UDC 621.396.962

A Method for Calculating and Compensating Frequency-Phase Distortions at the Junction and in Signal Fragments With Nonlinear Frequency Modulation

Kostyria O. O.[1], *Hryzo A. A.*[1], *Fedorov A. V.*[1], *Lisohorskyi B. A.*[1], *Ushakov S. I.*[2]

[1]Ivan Kozhedub Kharkiv National Air Force University, Kharkiv, Ukraine
[2]National Space Facilities Control and Test Center, Kyiv, Ukraine

E-mail: *kostyria1958@gmail.com*

The use of nonlinear frequency modulated signals in radar technology is due to the possibility of reducing the maximum level of the side lobes of their autocorrelation functions compared to linear frequency modulated signals. One of the promising areas of development of the theory of synthesis of signals with nonlinear frequency modulation is the study of a thin signal structure that is distorted when switching to a new signal fragment. In particular, it was found that jumps in instantaneous frequency and phase at the boundary between fragments cause additional distortions in the frequency-phase structure in subsequent fragments. These phenomena had previously been ignored by researchers. Previous work on the development and study of mathematical models of nonlinear frequency-modulated signals has revealed regularities that describe the change in the frequency-phase structure of the next fragment when the value or order of the oldest derivative of the instantaneous phase function changes. It is found that the number of components in the distortion spectrum is determined by the order of this derivative: for phase distortions – according to its value, for frequency distortions – by one less. Constant components have a physical interpretation and correspond to jumps in instantaneous frequency or phase at the boundary of fragments. The structure of the paper is determined by the research logic. The first section of the paper analyzes the available publications and shows that there is no research in this area. This substantiates the expediency and relevance of the research task set forth in the second section. The third section is devoted to the theoretical substantiation of the main provisions: the calculation expressions for determining the components of frequency-phase distortion in cases where the order of the instantaneous phase function does not change with the transition to a new fragment, increases by one or two. In further research, it is planned to consider the case when the order of the instantaneous phase function decreases with the transition to the next fragment of a nonlinear frequency-modulated signal.

Keywords: nonlinear frequency modulation; mathematical model; instantaneous frequency and phase jump; autocorrelation function; maximum level of side lobes

DOI: 10.20535/RADAP.2025.100.51-58

Introduction

The modernization of existing and creation of new models of radar equipment is associated with the widespread introduction of complex sensing signals based on the use of intra-pulse modulation (IPM) of their instantaneous frequency or phase. Among them, linear-frequency modulated (LFM) signals have become widely used, but their significant drawback is the relatively high maximum level of the side lobes (MLSL) of the autocorrelation function (ACF), which is approximately 13.5 dB relative to the maximum level of the main lobe (ML) of the ACF [1–9].

One of the directions of work on reducing the MLSL of ACF of frequency-modulated (FM) signals is the development and implementation of mathematical models (MM) of signals with nonlinear frequency modulation (NLFM), which can consist exclusively of LFM fragments or contain fragments with different FM laws from the linear one [10–31].

The research conducted by the authors of this article on the development of new MM NLFM signals has revealed previously unknown patterns that arise in the synthesis of such signals. In particular, it was proved that in a multi-fragment NLFM signal, instantaneous frequency and phase jumps occur at the junctions between fragments. These jumps cause a distortion of the resulting amplitude-frequency spectrum (AFS), which, in turn, leads to a distortion of the ACF and can cause an increase in the MLSL [29–35].

The results of studies of MM NLFM signals, which include not only LFM fragments but also fragments with quadratic (QFM) or cubic frequency modulation (CubFM), made it possible to identify and formulate generalizations common to all cases of using power laws of FM in IPM fragments. These generalizations relate to the determination of the composition and magnitude of frequency-phase distortions in the signal fragments following their junction [33–35].

In this paper, we analytically substantiate the existence of regularities that condition the appearance of additional components of instantaneous frequency and phase distortion in the fragments of the NLFM signal. This phenomenon occurs not only due to jumps in these parameters at the moment of transition between fragments, but also as a result of deeper changes in the frequency-phase structure of the signal.

1 Analysis of research and publications

The stages of development of radar technology to the modern level and the peculiarities of its implementation are discussed in detail in [1–9], which indicates a continuous improvement of the systems for generating sensing and processing received signals. The transition from simple to complex radar signals made it possible to reduce the peak power of transmitting devices by increasing the duration of radio pulses. At the same time, the realization of the echo-signal compression effect in the coordinated filter of the radio receiver provided the necessary range resolution.

One of the directions of further improvement of radio systems for various purposes is to reduce the MLSL of ACF signals with IPM, for which NLFM signals are widely used [3, 4, 10–35].

A considerable number of publications are devoted to the synthesis and processing of NLFM signals. In particular, they consider their applications in detecting and tracking airborne objects [3, 4, 10–35], in air- and space-based radio engineering systems [36–39], in medical diagnostics (ultrasound imaging) [40, 41], sonar [42], and in the field of electronic warfare [43].

The main efforts of researchers are aimed at reducing the MLSL of ACF signals with NLFM. For this purpose, both single-fragment and multi-fragment versions of the results of their synthesis are proposed, which involve the use of modulation functions with an S-shaped law of instantaneous frequency change [3, 18, 22–25, 32]. Such a law can be realized on the basis of trigonometric functions [18] or polynomials of varying complexity [32].

The effect of such modulation is manifested in a change in the frequency response rate (FRR) at the edges of the spectrum, which leads to a rounding of the AFS of the signal. This is equivalent to applying weighting in the time domain [44–46].

Rounding of the spectral shape can also be achieved by sequential synthesis of LFM fragments with different FRR, for example, when one of the two-fragment or two fragments of a three-fragment signal has a higher FRR than the others [4, 13–17, 19, 27, 29–31, 33–36].

In [29–31, 33–35] new MM NLFM signals consisting of LFM fragments and also including fragments with nonlinear FM were proposed. Unlike the known ones, these variants of the models provide special compensating components that reduce the effect of instantaneous time and phase jumps at the junctions of fragments, which in turn contributes to the reduction of the MLSL of ACF signals.

It was found that in the case of using LFM fragments, the main reason for the occurrence of frequency-phase jumps is the change in the frequency response during the transition between them. Further studies of MM NLFM signals containing combinations of LFM-LFM, LFM-QFM, and LFM-CubFM fragments [33–35] revealed regularities on the basis of which a method for detecting and compensating for frequency-phase distortions at the junctions of fragments was developed. This approach has not been previously considered in the known sources of literature.

2 Formulation of the research task

The aim of this work is to develop an analytical method for calculating and compensating frequency-phase distortions arising in NLFM signals containing combinations of fragments with different frequency modulation laws.

The achievement of this goal necessitates the following interrelated tasks:

– analytical research of the peculiarities of frequency-phase distortion formation at the junctions between fragments in NLFM signals of the LFM-LFM, LFM-QFM, and LFM-CubFM types;

– derivation of mathematical dependencies describing the nature of changes in instantaneous frequency and phase arising from transitions between fragments with different frequency modulation rates;

– structural analysis of the obtained dependencies and compensation of the existing distortions based on the use of special compensating components in the signal model.

Thus, each of the tasks is directly aimed at the step-by-step realization of the overall goal of the study, i.e., improving the quality characteristics of NLFM signals by eliminating the influence of interfragment distortions.

3 Description of the research material

3.1 Determination and compensation of the frequency and phase distortions of the MM of the current time of the NLFM signal as part of two LFM fragments

Paper [33] shows that frequency and phase jumps occur at the junctions of LFM fragments caused by an instantaneous change in the frequency response at the moment of transition from the first fragment to the second, which is the first derivative of the instantaneous frequency and, accordingly, the second derivative of the instantaneous phase, where $n = 1, 2$ is the number of the LFM signal fragment:

$$\beta_n(t) = \frac{df_n(t)}{dt}.$$

Let us consider in more detail the current time MM of a three-fragment NLFM signal introduced in [33]. For the purpose of this study, we will limit ourselves to its first two fragments.

A distinctive feature of this MM from the known ones [4, 10, 14–17, 19, 26, 27] is the presence of compensation components that take into account the occurrence of a jump in instantaneous frequency and phase at the junction of fragments caused by the change in the FRR from β_1 to β_2. For further analysis, we present the MM record for both the instantaneous frequency (1) and the instantaneous phase of the NLFM signal (2):

$$f(t) = \begin{cases} f_0 + \beta_1 t, & 0 \le t \le T_1; \\ f_0 + \beta_2 t - (\beta_2 - \beta_1)T_1, & T_1 < t \le T_1 + T_2, \end{cases} \tag{1}$$

$$\varphi(t) = 2\pi \begin{cases} f_0 t + \dfrac{\beta_1 t^2}{2}, \quad 0 \le t \le T_1; \\ [f_0 + (\beta_2 - \beta_1)T_1]\, t + \dfrac{\beta_2 t^2}{2} - \dfrac{(\beta_2 - \beta_1)T_1^2}{2}, \\ \qquad T_1 < t \le T_1 + T_2, \end{cases} \tag{2}$$

where f_0 is the initial frequency of the NLFM signal; β_1, β_2 is the frequency modulation rate of the first and second LFM fragments, which is equal:

$$\beta_1 = \frac{\Delta f_1}{T_1}; \quad \beta_2 = \frac{\Delta f_2}{T_2},$$

where $\Delta f_1, \Delta f_2$ is the deviation of the frequency of the corresponding LFM fragment; T_1, T_2 is the duration of the first and second fragments of the NLFM signal.

The compensation components of frequency-phase distortions at the junction of fragments for the instantaneous frequency are described by the second expression in (1) and, respectively, in (2), for the instantaneous phase.

That is, at the junction of the fragments, due to the change in the frequency domain, there was a jump in the instantaneous frequency, to compensate for which a component was introduced into the second expression of MM (1):

$$\delta f_{12} = (\beta_2 - \beta_1)T_1. \tag{3}$$

Similarly, the second expression of MM (2) has two compensating components, a linear one:

$$\delta f_{12} t = T_1(\beta_2 - \beta_1)t \tag{4}$$

and constant:

$$\delta\varphi_{12} = \frac{1}{2}T_1^2(\beta_2 - \beta_1). \tag{5}$$

Components (3), (5) are constant values and have the physical essence of a jump in instantaneous frequency and, accordingly, phase at the junction of the NLFM signal fragments.

For the situation under consideration, i.e., in the presence of only LFM fragments, the change in the instantaneous phase of the signal occurs according to the quadratic law, i.e., the instantaneous phase function has two time derivatives: the instantaneous frequency $f(t) = d\varphi/dt$ and the FRR $\beta n = df_n/dt = d^2\varphi_n/dt^2 = \Delta f_n/T_n$. The frequency response, which is the oldest derivative of the instantaneous phase of the LFM signal, is a constant value for each signal fragment.

Let us write the MMs of instantaneous frequency and phase (1), (2) using the definition of the FRR:

$$f(t) = \begin{cases} f_0 + \displaystyle\int_t \beta_1 dt = f_0 + \beta_1 t, \quad 0 \le t \le T_1; \\ f_0 + \Delta f_1 + \displaystyle\int_t \beta_2 dt = f_0 + \Delta f_1 + \beta_2 t + C_1, \\ \qquad T_1 < t \le T_1 + T_2, \end{cases} \tag{6}$$

$$\varphi(t) = 2\pi \begin{cases} f_0 t + \displaystyle\iint_t \beta_1 dt^2 = f_0 t + \frac{1}{2}\beta_1 t^2, \quad 0 \le t \le T_1; \\ (f_0 + \Delta f_1)t + \displaystyle\iint_t \beta_2 dt^2 = \\ = (f_0 + \Delta f_1)t + \dfrac{1}{2}\beta_2 t^2 + C_1 t + C_2, \\ \qquad T_1 < t \le T_1 + T_2. \end{cases} \tag{7}$$

The absence of constant integrations in the first expressions of MM (6) and (7) is due to the zero initial conditions for these components.

A comparative analysis of (1), (2) and (6), (7), respectively, allows us to establish the following definitions:

$$C_1 = -(\beta_2 - \beta_1)T_1;$$
$$C_1 t = -T_1(\beta_2 - \beta_1)t;$$
$$C_2 = -\frac{1}{2}T_1^2(\beta_2 - \beta_1).$$

Thus, it can be concluded that the integration constant C_1 has the physical essence of an instantaneous frequency jump at the moment of transition from the first LFM fragment to the second, the value of C_2 has the physical essence of an instantaneous phase jump at the junction of fragments, and the product $C_1 t$ forms an additional linear increase in the instantaneous phase caused by a frequency jump, which also needs to be compensated.

From the analysis of (1), (6) and (2), (7), it follows that when synthesizing an NLFM signal from two LFM fragments, the instantaneous phase expression of the second fragment will include two compensation components – a linear and a constant one. The linear component has the physical essence of an additional increase in the instantaneous phase due to a frequency jump at the junction of the fragments, and the constant component is equal to the value of the phase jump that occurs as a result of the specified frequency jump. The total number of frequency-phase distortion components is equal to the number of instantaneous phase derivatives. The instantaneous frequency function for the case of using the LFM has one derivative and, as a result, one compensation component in the instantaneous frequency expression (6) of the second LFM fragment.

It should be noted that the structural feature of the frequency-phase distortion components is the presence of constant integrations in their expressions, while the other terms determine the change in frequency-phase parameters in accordance with the given FM law.

3.2 Determination and compensation of the frequency and phase distortions of the current time MM of a two-fragment NLFM signal as part of the LFM and QFM fragments

The experience of developing MM NLFM signals for the case when one of the fragments has a different FM law from the linear one indicates the presence of law-dimensions that have not been considered in well-known academic sources.

The essence of these patterns can be explained by the following considerations. The analysis of MM (6), (7) of the previously considered NLFM signal consisting of two LFM fragments allows us to conclude that in order to find the components of the instantaneous frequency and phase expressions, as well as to calculate the frequency-phase jumps at the junction of the fragments, it is necessary to determine the oldest derivative of the instantaneous phase of the signal, which is a constant value. Younger derivatives are found by integrating the oldest one, and therefore have a power law dependence on time. These derivatives provide the determination of both fundamental and compensation components, such as linear, quadratic, and cubic for the instantaneous phase of the QFM fragment, the number of components for the instantaneous frequency and their degree is one less.

The existence of this regularity is confirmed by the example of the development of the MM of the current time of the NLFM signal as a part of the LFM and QFM fragments [34].

To simplify further mathematical calculations, we will consider only the second fragment of the NLFM signal without specifying its time intervals.

Provided that the instantaneous frequency of the second fragment of the NLFM signal changes in accordance with the quadratic law $f_2(t) = F(t^2)$, the value of the FRR will change linearly in time. By analogy with the theory of motion of physical objects, the derivative of the FRR is denoted as the FM acceleration (FMA). Let's apply the concept of the FMA to the third derivative of the instantaneous phase, which in this case has a constant value:

$$\alpha_2 = \frac{d^3\varphi_2(t)}{dt^3}.$$

Then the instantaneous frequency of the second fragment:

$$f_2(t) = f_0 + \Delta f_1 + \iint\limits_t \alpha_2 dt^2 = \\ = f_0 + \Delta f_1 + \alpha_2\left(\frac{t^2}{2} - T_1 t + \frac{T_1^2}{2}\right). \quad (8)$$

The composition of (8) includes constant integrations C_1, C_2, which are found based on the initial conditions $t = T_1$:

$$\begin{aligned} C_1 &= \alpha_2 t|_{t=T_1} = \alpha_2 T_1; \\ C_2 &= \delta f_{12} = \alpha_2 \frac{t^2}{2}\bigg|_{t=T_1} = \alpha_2\frac{T_1^2}{2}. \end{aligned} \quad (9)$$

The definition of integration constants is mandatory, since the component (9) is actually equal to the value of the frequency jump at the junction of the NLFM signal fragments. The analysis of (8) shows that the formation of instantaneous time-total values occurs with the participation of three components: quadratic, linear, and constant.

By integrating (8), we find the instantaneous phase of the second signal fragment:

$$\varphi_2(t) = 2\pi\left[(f_0+\Delta f_1)t + \alpha_2\left(\frac{t^3}{6} - \frac{T_1}{2}t^2 + \frac{T_1^2}{2}t - \frac{T_1^3}{6}\right)\right]. \quad (10)$$

The new integration constant has a physical interpretation as the value of the instantaneous phase jump at the moment of transition between fragments [34]:

$$C_3 = \delta\varphi_{12} = \alpha_2\frac{T_1^3}{6}. \quad (11)$$

To make (8)–(11) usable, it is necessary to find the FMA α_2, which is determined from the relation:

$$\Delta f_2 = \alpha_2 \iint\limits_{T_2} dt,$$

where:

$$\alpha_2 = \frac{2\Delta f_2}{T_2^2}. \tag{12}$$

Substitution of (12) into (8)–(11) provides the necessary calculation relations for mathematical modeling in accordance with (8), (10).

The presented variant of MM (8), (10) implements the increasing law of frequency change and provides compensation for frequency and phase jumps at the junction of fragments caused by the presence of FMA.

The results of testing the performance of these MMs are presented in [34].

3.3 Research of frequency-phase distortions based on the MM of the current time of a two-fragment NLFM signal of the LFM-CubFM type

The transition to the cubic law of the FM in the second fragment of the NLFM signal causes the time dependence of the FMA derivative, and the next derivative will no longer depend on time, and then, in accordance with the theory of motion, we will use the name "jerk" FM (JFM) for it [35], i. e:

$$\eta_2 = \frac{d\alpha_2(t)}{dt} = \frac{d^4\varphi_2(t)}{dt^4}.$$

Thus, for the case when the second fragment of the NLFM signal has a CubFM, we have the derivatives of the instantaneous phase functions from the first to the fourth – $f_2(t)$, $\beta_2(t)$, $\alpha_2(t)$ and η_2. In this case, the instantaneous phase depends on time to the fourth power, the JFM is the oldest derivative and is a constant with respect to which the time change of the younger derivatives can be determined.

We assume that the frequency-phase distortion at the junction of the first and second fragments is caused by the fourth derivative of the instantaneous phase of the second signal fragment, denoted by η_2. Finding $\alpha_2(t)$, $\beta_2(t)$, $f_2(t)$ and $\varphi_2(t)$ by integrating η_2, at each stage must be performed with further definition and taking into account the integration constants, since they significantly affect the value of the frequency-phase distortion.

Let us write down in general form the expressions for $f_2(t)$ and $\varphi_2(t)$ the CubFM fragment [35]:

$$f_2(t) = f_0 + \Delta f_1 + \iiint\limits_{t} \eta_2 dt^3 = \\ = f_0 + \Delta f_1 + \eta_2\frac{t^3}{6} + \frac{1}{2}C_1 t^2 + C_2 t + C_3; \tag{13}$$

$$\varphi_2(t) = 2\pi \int\limits_{t} f_2(t)dt = \\ = 2\pi\left[(f_0+\Delta f_1+C_3)t+\frac{1}{24}\eta_2 t^4+\frac{C_1}{6}t^3+\frac{C_2}{2}t^2+C_4\right]. \tag{14}$$

The integration constants C_1-C_4 are found by determining the initial conditions. Each of these constants has a corresponding physical interpretation. The beginning of the second fragment of the NLFM signal falls at the time $t = T_1$, at which the JFM η_2 causes a jump in the instantaneous frequency:

$$C_3 = \delta f_{12} = \frac{1}{6}\eta_2 T_1^3. \tag{15}$$

The frequency jump (15) causes additional linear (16) and quadratic (17) frequency increments:

$$C_2 t = \frac{1}{2}\eta_2 T_1^2 t; \tag{16}$$

$$C_1 t^2 = \frac{1}{2}\eta_2 T_1 t^2. \tag{17}$$

After substituting (15)–(17) into (13) for the instantaneous frequency of the second fragment, we obtain [35]:

$$f_2(t) = f_0+\Delta f_1+\eta_2\left(\frac{t^3}{6} - \frac{T_1}{2}t^2 + \frac{T_1^2}{2}t - \frac{T_1^3}{6}\right). \tag{18}$$

The instantaneous phase jump at the junction of C_4 fragments is determined based on the same initial condition as the yielding one:

$$C_4 = \delta\varphi_{12} = \frac{1}{24}\eta_2 T_1^4. \tag{19}$$

Substituting (15)–(17), (19) into (14) provides an expression for the instantaneous phase of the CubFM fragment [35]:

$$\varphi_2(t) = 2\pi\left[\left(f_0 + \Delta f_1 - \frac{\eta_2 T_1^3}{6}\right)t + \right. \\ \left. +\eta_2\left(\frac{t^4}{24} - \frac{T_1 t^3}{6} + \frac{T_1^2 t^2}{4} - \frac{T_1^4}{24}\right)\right]. \tag{20}$$

As in the case of the FMA α_2 (12), we will determine the JFM η_2 with respect to the parameters of the second signal fragment. To this end, based on (13), we will find the frequency deviation of the fragment with the CubFM:

$$\Delta f_2 = \eta_2 \iiint\limits_{T_2} dt^3,$$

where we already have the JFM η_2:

$$\eta_2 = \frac{6\Delta f_2}{T_2^3}. \tag{21}$$

In expression (20), one of the components has a physical interpretation of the instantaneous frequency jump (15), and the other (19), when multiplied by 2π, is the instantaneous phase jump at the junction of the fragments.

Taking into account the first expression of MM (2), (14)–(21), we finally write the MM for the instantaneous phase of the NLFM signal with the first LFM and the second CubFM fragments, indicating the time intervals for them [35]:

$$\varphi(t)=2\pi\begin{cases}\left[f_0t+\dfrac{\beta_1t^2}{2}\right], \quad 0\le t\le T_1;\\ \left[(f_0+\Delta f_1-\delta f_{12})\,t+\right.\\ \left.+\eta_2\left(\dfrac{t^4}{24}-\dfrac{T_1t^3}{6}+\dfrac{T_1^2t^2}{4}\right)-\delta\varphi_{12}\right],\\ T_1<t\le T_1+T_2.\end{cases} \tag{22}$$

Thus, the isolation of the JFM made it possible to synthesize the MM (22), which compensates for both frequency and phase jumps at the junction of the LFM and CubFM fragments and additional phase distortions in the CubFM fragment itself by introducing compensating components (15)-(17), (19). The validation of the functional performance (22) was performed in [35].

Conclusions

The paper proposes a method for determining the magnitude and subsequent compensation of frequency-phase distortions of two-fragment NLFM signals in which the first fragment has an LFM, and the second has one of three variants: a linear, quadratic, or cubic law of instantaneous frequency change. Thus, we investigate cases where the degree of the FM law remains unchanged or increases by one or two orders of magnitude when moving to the next fragment.

The development of the method became possible due to the establishment of regularities of instantaneous frequency and phase jumps at the junctions of the NLFM signal fragments, both in the cases of fragments with the same modulation law and when they change. It is proved that the reason for the appearance of additional frequency-phase distortions is a change in the value of the oldest derivative of the instantaneous phase, if the order of the derivatives in the fragments coincides, or the appearance of a new higher-order derivative in other cases.

The essence of the proposed method is that each of the lower derivatives of the instantaneous phase causes a corresponding distortion. The magnitudes of these distortions are determined by sequentially integrating the oldest derivative with the obligatory consideration of integration constants. In particular, the number of components of the instantaneous phase distortion is equal to the order of the oldest derivative, the number of components of the instantaneous frequency is one less.

Thus, two components arise for the LFM fragments, three for the quadratic fragment, and four for the cubic fragment. In this case, the constant component of phase distortion corresponds to the physical essence of the instantaneous phase jump, and the constant component of frequency distortion corresponds to the instantaneous frequency jump.

A structural sign of the presence of distortion is the appearance of a constant integration in the corresponding analytical expression. The other terms of the expression ensure the formation of the instantaneous frequency and phase of the signal in accordance with the given FM law.

The application of the proposed method is limited by the conditions of differentiability of the instantaneous phase function of the studied fragments, as well as the requirement of a finite number of its derivatives.

The obtained results can be useful for specialists engaged in the development and research of systems for the formation and processing of NLFM signals.

In future researches, it is planned to analyze mathematical models of two-fragment NLFM signals with the reverse order of fragments, i.e., from the highest to the lowest degree of the modulation law.

References

[1] Levanon N., Mozeson E. (2004). *Radar Signals*. John Wiley & Sons, Inc., 432 p. DOI: 10.1002/0471663085.

[2] Van Trees H. L. (2001). *Detection, Estimation, and Modulation Theory, Part III: Radar-Sonar Processing and Gaussian Signals in Noise*. John Wiley & Sons, Inc., 643 p.

[3] Skolnik M. I. (1981). *Introduction to Radar Systems. Second Edition*. McGraw-Hill Book Co., 581 p.

[4] Cook C. E. and Bernfeld M. (1993). *Radar Signals: An Introduction to Theory and Application*. Artech House, 552 p.

[5] Blackman S. S., and Popoli R. F. (1990). *Design and Analysis of Modern Tracking Systems*. Artech House, 1230 p.

[6] Richards M. A., Scheer J. A., Holm W. A. (2010). *Principles of modern radar, Vol. I: Basic Principles*. Sheridan Books, Inc., 962 p.

[7] Barton D. K. (2004). *Radar System Analysis and Modeling*. Artech House Publishers, 566 p.

[8] Melvin W. L., and Scheer J. A. (2013). *Principles of Modern Radar. Vol. II: Advanced techniques*. Sci Tech Publishing, 846 p.

[9] McDonough R. N., and Whalen A. D. (1995). *Detection of Signals in Noise (2nd. ed.)*. Academic Press, Inc., USA, 495 p.

[10] Adithya Valli N., Elizabath Rani D., Kavitha C. (2019). Windows for Reduction of ACF Sidelobes of Pseudo-NLFM Signal. *International Journal of Scientific & Technology Research*, Vol. 8, Iss. 10, pp. 2155-2161.

[11] Zhang Y., Wang W., Wang R., et al. (2020). A Novel NLFM Waveform with Low Sidelobes Based on Modified Chebyshev Window. *IEEE Geosci. Remote Sens. Lett.*, Vol. 17, Iss. 5, pp. 814-818. doi:10.1109/LGRS.2019.2930817.

[12] Saleh M., Omar S.-M., Grivel E., et al. (2021). A variable chirp rate stepped frequency linear frequency modulation waveform designed to approximate wideband non-linear radar waveforms. *Digital Signal Processing*, Vol. 109, Article id. 102884. doi: 10.1016/j.dsp.2020.102884.

[13] Zhuang R., Fan H., Sun Y., et al. (2021). Pulse-agile waveform design for nonlinear FM pulses based on spectrum modulation. *IET International Radar Conference (IET IRC 2020)*, pp. 964-969. doi:10.1049/icp.2021.0700.

[14] Adithya Valli N., Elizabath Rani D., Kavitha C. (2019). Modified Radar Signal Model using NLFM. *International Journal of Recent Technology and Engineering (IJRTE)*, Vol. 8, Iss. 2S3, pp. 513-516. doi: 10.35940/ijrte.B1091.0782S319.

[15] Ch Anoosha and Krishna B. T. (2022). Peak Sidelobe Reduction analysis of NLFM and Improved NLFM Radar signal with Non-Uniform PRI. *AIUB Journal of Science and Engineering (AJSE)*, Vol. 21, Iss. 2, pp. 125–131. doi: 10.53799/ajse.v21i2.440.

[16] Parwana S., Kumar S. (2015). Analysis of LFM and NLFM Radar Waveforms and their Performance Analysis. *International Research Journal of Engineering and Technology (IRJET)*, Vol. 02, Iss. 02, pp. 334-339.

[17] Chan Y. K., Yam C. M., and Koo V. C. (2009). Sidelobes Reduction using Simple Two and Tri-Stages Nonlinear Frequency Modulation (NLFM). *Progress in Electromagnetics Research (PIER)*, Vol. 98, pp. 33-52. doi: 10.2528/PIER09073004.

[18] Ping T., Song C., Qi Z., et al. (2024). PHS: A Pulse Sequence Method Based on Hyperbolic Frequency Modulation for Speed Measurement. *International Journal of Distributed Sensor Networks*, Vol. 2024, Article no 6670576, 11 p. doi: 10.1155/2024/6670576.

[19] Septanto H., Sudjana O., and Suprijanto D. (2022). A Novel Rule for Designing Tri-Stages Piecewise Linear NLFM Chirp. *2022 International Conference on Radar, Antenna, Microwave, Electronics, and Telecommunications (ICRAMET)*, IEEE, pp. 62–67. doi:10.1109/ICRAMET56917.2022.9991201.

[20] Jin G., Deng Y., Wang R., et al. (2019). An Advanced Nonlinear Frequency Modulation Waveform for Radar Imaging With Low Sidelobe. *IEEE Transactions on Geosciences and Remote Sensing*, Vol. 57, Iss. 8, pp. 6155–6168. doi:10.1109/TGRS.2019.2904627.

[21] Xu Z., Wang X., Wang Y. (2022). Nonlinear Frequency Modulated Waveforms Modeling and Optimization for Radar Applications. *Mathematics*, Vol. 10(21), Article id. 3939, pp. 1-11. doi:10.3390/math10213939.

[22] Wei T., Wang W., Zhang Y., et al. (2022). Novel Nonlinear Frequency Modulation Waveform With Low Sidelobes Applied to Synthetic Aperture Radar. *IEEE Geoscience and Remote Sensing Letters*, Vol. 19, pp. 1-5, Art no. 4515405. doi: 10.1109/LGRS.2022.3216340.

[23] Roy A., Nemade H. B., Bhattacharjee R. (2021). Radar Waveform Diversity using Nonlinear Chirp with Improved Sidelobe Level Performance. *AEU-International Journal of Electronics and Communications*, Vol. 136, Article id. 153768. doi:10.1016/J.AEUE.2021.153768.

[24] Ghavamirad J. R., Sadeghzadeh R.A., Sebt M. A. (2025). Sidelobe Level Reduction in the ACF of NLFM Signals Using the Smoothing Spline Method. *Electrical Engineering and Systems Science, Signal Processing*, arXiv:2501.06657 [eess.SP], 5 p. doi: 10.48550/arXiv.2501.06657.

[25] Zhang Y., Deng Y., Zhang Z., et al. (2022). Parametric NLFM Waveform for Spaceborne Synthetic Aperture Radar. *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 60, pp. 1-9, Article no. 5238909. doi: 10.1109/TGRS.2022.3221433.

[26] Adithya Valli N., Elizabath Rani D., Kavitha C. (2019). Doppler Effect Analysis of NLFM Signals. *International Journal of Scientific & Technology Research*, Vol. 8, Iss. 11, pp. 1817-1821.

[27] Adithya Valli N., Elizabath Rani D., Kavitha C. (2020). Performance Analysis of NLFM Signals with Doppler Effect and Background Noise. *International Journal of Engineering and Advanced Technology (IJEAT)*, Vol. 9, Iss. 3, pp. 737-742. doi: 10.35940/ijeat.B3835.029320.

[28] Milczarek H., Lesnik C., Djurovic I., et al. (2021). Estimating the Instantaneous Frequency of Linear and Nonlinear Frequency Modulated Radar Signals – A Comparative Study. *Sensors*, Vol. 21(8), Article no 2840. doi: 10.3390/s21082840.

[29] Kostyria, O. O., Hryzo, A. A., Khudov V. H., et al. (2024). Two-Fragment Non-Linear-Frequency Modulated Signals with Roots of Quadratic and Linear Laws Frequency Changes. *Radio Electronics, Computer Science, Control*, Vol. 1(68), pp. 17-27. doi:10.15588/1607-3274-2024-1-2.

[30] Kostyria O. O., Hryzo A. A., and Dodukh O. M. (2024). Combined two-fragment radar signals with linear and exponential frequency modulation laws. *Systems of Arms and Military Equipment*, Vol. 4(76), pp. 58-64. doi: 10.30748/soivt.2023.76.06.

[31] Kostyria O. O., Hryzo A. A., Dodukh O. M., et al. (2023). Method of Minimization Sidelobes Level Autocorrelation Functions of Signals with Non-Linear Frequency Modulation. *Radio Electronics, Computer Science, Control*, Vol. 4, pp. 39-48. doi:10.15588/1607-3274-2023-4-4.

[32] Hryzo A. A., Kostyria O. O., Khudov H. V., et al. (2024). Implementation of Structural-Parametric Synthesis of a Nonlinear Frequency Modulated Signal on the Basis of a Genetic Algorithm. *Science and Technology of the Air Force of Ukraine*, Vol. 1(54), pp. 77-82. doi: 10.30748/nitps.2024.54.10.

[33] Kostyria O. O., Hryzo A. A., Dodukh O. M., et al. (2023). Mathematical Model of the Current Time for Three-Fragment Radar Signal with Nonlinear Frequency Modulation. *Radio Electronics, Computer Science, Control*, Vol. 3(63), pp. 17-26. doi: 10.15588/1607-3274-2023-3-2.

[34] Kostyria, O. O., Hryzo, A. A., Khudov V. H., et al. (2024). Mathematical Model of Current Time of Signal from Serial Combination Linear-Frequency and Quadratically Modulated Fragments. *Radio Electronics, Computer Science, Control*, Vol. 2(69), pp. 24-33. doi: 10.15588/1607-3274-2024-2-3.

[35] Kostyria O. O., Hryzo A. A., and Dodukh O. M. (2023). Compensation for Distortions of the Frequency-Time Structure of the Combined Signal under the Condition of Different Number of Derivatives of the Instantaneous Phase of its Fragments. *Scientific Works of Kharkiv National Air Force University*, Vol. 4(78), pp. 70-75. doi: 10.30748/zhups.2023.78.10.

[36] Zhao Yu., Ritchie M., Lu X., et al. (2020). Non-Continuous Piecewise Nonlinear Frequency Modulation Pulse with Variable Sub-Pulse Duration in a MIMO SAR Radar System. *Remote Sensing Letters*, Vol. 11, Iss. 3, pp. 283-292. doi: 10.1080/2150704X.2019.1711237.

[37] Liu S., Jia Y., Liu Y., et al. (2022). Research on Ultra-Wideband NLFM Waveform Synthesis and Grating Lobe Suppression. *Sensors*, Vol. 24, Article no 9829. doi: 10.3390/s22249829.

[38] Xu W., Zhang L., Fang C., et al. (2021). Staring Spotlight SAR with Nonlinear Frequency Modulation Signal and Azimuth Non-Uniform Sampling for Low Sidelobe Imaging. *Sensors*, Vol. 21(19), Article no. 6487. doi: 10.3390/s21196487.

[39] Jiang T., Li B., Li H., et al. (2021). Design and Implementation of Spaceborne NLFM Radar Signal Generator. *2nd IYSF Academic Symposium on Artificial Intelligence and Computer Engineering*, Vol. 120792S. doi: 10.1117/12.2623222.

[40] Fan Z., and Meng H.-Y. (2020). Coded Excitation with Nonlinear Frequency Modulation Carrier in Ultrasound Imaging System. *2020 IEEE Far East NDT New Technology & Application Forum (FENDT)*, IEEE, pp. 31-35. doi: 10.1109/FENDT50467.2020.9337517.

[41] Cheng Z., Sun Z., Wang J., et al. (2024). Magneto-Acousto-Electrical Tomography using Nonlinearly Frequency-Modulated Ultrasound. *Phys Med Biol*, Vol. 69(8), PMID: 38422542. doi: 10.1088/1361-6560/ad2ee5.

[42] Prakash B. L., Sajitha G., and Rajeswari K. R. (2016). Generation of Random NLFM Signals for Radars and Sonars and their Ambiguity Studies. *Indian Journal of Science and Technology*, Vol. 9, Iss. 29, pp. 1-7. doi: 10.17485/ijst/2016/v9i29/93653.

[43] Li J., Wang P., Zhang H., et all. (2024). A Novel Chaotic-NLFM Signal under Low Oversampling Factors for Deception Jamming Suppression. *Remote Sens*, Vol. 16(1), Article no 35. doi: 10.3390/rs16010035.

[44] Galushko V. G. (2019). Performance Analysis of Using Tapered Windows for Sidelobe Reduction in Chirp-Pulse Compression. *Radio Physics and Radio Astronomy*, Vol. 24(4), pp. 300-313. doi: 10.15407/rpra24.04.300.

[45] Azouz A., Abosekeen A., Nassar S., et al. (2021). Design and Implementation of an Enhanced Matched Filter for Sidelobe Reduction of Pulsed Linear Frequency Modulation Radar. *Sensors*, Vol. 21(11), p. 3835. doi: 10.3390/s21113835.

[46] Muralidhara N., Velayudhan V., and Kumar M. (2022). Performance Analysis of Weighing Functions for Radar Target Detection. *International Journal of Engineering Research & Technology (IJERT)*, Vol. 11, Iss. 03, pp. 161-165.

[47] Van Zyl A. C., Wiehahn E. A., Cillers J. E., et al. (2022). Optimized Multi-Parameter NLFM Pulse Compression Waveform for low Time-Bandwidth Radar. *International Conference on Radar Systems (RADAR 2022)*, Vol. 2022, Iss. 17 pp. 289-294. doi: 10.1049/icp.2022.2332.

[48] Argenti F., Facheris L. (2020). Radar Pulse Compression Methods Based on Nonlinear and Quadratic Optimization. *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 59, Iss. 5, pp. 3904-3916. doi: 10.1109/TGRS.2020.3010414.

Метод обчислення і компенсації частотно-фазових спотворень на стику та у фрагментах сигналів з нелінійною частотною модуляцією

Костиря О. О., Гризо А. А., Федоров А. В., Лісогорський Б. А., Ушаков С. І.

Застосування нелінійно-частотно модульованих сигналів в радіолокаційній техніці обумовлено можливістю зниження максимального рівня бічних пелюсток їх автокореляційних функцій у порівнянні з лінійно-частотно модульованими сигналами. Одним із перспективних напрямів розвитку теорії синтезу сигналів з нелінійною частотною модуляцією є дослідження тонкої сигнальної структури, яка зазнає спотворень при переході до нового фрагмента сигналу.

Зокрема, було встановлено, що стрибки миттєвої частоти та фази на межі між фрагментами викликають додаткові спотворення частотно-фазової структури у наступних фрагментах. Ці явища раніше залишались поза увагою дослідників.

Попередні роботи з розробки та дослідження математичних моделей нелінійно-частотно модульованих сигналів дозволили виявити закономірності, які описують зміну частотно-фазової структури наступного фрагмента при зміні значення або порядку найстаршої похідної функції миттєвої фази. Виявлено, що кількість складових у спектрі спотворень визначається порядком цієї похідної: для фазових викривлень – згідно з її значенням, для частотних – на одиницю менше. Постійні складові мають фізичне трактування і відповідають стрибкам миттєвої частоти або фази на межі фрагментів.

Структура роботи обумовлена логікою дослідження. У першому розділі роботи проведено аналіз наявних публікацій, який засвідчив відсутність досліджень у цьому напрямку. Це обґрунтовує доцільність і актуальність поставленого у другому розділі завдання дослідження. Третій розділ присвячений теоретичному обґрунтуванню основних положень: наведено розрахункові вирази для визначення складових частотно-фазових спотворень у випадках, коли порядок функції миттєвої фази з переходом на новий фрагмент не змінюється, підвищується на одиницю або на два.

У подальших дослідженнях планується розглянути випадок, коли з переходом до наступного фрагмента нелінійно-частотно модульованого сигналу відбувається зниження порядку функції миттєвої фази.

Ключові слова: нелінійна частотна модуляція; математична модель; стрибок миттєвої частоти та фази; автокореляційна функція; максимальний рівень бічних пелюсток