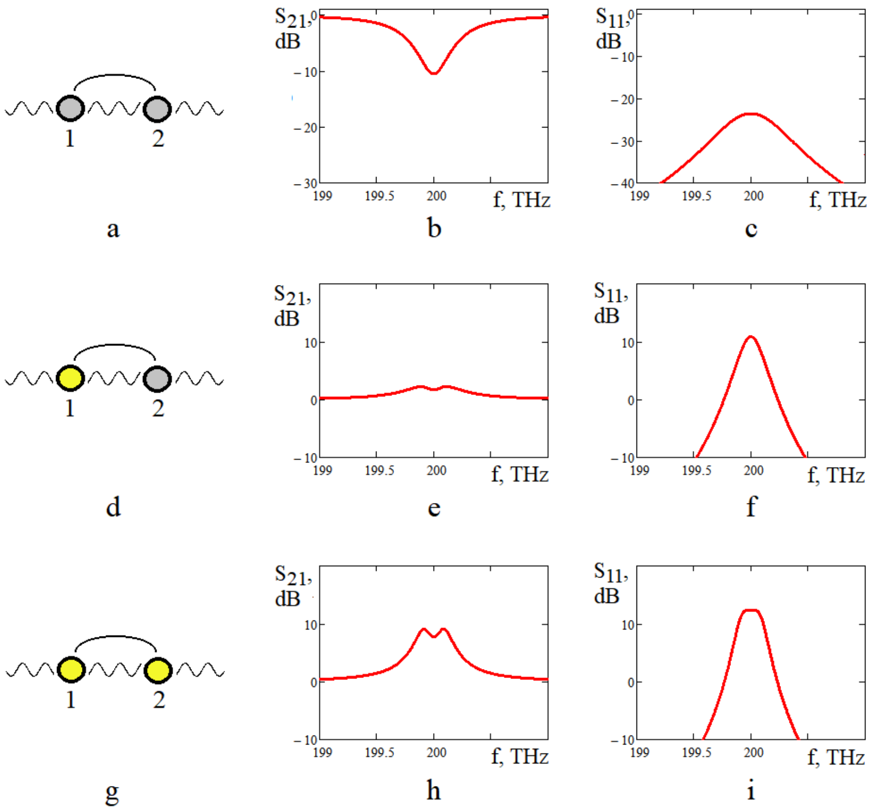

Fig. 4. Different loss and gain 2 DR system in a regular transmission line (a, d, g). Scattering characteristics of 2DRs in the transmission line ($f_0 = 200$ THz; distance between resonators $\Gamma\Delta z_{12} = 31\pi/2$; coupling coefficients of resonators with the line $\tilde{k}_1 = \tilde{k}_2 = 0,01$; mutual coupling coefficient $k_{12} = -0,0001$). $(\varepsilon''/\varepsilon')_1 = 0,03$; $(\varepsilon''/\varepsilon')_2 = 0,01$ (b, c); $(\varepsilon''/\varepsilon')_1 = -0,03$; $(\varepsilon''/\varepsilon')_2 = 0,01$(e, f); $(\varepsilon''/\varepsilon')_1 = -0,03$; $(\varepsilon''/\varepsilon')_2 = -0,01$(h, i).

On Fig. 5 shows an example of calculating the scattering characteristics of the band-pass filters made on DRs placed in a break of the line (a, d, g). The obtained result shows a partially similar scattering pattern: in the case of resonators with losses in the dielectric, the frequency dependences of the S-matrix correspond to the characteristics of a band-pass filter (b, c); for active and absorbing resonators, a well known [4] gain singularity (e, f) is observed at the resonance frequency, and for active resonators, the structure also demonstrates the possibility of amplifying both reflected and transmitted signals (h, i).

The constructed model makes it possible to calculate and optimize the characteristics of scattering and more complex structures, contained big number of active resonators including degenerate types of natural oscillations.

In Fig. 6 shows the scattering characteristics of known structures of coupled micro cavities located between two regular transmission lines [22]. Here coupling coefficients of the resonators with open space: $\tilde{k}_{OS} = 10^{-7}$; coupling coefficients with the transmission line of even types of oscillations of the resonators are: $\tilde{k}^e = 2 \cdot 10^{-4}$; $\tilde{k}^o = 1,5 \cdot 10^{-4}$ for odd oscillations: for the add-drop filter (Fig. 6, a); $\tilde{k}^e = 7 \times 10^{-5}$; $\tilde{k}^o = 1 \cdot 10^{-4}$ for the Double-channel SCISSOR (side-coupled integrated spaced sequence of optical resonators) (Fig. 6, f); for Twisted Double-channel SCISSOR $\tilde{k}^e = 3 \cdot 10^{-4}$; $\tilde{k}^o = 3 \cdot 10^{-4}$ (Fig. 6, k). Mutual coupling coefficients of neighboring microcavities of even types of oscillations of the add-drop filter $\tilde{k}^e_{s,s+1} = 2 \cdot 10^{-4}$; Double-channel SCISSOR $\tilde{k}^e_{s,s+1} = 7 \cdot 10^{-6}$; Twisted Double-channel SCISSOR $\tilde{k}^e_{s,s+1} = 1 \times 10^{-5}$; as well as odd oscillations for the add-drop filter $\tilde{k}^o_{s,s+1} = -2 \cdot 10^{-4}$; Double-channel SCISSOR $\tilde{k}^o_{s,s+1} = -4 \cdot 10^{-6}$; Twisted Double-channel SCISSOR $\tilde{k}^o_{s,s+1} = -1 \cdot 10^{-5}$. Relative values of dielectric constants of the resonator material are $(\varepsilon''/\varepsilon')_s = -10^{-4}$ for the add-drop filter; $(\varepsilon''/\varepsilon')_s = -1,4 \cdot 10^{-4}$ for the Double-channel SCISSOR; $(\varepsilon''/\varepsilon')_s = -8 \cdot 10^{-5}$ for the Twisted Double-channel SCISSOR ($s = 1, 2, 3$). The best result obtained for the Double-channel SCISSOR (Fig. 6, i).

The obtained calculation data indicate the possibility of constructing and optimizing amplifiers based on the use of DRs with an active dielectric on well known filter structures.

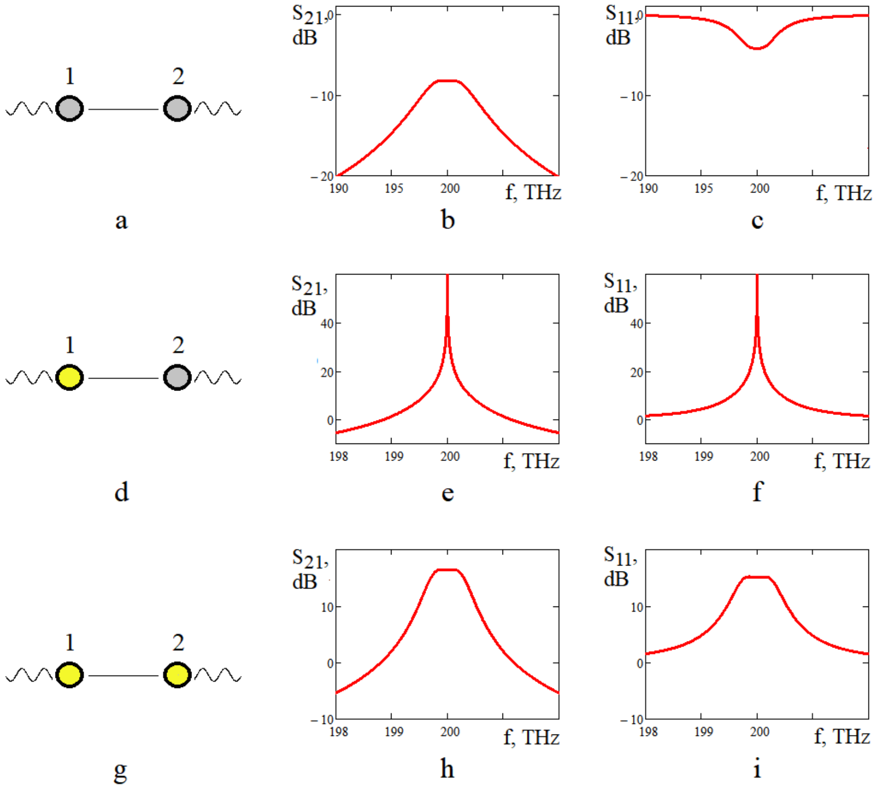

Fig. 5. Two loss and gain DR system in a break section of a transmission line (a, d, g). Scattering characteristics of 2 DRs ($f_0 = 200$ THz; coupling coefficients of resonators with the line $\tilde{k}_1 = \tilde{k}_2 = 0,01$; mutual coupling coefficient $k_{12} = 0,006$). $(\varepsilon''/\varepsilon')_1 = 0,013$; $(\varepsilon''/\varepsilon')_2 = 0,02$ (b, c); $(\varepsilon''/\varepsilon')_1 = -0,013$; $(\varepsilon''/\varepsilon')_2 = 0,02$ (e, f); $(\varepsilon''/\varepsilon')_1 = -0,013$; $(\varepsilon''/\varepsilon')_2 = -0,02$ (h, i)

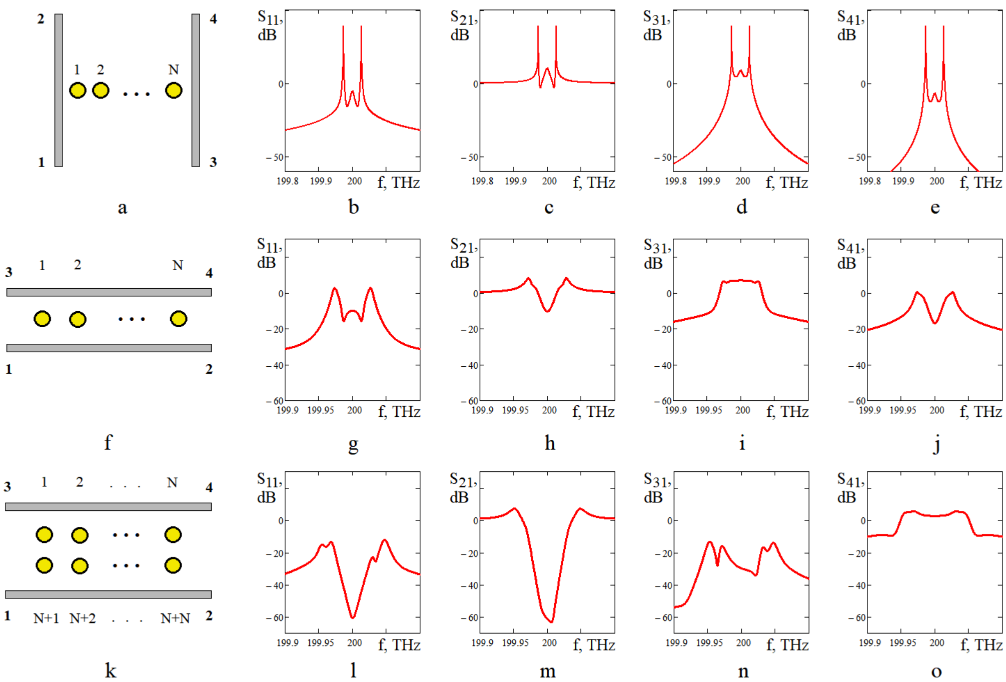

Fig. 6. Frequency dependences of the scattering matrix by 3 active cavity add-drop filters (a – e); Double-channel SCISSOR (f – j); Twisted Double-channel SCISSOR (k – o), calculated using formulas (13), (14)

Discussion and Conclusion

A general theory of scattering of electromagnetic waves by systems of resonators with loss and gain dielectric is developed. It is shown that in this case the stray field also can be expanded into a system of functions determined by the natural oscillations of the coupled resonator system. Using perturbation theory, system equations determining the frequencies and amplitudes of coupled oscillations of resonators in a transmission line are obtained. A system of equations that determin the amplitudes of forced oscillations, when line waves are incident on resonators, is found.

The resulting system of equations for the parameters of coupled oscillations differs from that found earlier [24] in that it takes into account the influence of the imaginary part of the complex dielectric constant on the frequencies of the resonators. As calculation data have shown, such an influence is significant for coupled oscillations of resonators made of an active dielectric. We also took into account the complex values of the dielectric constant when deriving the system of equations for the amplitudes of the scattered field, but they were not explicitly included in the final relations. This result can be interpreted by the fact that the influence of field losses (gain) in the dielectric was taken into account at the first stage, when solving the problem of natural oscillations of the system. The calculations of scattering characteristics demonstrated the correctness of such approach.

Thus, the possibility of constructing filters with amplification functions performed on resonators made of an active dielectric is shown. The developed theory makes it possible to simultaneously calculate and optimize the scattering characteristics of different bandpass and band-stop filters, built on dielectric resonators with loss and gain.

The results of this study can be used in the design and optimization of various frequency-selective elements of communication devices in the microwave, terahertz, infrared and optical ranges.

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Розсіювання електромагнітних хвиль на системах діелектричних резонаторів з активним та поглинаючим діелектриком

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Розглядається задача розсіювання хвиль на системі зв'язаних діелектричних резонаторів (ДР), виконаних із активного, або поглинаючого діелектрика. Розв'язання задачі розсіювання розкладається по полю власних коливань системи ДР. Поле, що описує власні коливання системи ДР, розкладається по полю парціальних резонаторів, які виконані із діелектрика з комплексною діелектричною проникністю. Виводиться модифікована система рівнянь, вирішення якої дозволяє визначити частоти та амплітуди власних коливань системи активних, або поглинаючих резонаторів. В роботі отримано нову систему лінійних рівнянь для амплітуд вимушених коливань резонаторів з активним діелектриком. Знайдено загальні рішення для поля розсіювання на резонаторах, що розташовуються в регулярній лінії передачі, або у розриві регулярної лінії. Наводиться декілька прикладів розрахунку частотних залежностей матриці розсіювання для двох ланцюгових режекторних та смугових фільтрів, що складаються зі зв'язаних активних або поглинаючих діелектричних резонаторів. Можливості запропонованого методу демонструються на прикладах оптимізації характеристик розсіювання на різноманітних смугових та режекторних фільтрах, які виконані із активного діелектрика. Показано, що використання резонаторів із активного діелектрика дозволить будувати та оптимізувати частотні характеристики нового класу пристроїв, які одночасно виконують функції фільтрів та підсилювачів. Визначені умови, при виконанні яких виникає можливість побудови фільтрів з функціями підсилювачів. У майбутньому запропоновані пристрої можуть знайти застосування у системах оптичного зв'язку.

Ключові слова: розсіювання; діелектричний резонатор; матриця розсіювання; активний діелектрик; смуговий фільтр; режекторний фільтр; add-drop filter; Double-channel SCISSOR