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Comparative Analysis of Spatial Spectral Estimation Algorithms: Advantages of the Maximum Entropy Method in Radio Direction Finding

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The current stage of development in radio monitoring systems is characterized by a transition toward Software Defined Radio and the requirement for high-precision direction-of-arrival (DoA) estimation under the constraints of limited antenna resources. The paper presents a comparative analysis of four spatial spectral estimation algorithms – Bartlett, Capon, MUSIC, and the Maximum Entropy Method (MEM) – implemented on the four-channel KerberosSDR hardware platform for direction finding of a real wideband DVB-T2 standard signal at a frequency of 482 MHz. The experimental results indicate that under the conditions of a limited antenna array aperture ($M = 4$) and real-world interference, the MEM algorithm achieved a higher level of spatial spectrum sidelobe suppression of $-20\ldots-40$ dB compared to the Bartlett, Capon, and MUSIC methods, for which the noise floor was $-6\ldots-11$ dB, with a bearing error of 5° relative to the true direction of 300°. A key practical limitation of the baseline DoA method implementations in the KerberosSDR environment was identified – excessive processing latency of the mathematical spatial spectrum computation block, caused by iterative loops at the Python interpreter level within the PyArgus library. It should be noted that the total spatial spectrum update time in the KerberosGUI environment is governed by a combination of hardware and software delays: IQ sample acquisition, data transfer over the USB interface, RAM-disk buffering, graphical user interface rendering, and spatial spectrum computation. Consequently, the proposed optimization targets specifically the acceleration of the algorithmic computation kernel. To overcome this limitation, an optimized computation kernel was developed based on tensor contraction using Einstein summation notation (`numpy.einsum`) and Tikhonov regularization regularization. Verification confirmed the mathematical equivalence of the tensor model: the mean square error without regularization was $1.05 \cdot 10^{-15}$, which lies within the range of double-precision floating-point computational accuracy. The transition from scalar loops to vectorized tensor operations provided a 31.4-fold acceleration in spatial spectrum computation – from 6.7 ms to 0.2 ms per snapshot — without any loss of accuracy.

Keywords: spatial spectral estimation, Maximum Entropy Method (MEM), tensor core, Einstein notation, Tikhonov regularization, KerberosSDR, super-resolution

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Introduction

The current stage of development in wireless communication and radio monitoring systems is characterized by a rapid transition toward Software Defined Radio (SDR) systems and the requirement for high-precision direction-of-arrival (DoA) estimation for radiation sources. Under the constraints of limited antenna resources, which is typical for mobile platforms, classical spectral analysis methods face a fundamental limitation: the broadening of the main peak due to the small array aperture [1–8]. This drives interest in super-resolution algorithms capable of overcoming the Rayleigh resolution limit [3, 9]. Traditional correlation algorithms, such as the Bartlett method and the adaptive Capon method, remain benchmarks for computational stability; however, they exhibit limited selectivity [2, 4, 10]. Subspace-based methods, particularly MUSIC (MUltiple SIgnal Classification), provide high accuracy but require a priori knowledge regarding the number of sources and are sensitive to phase distortions in real-world environments [5, 11]. In this context, the Maximum Entropy Method (MEM) becomes particularly relevant, as it allows for the formation of extremely narrow spatial peaks even with a critically low number of antenna elements [6, 12–14]. The objective of this work is a comparative performance evaluation of the Bartlett, Capon, MUSIC, and MEM algorithms within the KerberosSDR hardware platform for the direction-finding of a real wideband signal. The novelty of this work lies in the experimental validation of the advantages of the MEM entropy-based approach over correlation methods under real-world interference conditions. Particular attention is paid to the analysis of processing time delays, which enables the identification of computational bottlenecks in existing iterative algorithm implementations, laying the foundation for their subsequent tensor optimization.

1 Related Work

The development of non-linear spectral analysis methods in the second half of the 20th century became a prerequisite for the emergence of super-resolution algorithms, which allow for the estimation of spatial coordinates of radiation sources beyond the classical Rayleigh limit. Fundamental works [1, 2, 4, 5, 9, 11, 12, 15, 16] established the mathematical foundation for methods capable of resolving signals located at small angular distances from one another. Issues regarding the implementation of spatial signal processing algorithms in modern infocommunication systems are actively investigated by both domestic and international scientific schools [7, 8, 10]. However, the transition to super-resolution methods under conditions of limited antenna resources remains a pressing challenge. The fundamental limits of accuracy and the statistical efficiency of these methods – specifically the connection between the MUSIC algorithm and the Cramér-Rao Bound (CRB) — were analyzed in detail in the seminal work by Stoica P. and Nehorai A. [11]. The historical aspect of implementing the Maximum Entropy Method (MEM) in radar applications is presented in the works of Herring R. W. [12], where the advantages of MEM over classical Fourier-based methods when processing short data snapshots were first demonstrated. Modern research is directed toward refining the mathematical framework and adapting algorithms to new types of antenna arrays. The work of Qu X. et al. [17] confirms the superiority of tensor modeling over the classical matrix approach. The use of entropy-based approaches and correntropy for nested antenna arrays under impulsive noise conditions is examined by Zhao J. [18]. A promising but resource-intensive direction is the integration of Deep Learning methods, proposed by Zhao Y. [19], which enhances accuracy in low Signal-to-Noise Ratio (SNR) environments but requires significant computational power. A significant contribution to the development of iterative DoA estimation methods was made by Gong M. Y. and Lyu B. [20] through the adaptation of Expectation-Maximization (EM) statistical models. The issue of reducing computational complexity via covariance matrix decomposition was explored by Aounallah N. [21]. The current state of radio direction-finding algorithms and the trend toward Open Source Python-based software are systematized in a global review by Salama A. A. [6]. In the context of practical implementation, particular attention is given to the NumPy technology stack, which enables high-performance array operations, and specialized libraries such as pyArgus [22], which provide tools for signal processing based on SDR solutions like KerberosSDR [23]. We shall consider four algorithms that are currently the most technically mature and available for comparative study: Bartlett's method [9], the Capon method (MVDR) [4, 15], the MUSIC method [4,5,11,16], and the Maximum Entropy Method [12–14]. Bartlett's Method is a classical spectral approach based on maximizing the antenna array response in the direction of the source

$$P_{\text{Bartlett}}(\theta) = \mathbf{a}^H(\theta)\hat{\mathbf{R}}\mathbf{a}(\theta), \tag{1}$$

where θ denotes the direction of arrival (DoA) of the signal, $\mathbf{a}(\theta)$ is the steering vector of the antenna array with dimensions $M \times 1$, $(\cdot)^H$ denotes the Hermitian transpose operator, and $\hat{\mathbf{R}}$ is the estimated spatial covariance matrix of dimensions $M \times M$. The steering vector is assumed unnormalized, $\|\mathbf{a}(\theta)\|^2 = M$. The method exhibits limited angular resolution due to the Rayleigh resolution criterion, which restricts its ability to distinguish closely spaced signal sources located within the main lobe of the array radiation pattern.

The Capon Method (CAPON-MVDR – Minimum Variance Distortionless Response) [1, 15] is an algorithm that adaptively suppresses interference originating from side-lobe directions:

$$P_{\text{Capon}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta)}, \tag{2}$$

where $\hat{\mathbf{R}}^{-1}$ is the inverse of the estimated covariance matrix $\hat{\mathbf{R}}$. The method is sensitive to errors in the steering vector $\mathbf{a}(\theta)$ and requires a large number of samples for accurate estimation of the inverse covariance matrix $\hat{\mathbf{R}}^{-1}$.

The MUSIC method is a multiple signal classification algorithm based on the eigenvalues of the covariance matrix and the division of space into signal and noise subspaces:

$$P_{\text{MUSIC}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{a}(\theta)}, \tag{3}$$

where $\mathbf{E}_n$ denotes the noise subspace matrix composed of eigenvectors corresponding to the smallest eigenvalues of the covariance matrix. When the steering vector $\mathbf{a}(\theta)$ coincides with the actual signal direction, it becomes orthogonal to the noise subspace. As a result, the denominator approaches zero, leading to an infinitely sharp peak in the spatial spectrum. However, the MUSIC algorithm requires precise a priori knowledge of the number of signal sources and exhibits significant computational complexity due to the need to perform eigenvalue decomposition (EVD) of the covariance matrix.

The Maximum Entropy Method (MEM) [12–14] constructs the spectral estimate by extrapolating the autocorrelation function in a manner that maximizes the entropy of the underlying process. This approach yields high resolution by enforcing the most uniform spectral distribution consistent with the observed data. In contrast to subspace-based methods such as MUSIC, MEM does not require eigenvalue decomposition of the covariance matrix.

For a linear antenna array, the classical Herring formulation is given by:

$$P_{\text{MEM}}(\theta) = \frac{P_M}{\left|1 + \sum_{k=1}^{M-1} a_k e^{-j2\pi dk \sin\theta}\right|^2}, \quad (4)$$

where P_M denotes the prediction error power, a_k are the linear prediction (autoregressive) coefficients estimated using Burg's algorithm, d is the inter-element spacing normalized to the wavelength, $\sin\theta$ represents the spatial frequency corresponding to the angle of arrival relative to the array broadside, and M is the number of antenna elements.

It should be noted that the KerberosSDR implementation [22, 23] does not employ the classical MEM in its standard form. Instead, it utilizes a modified approach commonly referred to in the literature as a maximum entropy estimator based on a selected column of the inverse covariance matrix.

This approach is closely related to MEM through linear prediction theory. Specifically, for $j = 0$, the vector $\mathbf{c}_0$ corresponds to the first column of the inverse covariance matrix, which is equivalent to the linear prediction coefficients associated with the reference sensor.

The spatial spectrum is given by:

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\, \mathbf{c}_j \mathbf{c}_j^H\, \mathbf{a}(\theta)}, \quad (5)$$

where $\mathbf{c}_j = \mathbf{R}^{-1}[:, j]$ denotes the j-th column of the inverse covariance matrix, and $\mathbf{a}(\theta)$ is the steering vector.

Here, $\mathbf{c}_j\mathbf{c}_j^H$ represents a rank-one matrix formed via dyadic (outer) multiplication. The parameter j determines the reference element used for linear prediction, with $j = 0$ corresponding to the first antenna element.

Due to its dependence on the linear prediction vector $\mathbf{c}_j$, the method is sensitive to signal phase variations caused by target motion. While this property may allow faster adaptation in dynamic scenarios, it can also lead to performance degradation in the presence of phase instability.

2 Problem Statement

Despite the profound theoretical development of mathematical models for spectral estimation, their practical implementation in real-time systems across various hardware platforms remains a critical challenge. To determine the influence of each method's mathematical model on the quality of the spatial spectrum, an experimental comparative study will be conducted using a real DVB-T2 standard signal (482 MHz) via the KerberosSDR four-channel SDR platform [23]. To evaluate method selectivity in real-world environments, it is necessary to obtain spatial spectra that reveal the relationship between the correlation matrix processing type and the system's ability to form a stable Direction of Arrival (DoA) vector toward the radiation source. The results will allow for a comparison of the methods based on sidelobe suppression levels and angular accuracy under conditions of limited antenna array aperture. Most existing implementations of super-resolution methods rely on iterative loops, causing excessive computational load on the Central Processing Unit (CPU) and hindering high angular resolution in dynamic environments. This problem is particularly relevant when addressing the following complex tasks in radio direction finding:

- Source localization in complex interference environments, requiring high-precision DoA estimation in dense urban areas characterized by intensive multipath propagation.

- Time-critical and wideband processing, where handling wideband signals over very short time intervals requires fast DoA estimation algorithms for timely target identification.

- Stabilization under small sample sizes and correlated signals, where limited input data often leads to a singular covariance matrix $\mathbf{R}$, making classical inversion methods numerically unstable and causing artifacts in the spatial spectrum.

3 Research results

The radio direction-finding process in an SDR system is a multi-level digital signal processing chain, which can be divided into four key logical stages. Let us examine the stages of spatial spectrum estimation in detail, focusing on their computational implementation [22].

Stage 1 – Formation of the spatial covariance matrix. The first step involves collecting complex IQ samples from four coherent channels. The system forms a data vector

$$\mathbf{x}(n) = [x_1(n), x_2(n), x_3(n), x_4(n)]^T, \quad (6)$$

where n denotes the discrete time index, and $x_i(n)$ is the complex-valued signal $(I + jQ)$ received at the i-th antenna element.

The basis of all data processing algorithms is the estimation of the covariance matrix $\hat{\mathbf{R}}$. We calculate the values of the matrix elements using the formula:

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{x}(n)\mathbf{x}^H(n), \tag{7}$$

where $\mathbf{x}(n)$ is the vector of complex samples obtained from the antenna array at the n-th time instant, N is the number of samples, and $(\cdot)^H$ denotes the Hermitian transpose operator. This matrix contains all the information about the phase relationships between the antenna elements, and serves as the basis for angle estimation. The procedure of forming the covariance matrix is a fundamental stage in modern space-time signal processing algorithms.

Stage 2 – Generation of steering vectors. To scan the angular space θ, steering vectors $\mathbf{a}(\theta)$ are generated. For a Uniform Linear Array (ULA), where the elements are arranged along a single line with a spacing d. For the case of half-wavelength spacing ($d = \lambda/2$),

$$\mathbf{a}_{\mathrm{ULA}}(\theta) = \left[1,\ e^{j\pi \sin\theta},\ \ldots,\ e^{j(M-1)\pi \sin\theta}\right]^T, \tag{8}$$

where θ is the angle of arrival relative to the array broadside.

For a Uniform Circular Array (UCA), which is frequently used in KerberosSDR to achieve a 360° panoramic view, the mathematics of forming the steering vectors $\mathbf{a}(\theta)$ differs from that of a linear array. Unlike a linear array, where the phase shift depends solely on the distance along a single axis, in a circular array, each antenna has its own coordinates (x_m, y_m) in the plane. If M is the number of antennas and r is the radius of the circle in wavelengths λ, then the position of the m-th antenna is defined as:

$$x_m = r \cdot \cos\left(\frac{2\pi m}{M}\right), \quad y_m = r \cdot \sin\left(\frac{2\pi m}{M}\right). \tag{9}$$

When a plane wave arrives at an angle θ at an antenna with coordinates (x_m, y_m), the phase shift is determined by the projection of the wave vector onto the array plane:

$$\psi_m(\theta) = 2\pi\left(x_m \cos\theta + y_m \sin\theta\right), \tag{10}$$

For the KerberosSDR with antennas arranged in a circle of radius r, the phase shift depends on the coordinates given in (9) of each antenna, which is mathematically defined by the dot product of the wave direction vector and the antenna's radius vector:

$$\psi_m(\theta) = 2\pi r \cos(\theta - \phi_m), \quad \phi_m = \frac{2\pi m}{M}, \tag{11}$$

where r is the radius normalized to the wavelength λ, θ is the scanning angle and ϕ_m is the angular position of the m-th antenna on the circle. For the m-th element of the Uniform Circular Array (UCA), the steering vector formula is given by:

$$a_m(\theta) = e^{j\cdot 2\pi r\cdot\cos(\theta-\phi_m)}, \quad m = 0, \ldots, 3. \tag{12}$$

The steering vector for the Uniform Circular Array (UCA) is expressed as follows:

$$\begin{aligned}\mathbf{a}_{\mathrm{UCA}}(\theta) &= \left[a_0(\theta),\ a_1(\theta),\ a_2(\theta),\ a_3(\theta)\right]^T \\ &= \left[e^{j\psi_0(\theta)},\ e^{j\psi_1(\theta)},\ e^{j\psi_2(\theta)},\ e^{j\psi_3(\theta)}\right]^T.\end{aligned} \tag{13}$$

Stage 3 – Computation of spectral function values using a selected algorithm. The estimated covariance matrix $\hat{\mathbf{R}}$ [24] is processed using a selected direction-of-arrival estimation algorithm, such as Bartlett, Capon, MEM, or MUSIC, to compute the corresponding spatial spectrum values.

Stage 4 – Logarithmic transformation and normalization for visualization. The resulting spectral values $P(\theta)$ are normalized and converted to a logarithmic scale (dB) for visualization in KerberosGUI:

$$P_{\mathrm{dB}}(\theta) = 10\log_{10}\left(\frac{P(\theta)}{\max(P(\theta))}\right). \tag{14}$$

This normalization yields a relative power level rather than an absolute measure. Specifically, all values are expressed with respect to the maximum of the main lobe, such that the peak corresponds to 0 dB. This representation enables a clear comparison of the resolution between different algorithms, as it facilitates the evaluation of interference suppression levels relative to the desired signal. An experiment was conducted using a DVB-T2 standard radio signal transmitted from a television tower at a frequency of 482 MHz. The distance between the receiver and the source was approximately 2.8 km, and the true direction of arrival (DoA) was approximately 300°.

The hardware setup consisted of a KerberosSDR four-channel phase-synchronized direction finder. The antenna array geometry was a Uniform Circular Array (UCA) with a radius of $r = 0.333\,\lambda$ [23], which corresponds to an inter-element spacing of approximately 0.21 m ($0.333\,\lambda$). To form the four-element circular antenna array, high-gain wideband antennas (12 dBi) equipped with magnetic bases and SMA connectors were utilized. The vertical configuration of these whip antennas ensured compatibility with vertically polarized signals, consistent with terrestrial television broadcasting standards.

The processing parameters in KerberosGUI were configured as follows: the sample size for covariance matrix estimation was set to $2^{13} = 8192$ samples, providing sufficient statistical accuracy for super-resolution methods. The sampling rate was set to 1.024 MHz, as recommended for direction-finding operation. The receiver gain was fixed at 40.2 dB. A digital FIR filter was used for signal conditioning prior to processing, with a passband of 150 kHz, filter type 0, and a decimation factor equal to 1. The appearance of the deployed KerberosSDR experimental setup is shown in Fig. 1.

Fig. 1. Photo of the experimental setup

To correctly interpret the experimental results, it is necessary to consider the specific visualization features of the KerberosSDR GUI. Figure 2 presents the simultaneous outputs of the four algorithms, where each curve corresponds to a particular mathematical model used for processing the covariance matrix estimate $\hat{\mathbf{R}}$.

A key indicator of the reliability of the obtained results is the *Averaged Spectrum* curve, which combines the outputs of all active methods, thereby mitigating the effects of instantaneous phase distortions inherent in real-world environments. This is particularly important for wideband signals, where power fluctuations may lead to peak drift in individual algorithms. A detailed analysis of each spectral response in the vicinity of the estimated bearing (approximately 295°) confirms proximity to the true direction of 300°. The absolute bearing error relative to the true direction of $\theta_{\text{true}} = 300°$ was $|\hat{\theta} - \theta_{\text{true}}| = |295° - 300°| = 5°$.

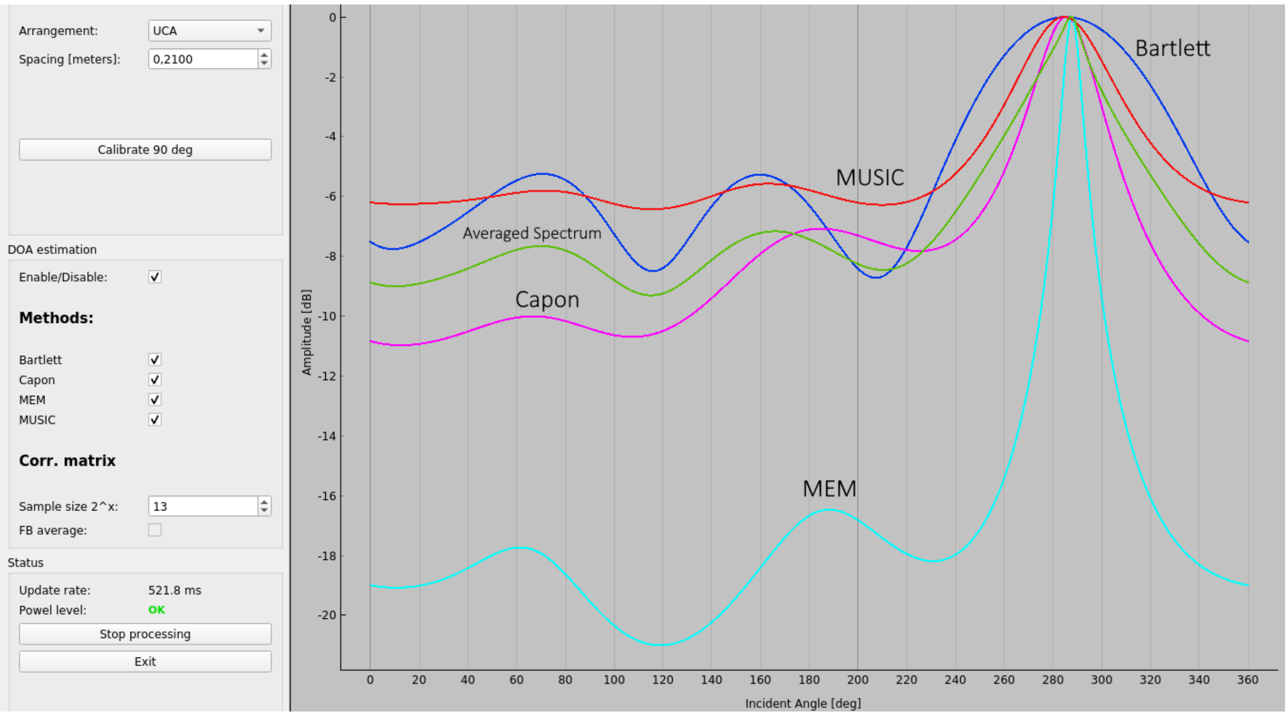


Fig. 2. Signal direction finding results (482 MHz) using the KerberosSDR system

Analysis of the obtained results (Fig. 2) confirms the significant advantage of the MEM method in radio direction-finding tasks involving a limited antenna array aperture. The sidelobe level of the spatial spectrum $P(\theta)$ for MEM reaches approximately -20 dB, whereas the Bartlett, Capon, and MUSIC methods exhibit a considerably higher background noise floor in the range of -6 to -11 dB. The main lobe formed by the MEM method is substantially narrower compared to its counterparts, providing super-resolution angular accuracy even when utilizing the budget-grade KerberosSDR platform. Upon activation of all four direction-of-arrival (DOA) estimation algorithms (Bartlett, Capon, MEM, MUSIC), the KerberosSDR software enters a parallel spatial power spectrum computation mode. Five vectors are displayed simultaneously: four correspond to the individual estimates of the spatial spectrum $P(\theta)$ obtained by each method, where the spectral maximum indicates the most probable direction to the signal source according to the respective mathematical model; the fifth vector, referred to as *Averaged Spectrum* (weighted average), represents a composite integral estimate.

The directional integral estimate is formed by synthesizing the angular solutions of all active methods, with each algorithm's contribution weighted proportionally to its statistical confidence, typically reflected by the amplitude of its spectral peak. The *Averaged Spectrum* estimate is formed using a circular weighted mean of the angular solutions provided by each active DoA method. Unlike a conventional arithmetic mean, which yields erroneous results in the vicinity of the 0°/360° boundary, the vector-based averaging approach operates on the unit circle and is therefore free from angular discontinuities. Formally, the composite bearing estimate θ_{avg} is defined as the argument of the resultant weighted vector:

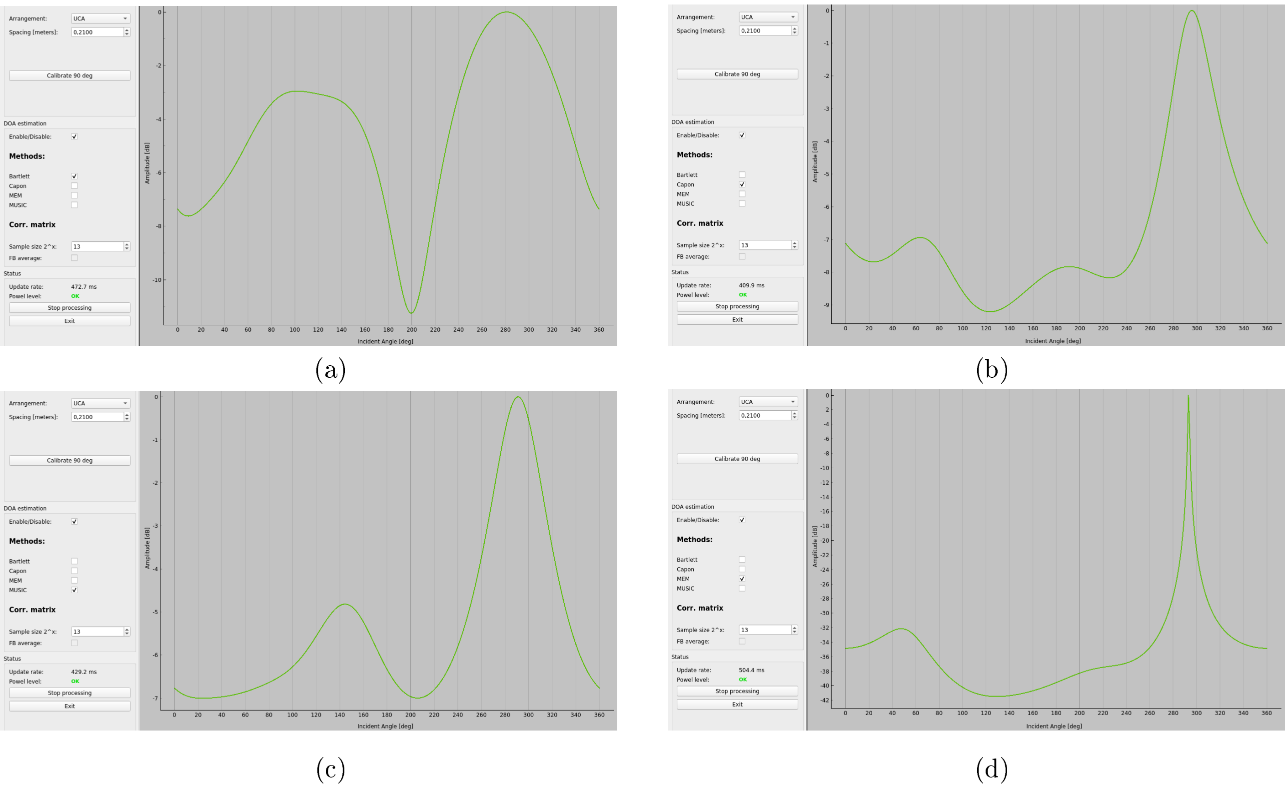
$$\theta_{avg} = \operatorname{atan2}\left(\sum_{i=1}^{n} w_i \sin(\theta_i),\ \sum_{i=1}^{n} w_i \cos(\theta_i)\right), \quad (15)$$

where n is the number of active estimation methods, θ_i is the bearing angle at which method i attains its spectral maximum, and w_i is the corresponding weight coefficient, proportional to the relative peak amplitude of the spatial spectrum in linear scale, serving as an SNR-like confidence metric. This formulation ensures that methods exhibiting higher spectral peak amplitudes — and thus greater statistical confidence in their angular estimates — contribute more significantly to the composite bearing solution. As a practical consequence, if one method yields 359° and another yields 1°, the resultant estimate correctly resolves to 0°, whereas a naive arithmetic mean would produce the erroneous value of 180°. This approach prioritizes methods with the highest signal-to-noise ratio (notably MEM), thereby suppressing random spectral fluctuations and enhancing bearing stability under multipath signal propagation conditions.

Evaluation of the temporal characteristics revealed that when all four algorithms operate simultaneously, the data update rate decreases to approximately 1.9 Hz, corresponding to a latency of 521.8 ms. The stability of the update time with an increasing number of active methods is attributable to the architectural design of the data processing pipeline: the covariance matrix of signals from all four channels is computed once per data block, after which all methods utilize it as a shared input.

Owing to the vectorized operations provided by linear algebra libraries (NumPy/SciPy), the computation of the objective functions $P(\theta)$ incurs minimal additional overhead. Consequently, the update time is governed primarily by the length of the I/Q sample block required to ensure statistical significance of the estimate, as well as by the data transfer time over the USB interface, rather than by the speed of the mathematical operations themselves. The inter-process communication between the `C++` signal processing backend and the Python/Dash frontend in KerberosSDR is implemented via shared files mounted on a `tmpfs` RAM-disk. The backend continuously acquires coherent IQ sample blocks of configurable size (2^{10}–2^{13} samples per channel per frame) from four RTL-SDR tuners, performs phase alignment, constructs the spatial covariance matrix $\hat{\mathbf{R}}$, and writes the resulting bearing estimate to a shared file at rates of several frames per second. The use of a RAM-disk, rather than a physical storage device, is necessitated by three factors: the elimination of I/O-induced latency that would otherwise cause buffer overflow in the real-time `C++` pipeline; the prevention of excessive write-cycle wear on solid-state storage; and the enabling of high-throughput, low-latency file access between concurrently executing processes within the Linux filesystem. It should be emphasized that the observed latency does not represent a simple summation of the individual computation times of each method; instead, the primary computational overhead arises from redundant iterative *for*-loops during data processing and subsequent data transfer to the Graphical User Interface (GUI) within the Python environment.

Thus, the MEM method provides an optimal trade-off between angular resolution and interference suppression, while the total system response time of approximately 0.5 s is additionally governed by a combination of hardware and software delays: IQ sample acquisition, data transfer over the USB interface, RAM-disk buffering, spatial spectrum computation from the estimated covariance matrix $\hat{\mathbf{R}}$, and rendering of the results in the graphical user interface. For a more in-depth performance analysis, spatial spectra were recorded for each method in individual operating mode during direction-finding of the 482 MHz signal (Fig. 3).



(a) (b) (c) (d)

Fig. 3. Spatial power spectra of signals obtained by the methods: a) Bartlett; b) Capon; c) MUSIC; d) MEM

A comparative analysis of the obtained spatial spectra (Fig. 3) allows for the evaluation of each algorithm's capability to resolve radiation sources under limited aperture conditions. The Bartlett method (Fig. 3a) demonstrates the lowest resolution. The wide main lobe and significant energy leakage into the sidelobe regions (approximately 60°—140°) indicate poor selectivity, making precise direction finding challenging when multiple sources are located at closely spaced angular positions.

The Capon method (Fig. 3b) provides improved interference suppression. The spectral curve is smoother, and the main peak is noticeably narrower compared to the classical beamformer. The algorithm effectively suppresses noise in the rear hemisphere (approximately 120°—240°), but is prone to forming shallow sidelobes, which slightly reduces the overall bearing contrast. The MUSIC method (Fig. 3c) exhibits high angular resolution but reveals sensitivity to signal propagation conditions. The presence of an artifact (false peak) near 145° suggests that, under multipath conditions, subspace-based methods may interpret reflected signals as distinct sources, requiring additional filtering. The Maximum Entropy Method (MEM) (Fig. 3d) demonstrates the highest selectivity. The peak in the direction of the source (approximately 295°) is the sharpest among all studied methods, and the noise floor decreases to approximately −40 dB. This confirms the superiority of Burg's autoregressive approach in tasks involving the extraction of weak signals in dense interference environments. Although MUSIC is often considered the industry standard, the experimental results obtained using the KerberosSDR platform indicate the superior robustness of the MEM method. It exhibits improved temporal stability of the peak position and the highest energy concentration capability, which is particularly important for compact antenna arrays. The generalized results of the comparative analysis based on key technical metrics are presented in Table 1.

TABLE 1 Comparative analysis of spatial spectra of DoA methods

Method	**Resolution**	**Selectivity (peak width)**	**Noise level (background)**	**Features at 482 MHz**	**Update time**
Bartlett	Low	Low (wide)	-8...-11 dB	High noise immunity, but low accuracy	472.7 ms
Capon	Medium	Medium	-9...-11 dB	Good side noise rejection, but blurs the peak	409.9 ms
MUSIC	High	High	-6...-7 dB	Sensitive to multipath (appearance of false peaks)	429.2 ms
MEM	Ultra High	Ultra High	-20...-40 dB	Best resolution and spectral purity.	504.4 ms

*Update time includes IQ sample acquisition, USB interface transfer, RAM-disk buffering, mathematical computation, and real-time GUI rendering.

Since the accuracy and stability of radio direction finding results directly depend on the chosen mathematical apparatus, this work provides a comprehensive comparative analysis of algorithms integrated into the KerberosSDR GUI software environment via the PyArgus library. The implementation of the algorithms of the 4 methods considered is given in Table 2.

TABLE 2 Table of algorithmic solutions of the considered methods

Algorithm 1: DoA estimation using Bartlett's method	**Algorithm 2: DoA estimation using the Capon method**
Input: Covariance matrix $\mathbf{R}$ of size $M \times M$, matrix of steering vectors $\mathbf{A}$. **Output:** Spatial power spectrum $S_{Bartlett}$. 1. Initialize the result vector $S_{Bartlett}$ with complex zeros, with a dimension corresponding to the number of scanning angles. 2. Loop for each steering vector $\mathbf{a}$ in matrix $\mathbf{A}$: a. Calculate the scalar power value using the quadratic form: $P(\theta) = \mathbf{a}^H \mathbf{R} \mathbf{a}$, where $\mathbf{a}^H$ denotes the Hermitian transpose of the vector $\mathbf{a}$ (implemented as np.conj(a).T). b. Store the obtained value in the corresponding spectrum index. 3. End of loop. 4. **Return** the vector $S_{Bartlett}$.	**Input:** Covariance matrix $\mathbf{R}$ of size $M \times M$, matrix of steering vectors $\mathbf{A}$. **Output:** Spatial power spectrum S_{Capon}. 1. Singularity check: Calculate the inverse covariance matrix $\mathbf{R}^{-1}$. If the matrix is singular (ill-conditioned), exit with an error. 2. Initialize the result vector **ADSINR** (Angular Dependent SINR). 3. Loop for each steering vector $\boldsymbol{a}$ in matrix $\mathbf{A}$: a. Calculate the quadratic form using the inverse matrix: $D(\theta) = \mathbf{a}^H \mathbf{R}^{-1} \mathbf{a}$. b. Store the intermediate result in an array. 4. End of loop. 5. Final calculation: Apply a reciprocal operation to each element of the array: $S_{Capon}(\theta) = \frac{1}{\mathbf{a}^H \mathbf{R}^{-1} \mathbf{a}}$. 6. **Return** the vector S_{Capon}.
Algorithm 3: DoA estimation using the MUSIC method	**Algorithm 4: DoA estimation using the MEM method**
Input: Covariance matrix $\mathbf{R}$ of size $M \times M$, steering vector matrix $\mathbf{A}$, number of signal sources $\mathbf{K}$ (signal_dimension). **Output:** Spatial power spectrum S_{MUSIC}. 1. Eigenvalue Decomposition (EVD): Compute the eigenvalues σ_i and eigenvectors $\mathbf{v}_i$ of matrix $\mathbf{R}$. 2. Sorting: Sort the resulting pairs in ascending order based on the magnitudes of the eigenvalues. 3. Formation of the Noise Subspace: Extract the first $\mathbf{M} - \mathbf{K}$ eigenvectors, which correspond to the smallest eigenvalues, to form the noise subspace matrix $\mathbf{E}_n$. 4. Orthogonality Matrix Calculation: Form the projection matrix of the noise subspace: $\mathbf{P}_n = \mathbf{E}_n \mathbf{E}_n^H$. 5. Initialize the result vector ADORT (Angular Dependent Orthogonality). 6. Loop for each steering vector $\mathbf{a}$ in matrix $\mathbf{A}$: a. Calculate the orthogonality measure of the steering vector relative to the noise subspace: $f(\theta) = \mathbf{a}^H \mathbf{P}_n \mathbf{a}$. b. Calculate the pseudospectrum value: $S_{MUSIK}(\theta) = \frac{1}{\lvert f(\theta) \rvert}$. 7. End of loop. 8. **Return** the vector S_{MUSIC}.	**Input:** Covariance matrix $\mathbf{R}$ of size $M \times M$, steering vector matrix $\mathbf{A}$, selected column index j (default $j=0$). **Output:** Spatial power spectrum S_{MEM}. 1. Matrix Inversion: Compute the inverse covariance matrix $\mathbf{R}^{-1}$. 2. Projection Matrix Formation: Extract the j-th column of the matrix $\mathbf{R}^{-1}$ (denoted as c_j) and calculate its outer product (dyadic product): $\mathbf{R}_{invc} = \mathbf{c}_j \mathbf{c}_j^H$, where $\mathbf{c}_j^H$ is the complex-conjugate row. 3. Initialize the result vector PAD (Projection-based Angular Distribution). 4. Loop for each steering vector $\mathbf{a}$ in matrix $\mathbf{A}$: a. Calculate the spectral response via the quadratic form: $P(\theta) = \mathbf{a}^H \mathbf{R}_{invc} \mathbf{a}$. 5. End of loop. 6. Final Calculation: Take the reciprocal of the result to obtain sharp peaks at the denominator's minima: $S_{MEM}(\theta) = \frac{1}{\mathbf{a}^H \mathbf{R}_{invc} \mathbf{a}}$. 7. **Return** the vector S_{MEM}.

Analysis of the baseline implementation of Direction-of-Arrival (DoA) estimation algorithms in hardware-software complexes using the PyArgus library (Table 2) revealed a significant limitation: the reliance on iterative *for*-loops to calculate the spatial spectrum. Specifically, the Maximum Entropy Method (MEM), despite its high resolution and low sidelobe levels, demonstrated an excessive mathematical kernel processing delay, which contributes to the total observed system latency of 504.4 ms alongside IQ acquisition, USB transfer, RAM-disk buffering, and GUI rendering overhead. Although the theoretical computational complexity of the method is $O(M^3 + N \cdot M^2)$ [6], where N is the number of scanning angles and M is the number of antenna elements, its practical implementation within the interpreted Python environment creates unacceptable latencies for real-time radar systems. To overcome this computational barrier and ensure system operation in a real-time processing mode, the authors transitioned from an iterative computing paradigm to a tensor one. Based on the core algorithms of the PyArgus library [22], which is built upon NumPy [25, 26], an optimized computing kernel was developed. The vectorized tensor implementation must preserve mathematical equivalence with the classical iterative formulation:

$$\max_i |P_{\text{legacy}}(\theta_i) - P_{\text{tensor}}(\theta_i)| < 10^{-10}, \qquad (16)$$

where $P_{\text{legacy}}(\theta_i)$ is the spatial spectrum value obtained using the classical iterative implementation for the scanning angle θ_i, $P_{\text{tensor}}(\theta_i)$ is the corresponding spectrum value obtained using the proposed tensor-vectorized implementation, and the inequality threshold 10^{-10} defines the maximum numerical deviation required to ensure mathematical equivalence between the implementations. The key feature of this kernel is the replacement of resource-intensive scalar FOR loops with vectorized tensor operations using Einstein notation (the numpy.einsum library [25, 27, 28]). This approach effectively maps the spatial spectrum computation onto optimized linear algebra kernels, leveraging the General Matrix-Matrix Multiplication (GEMM) paradigm. This allowed the processing of spatial spectra to be moved into the domain of multidimensional arrays, radically reducing the computational overhead of the Python interpreter.

Mathematical Description of the Optimized MEM Algorithm

The fundamental distinction of the developed kernel is the integration of the Tikhonov-Phillips regularization procedure [29, 30], which guarantees the numerical stability of the solution when processing covariance matrices. The application of full computational vectorization via Einstein tensor notation enables the implementation of this complex mathematical framework without compromising real-time processing throughput.

DoA Estimation Algorithm Using the Optimized MEM Method

Input: Covariance matrix $\mathbf{R}$ of size $M \times M$, steering vector matrix $\mathbf{A}$ of size $M \times P$, regularization parameter $\alpha = 10^{-6}$, column index j.

Output: Spatial power spectrum S_{MEM}.

1. **Correlation Matrix Regularization [29, 30]:** To prevent ill-conditioning of the matrix $\mathbf{R}$, a regularization parameter α (Tikhonov coefficient) is added:

$$\mathbf{R}_{\text{reg}} = \mathbf{R} + \alpha \mathbf{I},$$

where $\mathbf{I}$ is the identity matrix.

2. **Formation of the Weight Vector and Matrix [24]:** Based on the inverse regularized matrix, a weight vector $\mathbf{c}_j$ (the j-th column) is constructed, and its outer product is computed to obtain the intermediate matrix $\mathbf{R}_{\text{invc}}$:

$$\mathbf{R}_{\text{inv}} = \mathbf{R}_{\text{reg}}^{-1}, \quad \mathbf{c}_j = \mathbf{R}_{\text{inv}}[:, j], \quad \mathbf{R}_{\text{invc}} = \mathbf{c}_j \mathbf{c}_j^H.$$

3. **Tensor Vectorization [22, 25–28]:** Simultaneous computation of the denominator for all angles using Einstein notation. Instead of iterating over steering vectors in matrix $\mathbf{A}$, a tensor contraction is evaluated:

$$D_p = \sum_m \sum_k \overline{\mathbf{A}}_{mp} \, (\mathbf{R}_{\text{invc}})_{mk} \, \mathbf{A}_{kp}.$$

where p is the angle index ($p = 1, \ldots, P$), and $m, k \in \{1, \ldots, M\}$ are the spatial indices of the antennas (with M being the total number of antennas). Here, $\overline{\mathbf{A}}_{mp}$ represents the complex conjugate of the steering matrix element corresponding to the m-th antenna and the p-th angle, and $\mathbf{A}_{kp}$ is the steering matrix element for the k-th antenna and the p-th angle.

4. **Final Calculation:** To obtain sharp peaks at energy maxima, the reciprocal of the resulting value is computed:

$$S_{MEM}(p) = \frac{1}{D_p}.$$

5. **Return** the vector S_{MEM}.

The proposed transition from a vector-matrix to a tensor paradigm has transformed the MEM method from a theoretically accurate but computationally heavy algorithm into a highly efficient tool for radio direction finding. This approach minimizes interpreter overhead by offloading computations to low-level linear algebra libraries.

To verify the developed algorithm, a series of numerical experiments was conducted. The objectives were to confirm the mathematical equivalence of the tensor model relative to the classical iterative MEM implementation and to evaluate the net performance gain of the computing kernel. Testing was carried out in an isolated virtual environment (venv, Python 3.12.3) to minimize the impact of background operating system processes and ensure the reproducibility of the benchmarking results. The simulations were performed in Python 3.12.3 on a Ryzen 7 2700 processor, mimicking the computing node of the KerberosSDR system.

Testing Methodology

1. **Reference Data Generation:** Creation of a synthetic correlation matrix and a set of steering vectors that fully replicate the physical parameters of a four-element antenna array ($M = 4$). The signal length was set to $N = 256$ snapshots per frame, consistent with the operational parameters of the KerberosSDR system.

2. **Mathematical Validation:** Comparing the performance of the iterative MEM algorithm (FOR loops) and the tensor-based MEM model [22, 25–28] using identical input data. Scanning was performed across an angular range of $\theta \in [-90°; 90°]$ with a step of 0.1°, forming a spectrum vector of 1801 points. The identity of the output spectra $P(\theta)$ was strictly monitored: results obtained from both methods were compared using the maximum absolute deviation metric, and a result was considered mathematically valid under the condition (16), confirming the absence of distortions introduced by vectorization.

3. **Benchmarking:** Timing analysis included a preliminary "cold" start to account for library initialization and instruction caching. Each measurement cycle consisted of 100 iterations per method. The primary metrics were the average frame processing time (T_{avg}, ms) and the equivalent spectrum update frequency (frames per second). Results are presented in Table 3.

TABLE 3 Results of mathematical modeling of computational efficiency of MEM methods

Simulation parameter	**MEM algorithm (PyArgus)**	**Optimized MEM algorithm**	**Results**
Computation method	Python for loop iterative	Vectorization einsum (tensor)	—
Throughput	∼148 frames/s	∼4651 frames/s	31.4× speedup
Frame processing time (without regularization)	0.006753 s	0.000215 s	speedup 31.4× (-96.81%) time
Frame processing time (with regularization)	0.006437 s	0.000227 s	speedup 28.3× (-96.47%) time
Maximum difference of results (without regularization)	0 (benchmark)	$1.21 \cdot 10^{-12}$	Same
Mean square error (MSE) (without regularization)	0 (benchmark)	$1.05 \cdot 10^{-15}$	Same
Maximum variance of results (with regularization)	0 (benchmark)	$1.59 \cdot 10^{-3}$	Same
Mean square error (MSE) (with regularization)	0 (benchmark)	$1.13 \cdot 10^{-5}$	Same

The mathematical modeling conducted allows us to draw the following conclusions:

1. **Mathematical Equivalence:** The maximum discrepancy between the results of the baseline and optimized algorithms ($1.21{\cdot}10^{-12}$) is within the range of floating-point machine epsilon for double-precision numbers. This proves that the transition to tensor contraction introduces no methodological errors and fully preserves the physical validity of the spectral estimate.

2. **Regularization Effectiveness:** Upon activating the Tikhonov method, a controlled deviation in the results ($1.59{\cdot}10^{-3}$) is observed, which is explained by the intentional modification of the covariance matrix structure to improve its numerical conditioning. From an engineering perspective, this ensures the cleansing of the spectral background from artifacts and noise that typically arise during the inversion of ill-conditioned matrices.

3. **Efficiency of the Vectorized Tensor-Based Approach:** The computational complexity analysis of the baseline `directionEstimation.py` module from the PyArgus library revealed that the primary performance bottleneck originates from repeated Python-level iterations during the angular spectrum evaluation process across the full 1801-point scanning range described above. In the original implementation, steering vectors $\mathbf{a}(\theta)$ are processed sequentially, which introduces considerable loop overhead and leads to suboptimal CPU cache utilization.

The proposed implementation replaces iterative processing with a vectorized computational scheme in which the complete steering matrix $\mathbf{A}$, containing steering vectors for the entire scanning range, is processed simultaneously using NumPy array operations and the `np.einsum` function. This approach enables batch computation and improves memory access locality while allowing more efficient utilization of SIMD (Single Instruction, Multiple Data) instructions supported by modern processor architectures.

Although the asymptotic computational complexity of the algorithm remains unchanged, vectorization significantly reduces the execution overhead associated with repeated function calls and intermediate memory access operations. Instead of sequentially evaluating $P(\theta_1), P(\theta_2), \ldots, P(\theta_n)$, the proposed method performs a unified matrix transformation over the entire angular scanning space within a single high-level numerical operation.

Experimental evaluation demonstrates a substantial reduction in processing latency, enabling near real-time operation of the KerberosSDR system under dynamically changing electromagnetic conditions. The proposed optimization strategy is consistent with contemporary trends in high-performance array signal processing based on vectorized numerical computations and parallel data processing techniques, allowing efficient utilization of modern computing architectures while minimizing processing delays [6,17].

Conclusions

Based on the conducted research and experimental analysis of four spatial spectral estimation algorithms (Bartlett, Capon, MUSIC, and MEM) implemented within the KerberosSDR coherent direction finder, the Maximum Entropy Method (MEM) demonstrated superior performance for systems with a small number of antennas ($M = 4$) in dense urban environments. It provides the highest energy concentration and angular selectivity. The noise floor for MEM reaches levels of -20 to -40 dB, while correlation- and subspace-based methods exhibit significantly higher background noise in the range of -6 to -11 dB. Unlike MUSIC, MEM effectively extrapolates the available information beyond the observation window, enabling the resolution of sources with small angular separation. The developed mathematical kernel based on Einstein tensor notation addresses the primary drawback of iterative implementations in modern compact direction finders, namely high computational latency of the mathematical spatial spectrum estimation kernel. Verification confirmed the mathematical equivalence of the tensor model: the mean square error (MSE) without regularization was $1.05 \cdot 10^{-15}$, which lies within the range of double-precision floating-point accuracy, while the MSE with Tikhonov regularization was $1.13 \cdot 10^{-5}$, reflecting the intentional modification of the covariance matrix structure rather than any algorithmic distortion. The transition from scalar loops to vectorized tensor operations provided a 31.4-fold acceleration in spatial spectrum computation per single data snapshot —from 6.7 ms to 0.2 ms —without any loss of result accuracy. Since the MEM method is sensitive to the rank deficiency of the covariance matrix $\mathbf{R}$, Tikhonov regularization was integrated into the tensor kernel. This ensured mathematical stability against singular matrices and reduced the probability of spectral artifacts. Even accounting for regularization overhead, the kernel saved 90% of CPU time. The freed-up processor resources allow for the implementation of complex stabilization methods without compromising real-time performance. The recorded acceleration pertains to the internal mathematical operations of the direction finder—specifically, the formation of the weight vector and tensor contraction. The use of optimized MEM implementations within a virtual environment enabled efficient utilization of SIMD instructions on modern processors. This demonstrates that transitioning from a scalar to a tensor-based computational approach enables real-time processing of the direction-finding algorithmic core even on standard processors. The present work proposes a unified approach to the systemic modernization of the computational kernel of the PyArgus library. Future work will focus on extending the proposed approach to the full set of considered direction-finding algorithms. Replacing outdated scalar loops with parallel processing of the spatial spectrum as a unified tensor object transforms widely used libraries into high-performance tools capable of fully exploiting the potential of modern compact radio direction finders.

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Порівняльний аналіз алгоритмів оцінки просторового спектру: переваги методу максимальної ентропії у радіопеленгуванні

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Сучасний етап розвитку систем радіомоніторингу характеризується переходом до програмно визначеного радіо та необхідністю високоточної оцінки напрямку приходу сигналу за умов обмежених антенних ресурсів. У статті проведено порівняльний аналіз чотирьох алгоритмів просторової спектральної оцінки – Bartlett, Capon, MUSIC та методу максимальної ентропії (MEM) – реалізованих на чотириканальній апаратній платформі KerberosSDR для пеленгування реального широкосмугового сигналу стандарту DVB-T2 на частоті 482 МГц.

Отримані експериментальні результати свідчать про те, що в умовах обмеженої апертури антенної решітки ($M = 4$) та реальних завад метод MEM забезпечив вищий рівень придушення бічних пелюсток просторового спектру $-20 \ldots -40$ дБ порівняно з методами Bartlett, Capon та MUSIC, для яких шумовий фон становив $-6 \ldots -11$ дБ при похибці пеленгування 5° відносно істинного напрямку 300°.

Виявлено ключовий практичний недолік базової реалізації методів DoA у середовищі KerberosSDR — надмірні затримки математичного блоку обчислення просторового спектру, зумовлені застосуванням ітераційних циклів на рівні інтерпретатора Python у бібліотеці PyArgus. Слід зазначити, що загальний час оновлення просторового спектру у середовищі KerberosGUI визначається сукупністю апаратно-програмних затримок: збором IQ-вибірок, передачею даних через USB-інтерфейс, буферизацією у RAM-диску, відображенням результатів у графічному інтерфейсі та обчисленням просторового спектру. Тому пропонована оптимізація спрямована саме на прискорення обчислювального ядра алгоритму.

Для подолання цього обмеження розроблено оптимізоване обчислювальне ядро на основі тензорної згортки з використанням нотації Ейнштейна (`numpy.einsum`) та регуляризації Тихонова. Верифікація підтвердила математичну еквівалентність тензорної моделі: середньоквадратична похибка без регуляризації склала $1{,}05 \cdot 10^{-15}$, що знаходиться в межах діапазону точності обчислень з плаваючою комою подвійної точності. Перехід від скалярних циклів до векторизованих тензорних операцій забезпечив прискорення обчислення просторового спектру для одного знімку даних (snapshot) у 31,4 рази – з 6,7 мс до 0,2 мс – без втрати точності результатів.

Ключові слова: оцінка напрямку приходу сигналу; програмно визначене радіо; KerberosSDR; метод максимальної ентропії; MUSIC; тензорна оптимізація; `numpy.einsum`; регуляризація Тихонова; просторова спектральна оцінка