UDC 621.396.962

Features of Functioning and Synthesis of Mathematical Models with Time-Shift Multi-Fragment Nonlinear-Frequency-Modulated Signals

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A known method of reducing the maximum level of side lobes of the autocorrelation function of signals with intra-pulse frequency modulation, for example, linearly-frequency-modulated signals, is using their weight processing in a radio receiving device. An alternative to weight processing is rounding the amplitude-frequency spectrum, for which nonlinear-frequency-modulated signals are used. Many varieties of mathematical models of such signals are known, but the task of synthesizing new mathematical models of signals with nonlinear frequency modulation does not lose its relevance today.

The authors of the article previously synthesized mathematical models of two- and three-fragment nonlinear-frequency-modulated signals, which provide a decrease in the level of side lobes of the correlation function due to compensation for frequency-phase distortions at the joints of fragments. The reasons for the occurrence of these distortions are determined, substantiated analytically, and verified by modeling the mechanisms of their compensation.

To further reduce the maximum level of side lobes, it is proposed to increase the number of linearly-frequency-modulated fragments from three to five, for which the components of the mathematical model are calculated, which provide compensation for the instantaneous phase jumps of the resulting signal at the joints of all its fragments.

The structure of the work is due to the logic of the study. In the first section of the work, an analysis of known publications was carried out, which indicates the absence of mathematical models of shifted time of five-fragment nonlinear-frequency-modulated signals. From this, the task of the study is formulated in the second section of the article. The third section of the work is theoretical; it is devoted to the analytical definition of compensation components to avoid distortions of the instantaneous phase at the moments of transition from one fragment of the signal to the next. It is determined that the frequency-time parameters of all previous fragments contribute to forming phase jumps at the joints. The validity of the obtained theoretical results was checked by modeling.

Keywords: nonlinear frequency modulation; mathematical model; instantaneous phase jump; autocorrelation function; maximum level of side lobes

DOI: 10.64915/RADAP.2025.102.25-32

Introduction

The improvement of radar systems is closely related to developing new sounding signals capable of providing high resolution and noise immunity in a complex interference environment. Among such signals, a special place is occupied by signals with intra-pulse modulation of frequency, in particular linear-frequency-modulated (LFM) and nonlinear-frequency-modulated (NLFM) signals [1–4]. Although LFM signals allow to provide the required energy of the sounding signal by increasing its duration, they are characterized by a relatively high peak of side lobe level (PSL) of the autocorrelation function (ACF), which reduces the quality of target detection against the background of other powerful signals, for example, reflected from local objects [5].

To reduce the maximum PSL (MPSL), researchers have proposed various approaches, in particular, weight processing of the signal [4, 6–8] and rounding of the amplitude-frequency spectrum (AFS) of the signal by reducing its spectral power density in the low and high frequencies [9–12]. Both approaches can be used in parallel, since they have the exact physical nature, but the difference between them lies in the implementation area: the first works in the time plane, and the second in the frequency plane.

A practical alternative is to use multi-fragment NLFM signals, the structure of which allows you to adjust the frequency characteristics by selecting variations in the parameters of the fragments. NLFM signals with a serial combination of LFM fragments and a combination of fragments with linear and nonlinear frequency modulation laws have become widely used [13–17]. However, instantaneous phase jumps occur at the joints of such fragments, which negatively affect the spectral characteristics of the signal and cause an increase in MPSL ACF.

This study considers the mechanism of occurrence of phase distortions in multi-fragment NLFM signals, and an analytical approach to their compensation is proposed.

To further reduce MPSL NLFM signals, it is proposed to extend this approach to the three-fragment NLFM signal introduced in mathematical model (MM), shifted in time, and by structural synthesis to obtain a new MM five-fragment NLFM signal that provides less MPSL ACF and improved signal spectral characteristics.

1 Analysis of studies and publications

The creation of modern radar technology is based on the widespread use of complex and straightforward probing signals, the type of which is determined by the complexity of the air situation and depends on the mode of operation of the electronic means. Known works [1–4] are devoted to the problems of formation and formation of radar signals; research in this direction is actively continuing. NLFM signals are widely used in various fields of activity, for example, air and space control [2,4,17], where it is essential to ensure low MPSL and minimize coordinate measurement errors due to Doppler frequency shift. To this end, various methods are proposed, which include [18–20].

Several authors propose to perform nonlinear intrapulse modulation frequencies based on polynomial or piecewise-continuous functions for rounding the AFS of probing signals [13–17].

Among the variety of NLFM, it is necessary to distinguish a class of multi-fragment signals, consisting of fragments exclusively from LFM and combinations of LFM and NLFM. Note that the wide distribution of signals based on frequency modulated (FM) fragments is due to their tolerance to Doppler frequency shift, significantly simplifying the coordinated processing [10,12,13,15–17].

The increased attention to NLFM is due to the additional possibility of reducing the MPSL ACF of such signals. Researchers consider two- and three-fragment NLFM signals and investigate methods of their synthesis based on optimization of FM laws and signal structure, especially processing for various applications, which are discussed in publications [18–23].

It is proved in works [5, 23] that phase jumps occur at the junctions of multi-fragment NLFM signals caused by a change in the speed value FM (SFM), which is determined through the ratio of the frequency deviation of the LFM fragment to its duration and is the second derivative of the instantaneous phase – a constant value for each of the fragments. Failure to account for these phase jumps leads to a distortion of the AFS and, as a result, to a possible increase in MPSL and the nature of the change in MPSL ACF. For the case of two- and three-fragment NLFM signals, a mechanism for compensating for these jumps has been developed, which provides a stable decrease in MPSL ACF compared to conventional LFM.

Analysis of studies and publications showed that for five-fragment NLFM signals, the issue of developing and checking the operability of the MM shifted time remains unenlightened. Therefore, further research in this direction can significantly expand the capabilities of modern radar technology and increase its effectiveness in various fields of application.

2 Formulation of the research task

The work aims to develop and study the mathematical models (MMs) shifted time of the NLFM signal, consisting of five LFM fragments with phase jump compensation at the joints of the fragments to reduce the MPSL of its ACF.

3 Presentation of the study material

3.1 Study of MM functioning peculiarities of shifted time of two- and three-fragment NLFM signals

In the works of many authors, for example [10, 12, 17–20], for the study of two- and three-fragment NLFM signals, time-shifted MMs are used, the peculiarity of which is that for each subsequent fragment, the time scale is shifted by zero. For the case of three LFM fragments with increasing frequency, these MMs for instantaneous frequency and phase, respectively, have the form:

$$f_n(t)|_{n=[1,3]} = \begin{cases} f_0 + \beta_1 t, \quad 0 \le t \le T_1; \\ f_0 + \Delta f_1 + \beta_2 (t - T_1), \\ T_1 < t \le T_1 + T_2; \\ f_0 + \Delta f_1 + \Delta f_2 + \beta_3 (t - T_1 - T_2), \\ T_1 + T_2 < t \le T_S, \end{cases} \quad (1)$$

$$\varphi_n(t)|_{n=[1,3]} = = 2\pi \begin{cases} f_0 t + \frac{\beta_1 t^2}{2}, \quad 0 \le t \le T_1; \\ (f_0 + \Delta f_1)(t - T_1) + \beta_2 \left(\frac{t^2}{2} - T_1 t\right), \\ T_1 \le t \le T_1 + T_2; \\ (f_0 + \Delta f_1 + \Delta f_2)(t - T_1 - T_2) + \\ + \beta_3 \left(\frac{t^2}{2} - (T_1 + T_2)t\right), \\ T_1 + T_2 \le t \le T_S, \end{cases} \quad (2)$$

where f_0 is an initial NLFM signal frequency;
β_1, β_2, β_3 – SFM of the first, second, and third LFM fragments, which is determined from the relation $\beta_n = \Delta f_n / T_n$;
n – fragment number of NLFM signal;
Δf_n is a frequency deviation (difference between upper and lower frequencies) of the corresponding LFM fragment;
T_n – duration of each signal fragment;
$T_S = T_1 + T_2 + T_3$ – sum of fragment duration of NLFM signal.

In the future, when obtaining intermediate results to reduce mathematical records, we will omit the values of the time intervals of the fragments of NLFM signals, since they are fixed.

The feature of MM (1), (2) functioning is automatic compensation of instantaneous frequency jumps at moments of transition from one LFM fragment to the next. However, work [23] shows that instantaneous phase jumps occur at the joints of fragments, which distort the frequency-time structure of the NLFM signal, as a result of which the MPSL of its ACF can increase. The mechanisms for compensating for frequency jumps and phase distortions have not been investigated in well-known academic sources.

For further research, we will transition expressions for the second and third-MM fragments (1) and (2) from the shifted time scale to the current one. After simplification, we get:

$$f_2(t) = f_0 + \beta_1 T_1 + \beta_2 t - \beta_2 T_1, \quad (3)$$

$$f_3(t) = f_0 + \beta_1 T_1 + \beta_2 T_2 + \beta_3 t - \beta_3 T_1 - \beta_3 T_2, \quad (4)$$

$$\varphi_2(t) = 2\pi \times \times \left(-(f_0 + \Delta f_1) T_1 + (f_0 + \Delta f_1) t + \frac{\beta_2 t^2}{2} - \beta_2 T_1 t\right), \quad (5)$$

$$\varphi_3(t) = 2\pi \times \times (-f_0 (T_1 + T_2) - (\Delta f_1 + \Delta f_2)(T_1 + T_2) + + \left(f_0 - (\beta_3 - \beta_1) T_1 - (\beta_3 - \beta_2) T_2\right) t + \frac{t^2}{2}\beta_3 \Big). \quad (6)$$

Analysis (3) shows that compensation for the frequency jump at the junction of fragments in a model with a sloped time scale is provided by a harmful component $\beta_2 T_1$, and physically considering the additional frequency gain throughout the first LFM fragment. In (4), we already have two compensating components $\beta_3 T_1$ and $\beta_3 T_2$, which are also negative, and feel the additional frequency gain due to SFM β_3 at durations of the first and second fragments.

As a result of analysis (5) and (6), it follows that compensation for phase incursions in MM with a shifted time scale is tried to ensure that the slit sets the zero value of the initial phase of each subsequent NLFM signal fragment. However, although they were compensated, frequency jumps caused additional phase jumps at the joints of fragments that are not compensated in any way.

As indicated in [10, 18–20], MM (2) is obtained by finding an indefinite integral over time from (1), and the integration operation itself is given. From the analysis of analytical dependencies, it turns out that during the operation, as is mainly practiced, the presence of integration constants is neglected, and this in this case is decisive, since it allows us to find the value of phase jumps caused by frequency jumps. We prove this statement analytically, for which we integrate (1):

$$\varphi_n(t)|_{n=[1,3]} = 2\pi \times \times \begin{cases} \int_t f_1(t)dt = f_0 t + \frac{\beta_1 t^2}{2} + C_1; \\ \int_t f_2(t - T_1)dt = (f_0 + \beta_1 T_1)(t - T_1) + \\ + \beta_2 \left(\frac{t^2}{2} - T_1 t\right) + C_2; \\ \int_t f_3(t - T_1 - T_2)dt = \\ = (f_0 + \beta_1 T_1 + \beta_2 T_2)(t - T_1 - T_2) + \\ + \beta_3 \left(\frac{t^2}{2} - (T_1 + T_2)t\right) + C_3. \end{cases} \quad (7)$$

Integration constants $C_1 - C_3$ in (7) are calculated according to the initial conditions, which are the time values t at the beginning of each of the fragments, based on this, $C_1 = 0$, and for C_2 we have:

$$C_2 = \varphi_2(t)|_{t=T_1} = -\frac{1}{2} T_1^2 (\beta_2 + \beta_1). \quad (8)$$

Accordingly, find C_3:

$$C_3 = \varphi_3(t)|_{t=T_2} = -\frac{1}{2}\left[T_1^2(\beta_3 + \beta_1) + T_2^2(\beta_3 + \beta_2)\right]. \quad (9)$$

The integration constants C_2 and C_3 have a physical interpretation of the jumps of the instantaneous phase at the moments of transition from the first to the second and from the second to the third fragments. To compensate them in the model, we must introduce negative compensation components that we denote $\delta\varphi_{12} = C_2$, and $\delta\varphi_{23} = C_3$.

Thus, it was found that applying the operation of shifting the time scale by zero mark for the second and third LFM fragments in MM (1) is equivalent to the introduction of compensation components of instantaneous frequency jumps at the junction of fragments. Model (2) is obtained without considering the integration constants, which led to the loss of the compensation components of the phase distortions caused by these frequency jumps. As shown in [23], eliminating these instantaneous phase jumps at fragment junctions in the vast majority of cases leads to an improvement in the AFS of the resulting NLFM signal and, as a result, a decrease in the MPSL of their ACF.

Studies conducted by many authors [10, 12, 17] on the example of two- and three-fragment signals show that an effective way to reduce MPSL ACF NLFM signals is to increase the number of fragments. However, ignoring the detected mechanism of instantaneous phase jumps at the joints of fragments of NLFM signals significantly complicates and narrows the range of choice of their frequency-time parameters.

Thus, the results obtained in this subsection are the basis for the structural synthesis of a signal with many LFM fragments. For example, for the case of five, the MPSL of its ACF is expected to be lower than that of the considered MM.

The next section of the article is the actual continuation of the studies performed in [23], and is based on the results obtained in this work.

3.2 Synthesis of the mathematical model of the shifted time of the five-fragment NLFM signal

To test the hypothesis, we will use the approach outlined in [23] and the results of the previous subsection. As a base, we use the received MM three-fragment NLFM signal (7), in which phase jumps are compensated at the joints of LFM fragments due to the input of compensation components (8) and (9).

Increasing the number of fragments of the NLFM signal to five implies calculating the value of two additional instantaneous phase jumps. Using the approach used in [23] and based on the analysis (8), (9), we conclude that the instantaneous phase jump at the junction of LFM fragments is proportional to the sum of the products of the square of the duration of each of the previous fragments by the sum of SFM of the corresponding previous and current fragments. Following (8) and (9), we can write the expressions for the compensation components of the phase jump at the junction between the third and fourth fragments:

$$\delta\varphi_{34} = \frac{1}{2}\left(T_1^2(\beta_4+\beta_1) + T_2^2(\beta_4+\beta_2) + T_3^2(\beta_4+\beta_3)\right). \tag{10}$$

By analogy, for the moment of transition from the fourth to the fifth LFM fragment, we have:

$$\delta\varphi_{45} = \frac{1}{2}\left(T_1^2(\beta_5+\beta_1) + T_2^2(\beta_5+\beta_2) + T_3^2(\beta_5+\beta_3) + T_4^2(\beta_5+\beta_4)\right). \tag{11}$$

Thus, using (7) and taking into account (10) and (11), we obtain MM in the shifted time of the five-fragment NLFM signal:

$$\varphi_n(t)|_{n=[1,5]} = 2\pi \times \\ \times \begin{cases} f_s t + \dfrac{\beta_1}{2}t^2, \quad 0 \le t \le T_1; \\ (f_s + \Delta f_1)(t - T_1) + \beta_2\left(\dfrac{t^2}{2} - T_1 t\right) - \\ - \delta\varphi_{12}, \quad T_1 < t \le \sum\limits_{n=1}^{2} T_n; \\ \left(f_s + \sum\limits_{n=1}^{2}\Delta f_n\right)\left(t - \sum\limits_{n=1}^{2} T_n\right) + \beta_3\left(\dfrac{t^2}{2} - \sum\limits_{n=1}^{2} T_n t\right) - \\ - \delta\varphi_{23}, \quad \sum\limits_{n=1}^{2} T_n < t \le \sum\limits_{n=1}^{3} T_n; \\ \left(f_s + \sum\limits_{n=1}^{3}\Delta f_n\right)\left(t - \sum\limits_{n=1}^{3} T_n\right) + \beta_4\left(\dfrac{t^2}{2} - \sum\limits_{n=1}^{3} T_n t\right) - \\ - \delta\varphi_{34}, \quad \sum\limits_{n=1}^{3} T_n < t \le \sum\limits_{n=1}^{4} T_n; \\ \left(f_s + \sum\limits_{n=1}^{4}\Delta f_n\right)\left(t - \sum\limits_{n=1}^{4} T_n\right) + \beta_5\left(\dfrac{t^2}{2} - \sum\limits_{n=1}^{4} T_n t\right) - \\ - \delta\varphi_{45}, \quad \sum\limits_{n=1}^{4} T_n < t \le \sum\limits_{n=1}^{5} T_n. \end{cases} \tag{12}$$

MM (12) can be written in compact form:

$$\varphi_n(t)|_{n=[1,5]} = 2\pi \times \\ \times \left\{\left(f_0 + \sum_{1}^{n-1}\Delta f_{n-1}\right)\left(t - \sum_{1}^{n-1} T_{n-1}\right) + \right. \\ \left. + \beta_n\left(\frac{t^2}{2} - t\sum_{1}^{n-1} T_{n-1}\right) - \delta\varphi_{(n-1)n}\right\}, \\ \sum_{1}^{n-1} T_{n-1} \le t \le \sum_{1}^{n} T_n. \tag{13}$$

Verification of theoretically obtained results is performed by construction and analysis of AFS and ACF NLFM signals using MMs (1), (2), (7), (13) and a known MM LFM signal [1–4].

3.3 Results of mathematical modeling

The performance check of the proposed MM (13) and the comparative analysis (1), (2), (7) were carried out using the MATLAB application software package. When modeling, the frequency-time parameters of the signals were chosen to be identical, that is, the total deviation and duration of the NLFM signals coincided with the duration and deviation of the LFM signal frequency.

During the studies, the following were modeled:

- LFM signal duration 120 μs with frequency deviation 500 kHz;

- three-fragment NLFM signals by MM (1), (2) (Fig. 1, 2) and by MM (7) (Fig. 3) with fragment duration $T_1 = T_3 = 15$ μs, $T_2 = 90$ μs and their frequency deviations $df_1 = df_3 = 100$ kHz, $df_2 =$ 300 kHz;

- five-fragment NLFM signal (13), duration of fragments of which $T_1 = T_2 = T_4 = T_5 = 10$ μs, $T_3 = 80$ μs, and their frequency deviations $df_1 = df_5 =$ 100 kHz, $df_2 = df_4 = 50$ kHz, $df_3 = 200$ kHz (Fig. 4).

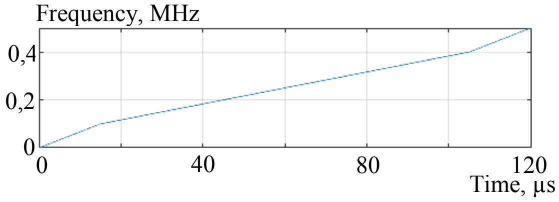
Only ACF measurement results were used for the LFM signal; no graphic material was given.

The graph of the change in the instantaneous frequency of the NLFM signal over MM (1), which is shown in Fig. 1a, allows us to conclude that there is no frequency distortion at the junction of fragments, which confirms the fact of automatic compensation of frequency jumps in MM shifted time, this graph also indicates that the change in the frequency of the synthesized signal occurs in a particular range.

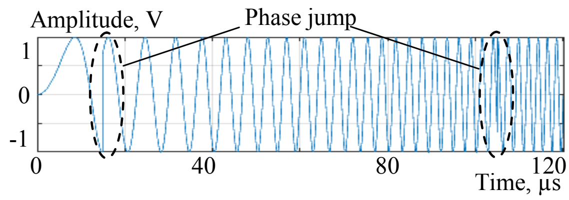
The oscillogram of the signal (Fig. 1b) shows significant instantaneous phase jumps at the joints of fragments, which cause substantial distortion of the resulting signal's AFS (Fig. 2a). The presence of phase jumps causes significant AFS dips at the fragment docking frequencies; ripples on their slopes are another sign of such jumps.

In the presence of large jumps of the instantaneous phase, the ACF in Fig. 2b has a high MPSL, the side lobe level has a chaotic distribution, which is a sign of phase distortions, and the phase change is also indicated by the presence of "steps" of ACF slopes.

The results of studies MM (7) and (13) confirm the assumption of the effect of phase jumps at the joints of fragments and the number of fragments on the appearance of the AFS and MPSL ACF of the resulting signal.

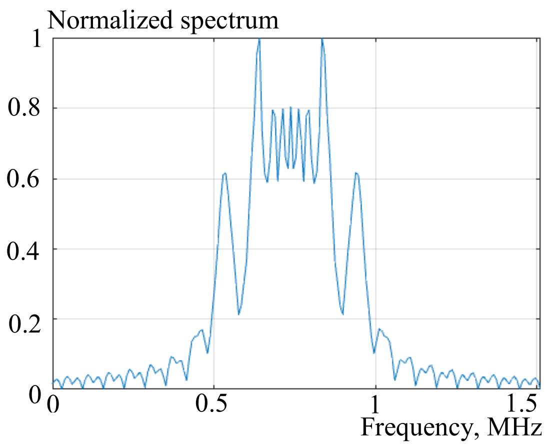


(a)

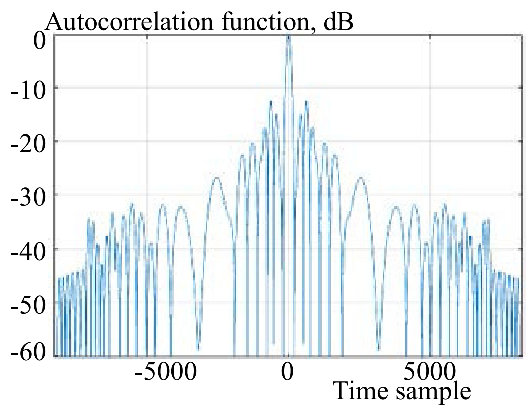


(b)

Fig. 1. Plot of the instantaneous frequency variation (a), MM (1), and the oscillogram signal (b), MM (2)



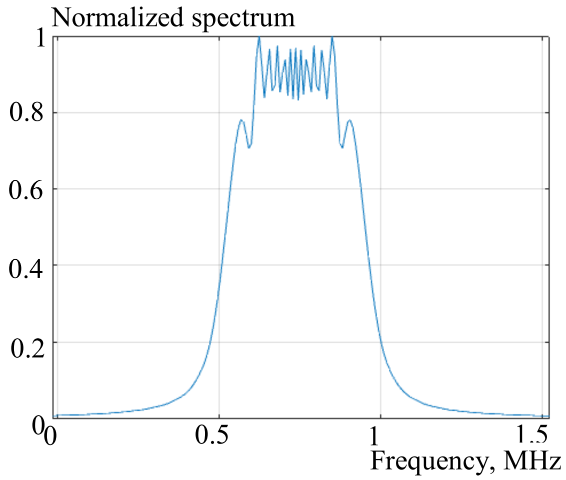
(a)



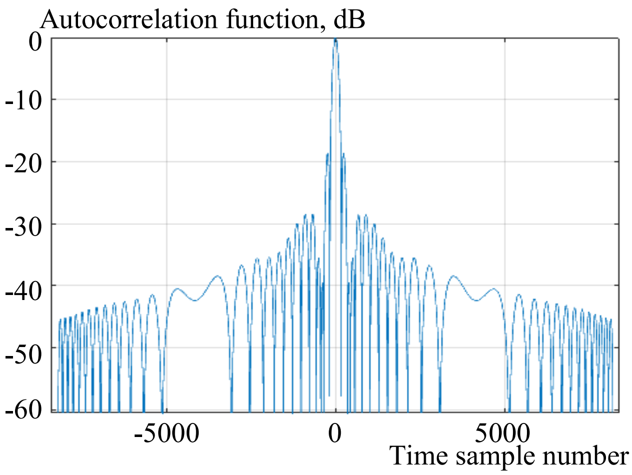
(b)

Fig. 2. Normalized spectrum (a) and ACF (b) of a three-fragment signal, MM (2)

The signal spectrum MM (7) of Fig. 3a was rounded, which ensured a decrease in MPSL of its ACF (Fig. 3b).



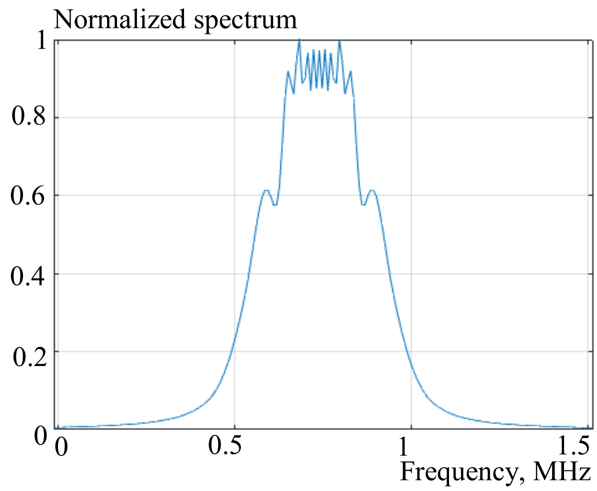
(a)



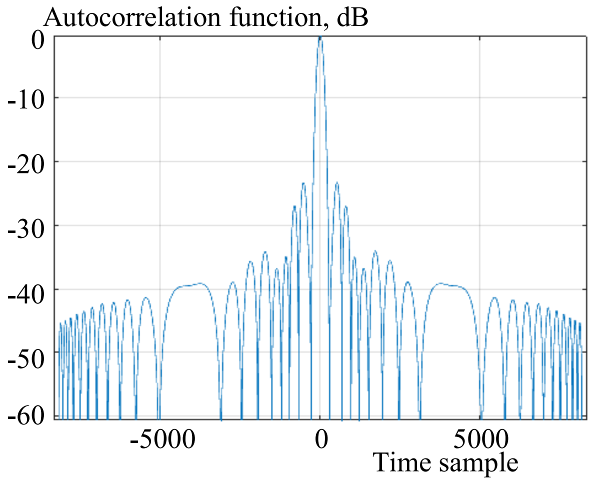
(b)

Fig. 3. Normalized spectrum (a) and ACF (b) of a three-fragment signal, MM (7)

MM (13) is investigated for the first time, since in the known literature, there is no information about the existence of MM shifted time of five-fragment NLFM signals. As predicted, increasing the number of fragments of the NLFM signal to five decreased the MPSL of its ACF (Fig. 4b). This is due to an even greater rounding of its spectrum (Fig. 4a) due to a smoother change in the SFM of the resulting signal.



(a)



(b)

Fig. 4. Normalized spectrum (a) and ACF (b) of a five-fragment signal, MM (13)

Analysis of the results shown in Table 1 supports the hypothesis that an increase in the number of LFM fragments of the NLFM signal can provide a significant decrease in the MPSL of its ACF, this is confirmed by the results obtained for MM shifted time NLFM signals with compensation for instantaneous phase jumps at the joints of fragments. The decrease in MPSL is accompanied by an expansion of the main lobe (ML) ACF, which is caused by a reduction of the effective spectrum width of NLFM signals. It is impossible to predict the nature of the change in the PSL decay rate, since it depends on the distribution of side lobe level and can change significantly with the change in the parameters of NLFM signals, both upwards and downwards.

Table 1 Comparison of ACF signal parameters

Parameter name / Signal type	LFM	NLFM-3 MM (2)	NLFM-3 MM (7)	NLFM-5 MM (13)
MPSL, dB	-13.47	-12.5 (+7%)	-18.7 (-39%)	-23.3 (-73%)
ML ACF width at level 0.707, μs	1.77	2.02 (+14%)	2.05 (+16%)	2.44 (+38%)
Rate of decline PSL, dB/dec	19.8	20.9 (+6%)	13.6 (-31%)	18.0 (-9%)

In the table, the three-fragment signal is marked as NLFM-3, five – NLFM-5, indicating which MM is used. The table charts indicate the percentage of the current parameter to the LFM signal case in brackets.

Conclusions

For the first time, a mathematical model in the shifted time of a five-fragment NLFM signal has been proposed and analytically substantiated, compensating for phase jumps at the joints of fragments. By increasing the number of fragments, it was possible to provide a better rounding of the resulting AFS signal, that is, a smoother change in its SFM.

It is analytically shown that the values of phase jumps at the joints of fragments depend on the duration and SFM of all previous fragments. For the first time, a relationship was obtained to determine the compensation components of instantaneous phase jumps at the joints between the third-fourth and fourth-fifth LFM fragments.

The results demonstrate an improvement in the spectral characteristics of the signal and a decrease in the MPSL of its ACF to -23.3 dB, which is almost 10 dB better than the standard LFM signal. The cost of lowering MPSL is to extend the maximum level ACF signal by 38% and reduce the PSL drop rate by 9%.

Such behavior is a well-known phenomenon: the reduction of the side lobe level is typically achieved at the expense of a partial degradation of the signal's range resolution, which manifests as an increase in the width of the mainlobe of the ACF. Determining the optimal balance between the mainlobe width and the side lobe level is a multi-objective optimization problem in the Pareto sense, and the resulting solution is always a compromise. Typically, to ensure the required range resolution, it is necessary to increase the signal bandwidth in advance, or, depending on the current radar task, to use different types of probing signals. Within the scope of the present study, the primary focus was on reducing the side lobe level, as this factor directly affects the quality of detecting weak target echoes against strong clutter from the ground. In particular, in radar systems for detecting low-altitude targets, this factor is dominant.

The model is of practical importance for radar systems detecting low-altitude targets, especially in non-stationary passive interference. Future research will focus on determining and justifying the optimal number of fragments, as well as on analyzing the effect of their number on the spectral characteristics of the signal.

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Особливості функціонування та синтезу математичних моделей зсунутого часу багатофрагментних нелінійно-частотно модульованих сигналів

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Відомим методом зниження максимального рівня бічних пелюсток автокореляційної функції сигналів з внутрішньо-імпульсною модуляцією частоти, наприклад, лінійно-частотно модульованих сигналів, є застосування їхньої вагової обробки в радіоприймальному пристрої. Альтернативою ваговій обробці є округлення амплітудно-частотного спектру, для чого застосовуються нелінійно-частотно модульовані сигнали. Відомо багато різновидів математичних моделей таких сигналів, однак задача синтезу нових математичних моделей сигналів з нелінійною частотною модуляцією не втрачає своєї актуальності і на сьогодні.

Авторами статті раніше синтезовано математичні моделі дво- та трифрагментних нелінійно-частотно модульованих сигналів, які забезпечують зниження рівня бічних пелюсток функції кореляції за рахунок компенсації частотно-фазових спотворень на стиках фрагментів. Визначено причини виникнення цих спотворень, обґрунтовано аналітично та перевірено шляхом моделювання механізми їх компенсації.

З метою подальшого зниження максимального рівня бічних пелюсток у роботі пропонується збільшити кількість лінійно-частотно модульованих фрагментів з трьох до п'яти, для чого розраховуються складові математичної моделі, які забезпечують компенсацію стрибків миттєвої фази результуючого сигналу на стиках всіх його фрагментів.

Структура роботи обумовлена логікою дослідження. У першому розділі роботи проведено аналіз відомих публікацій, який свідчить про відсутність математичних моделей зсунутого часу п'ятифрагментних нелінійно-частотно модульованих сигналів. З зазначеного витікає завдання дослідження, яке формулюється у другому розділі статті. Третій розділ роботи – теоретичний, його присвячено аналітичному визначенню компенсаційних складових для уникнення спотворень миттєвої фази у моменти переходу від одного фрагменту сигналу до наступного. Визначено, що вклад у формування фазових стрибків на стиках вносять частотно-часові параметри всіх попередніх фрагментів. Достовірність отриманих теоретичних результатів перевірено шляхом моделювання.

Ключові слова: нелінійна частотна модуляція; математична модель; стрибок миттєвої фази; автокореляційна функція; максимальний рівень бічних пелюсток