

Optimal Basis Functions in the Electromagnetic Analysis of Inductive Irises Accounting for Meixner's Edge Condition

Piltyay S. I. 

National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine

E-mail: s.piltia.i@kpi.ua

Inductive irises are fundamental components widely applied in modern microwave filters, duplexers, polarizers, and rotators based on waveguides, as their precise electromagnetic characterization is crucial for high-performance device design. This paper presents a rigorous and comprehensive numerical investigation of the optimal selection of basis functions for the electromagnetic analysis of such irises using a hybrid mode-matching and integral equation technique. The study focuses on a comparative analysis of three distinct types of basis functions used to represent the aperture field: classical trigonometric functions, orthogonal basis functions based on Gegenbauer polynomials with a weighting function of power $1/2$, and those with a weighting function of power $2/3$. A key contribution of this work is the explicit demonstration that incorporating Meixner's edge condition through the use of Gegenbauer polynomials with a weighting function of power $2/3$ significantly accelerates convergence rate and improves the numerical stability of the solution. The convergence of reflection coefficient solutions is analyzed in detail, revealing that the indicated set of basis functions outperforms other sets across various iris geometries. Specifically, it is established that to achieve a residual calculation error of less than 0.5% , it is sufficient to utilize only 4 such basis functions, whereas 10 functions are required for Gegenbauer polynomials with a weighting function of power $1/2$, and up to 40 functions are necessary when using trigonometric expansions. Furthermore, the paper investigates the influence of the iris window's physical dimensions, such as thickness and width, on the effectiveness of these basis functions. In addition to the convergence study, detailed amplitude distributions of the electric field within the iris window and adjacent regions are analyzed for narrower and wider windows. These distributions confirm the high physical fidelity of the model, particularly its ability to accurately capture field singularities at the sharp metallic edges. The numerical results show that at a distance of a quarter-wavelength from the inductive iris in a rectangular waveguide, the relative amplitude of the electric field of the fundamental electromagnetic mode TE_{10} exceeds the amplitudes of the electric field of the evanescent higher-order modes by more than 28 times. The developed mathematical model with optimal basis functions and the recommendations provided can be widely applied for the efficient computer-aided design and optimization of iris-based waveguide components such as filters, duplexers, polarizers, and rotators.

Keywords: electromagnetic modeling; mode-matching technique; integral equations; iris; rectangular waveguide; field distribution; basis functions; Gegenbauer polynomials; convergence; residual error; waveguide components

DOI: [10.64915/RADAP.2026.105.12-21](https://doi.org/10.64915/RADAP.2026.105.12-21)

Introduction

The rigorous electromagnetic analysis of waveguide discontinuities, such as conducting irises, remains a cornerstone in the design of modern high-precision microwave components, including filters [1–4], duplexers [5, 6], polarizers [7, 8], and rotators [9–11]. Recently, a highly accurate electromagnetic analysis of an inductive iris in a rectangular waveguide was performed in [12] via a hybrid mode-matching and integral equation technique. While the general theory of iris-based waveguide components is well-established, the efficiency of their numerical modeling

critically depends on the treatment of field singularities at the sharp edges of the conducting elements [13, 14]. According to Meixner's edge condition [15], the electromagnetic field components exhibit singular behavior near these edges, which poses a significant challenge for standard numerical methods in terms of convergence and computational cost.

To overcome these difficulties, the integral equation method combined with the Galerkin technique is frequently employed. The key to the effectiveness of this approach lies in the choice of basis functions used to expand the unknown electric field distribution within the iris aperture. In some cases, direct field-

matching or simple trigonometric expansions without the use of specialized polynomials suffice for analyzing waveguide structures, as demonstrated in [16] for sectoral coaxial ridged waveguides and in [17] for thin asymmetric inductive diaphragms. However, although various sets of orthogonal functions can be used, only those that explicitly incorporate the physical nature of the field singularity can guarantee rapid convergence and high accuracy.

In particular, basis functions based on Gegenbauer polynomials [18] with tailored weighting functions are particularly noteworthy, as they allow for a mathematically sound representation of the field near the metal edges [19]. The numerical effectiveness of such basis functions is especially advantageous in the analysis of components with complex geometries, including ridged waveguides [20, 21] and septum polarizers [22–24], where the singular behavior of the field significantly impacts the results.

This paper is dedicated to the systematic investigation and optimization of basis functions for the electromagnetic analysis of inductive irises in rectangular waveguides. Unlike general studies, this work focuses on the comparative performance of different types and numbers of basis functions, particularly in terms of the convergence of reflection coefficients. Special attention is given to the influence of the geometric parameters of the iris, such as its thickness and window width, on the accuracy of the solution. Furthermore, the study provides a detailed visualization of the electric field amplitude distributions within and adjacent to the iris window, offering a deeper insight into the physical behavior of the field when Meixner's condition is rigorously satisfied. By identifying the most effective basis sets, this research aims to provide practical recommendations for enhancing CAD tools used in the development of modern waveguide technology.

1 Purpose of the research and problem statement

The purpose of current research is to perform a rigorous numerical investigation and optimization of the basis functions used in the electromagnetic analysis of a symmetric inductive iris to ensure maximum computational efficiency. The study aims to justify the selection of basis sets that explicitly account for Meixner's edge condition, providing high accuracy with minimal computational cost for the design of waveguide components.

To achieve the research objective, the following problems must be solved:

- 1) A comparative analysis of various orthogonal basis functions, including classical trigonometric functions and Gegenbauer polynomials with different weighting functions, within the framework of the

existing hybrid mode-matching and integral equation method;

- 2) The evaluation of the convergence stability for the reflection coefficients of the fundamental TE_{10} mode and higher-order modes depending on the type and number of basis functions;
- 3) The quantitative determination of the numerical advantage (in terms of the number of required functions) of using Gegenbauer polynomials with a weighting function of power $2/3$ to achieve a predefined calculation error;
- 4) An investigation of the correlation between the iris physical dimensions (thickness and aperture width) and the effectiveness of the chosen expansion basis set;
- 5) The numerical reconstruction and analysis of the electric field amplitude distributions to confirm the physical fidelity of the model in capturing field singularities at the iris edges.

2 Orthogonal basis functions for expansion of electric field at iris windows

Segment of a rectangular waveguide containing a symmetric inductive iris is shown in Fig. 1. Designations of all inner dimensions are also presented in Fig. 1. Transversal inner sizes of the rectangular waveguide are $a \times b$. The width of the iris window is d , and its thickness is L .

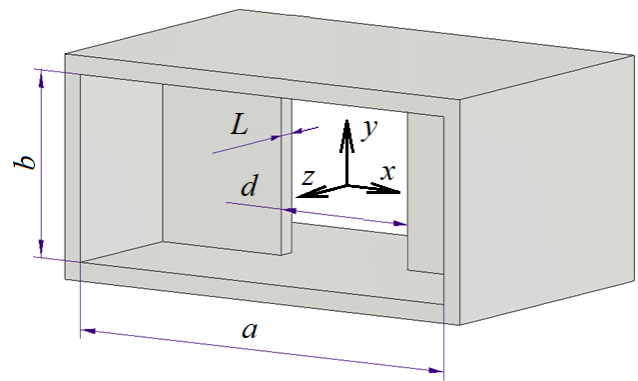


Fig. 1. Segment of a rectangular waveguide with a symmetric inductive iris and designations of the sizes

The explicit forms of expansion sums of the transverse electric fields at both iris windows were obtained in [12]. In the present study, these representations are utilized to satisfy the boundary conditions across the junctions and to formulate the matrix equations. For the sake of completeness and to facilitate the subsequent numerical convergence analysis, these field components are explicitly summarized below:

$$\begin{aligned}
\dot{E}_1^{(1)}(x) &= \begin{cases} \sum_{j=1}^{\infty} \frac{\dot{s}_j - \dot{p}_j}{\sqrt{2d}} \cos \frac{(2j-1)\pi x}{d}, & |x| \leq d/2 \\ 0, & d/2 \leq |x| \leq a/2 \end{cases}; \\
\dot{E}_2^{(1)}(x) &= \begin{cases} \sum_{j=1}^{\infty} \frac{\dot{s}_j + \dot{p}_j}{\sqrt{2d}} \cos \frac{(2j-1)\pi x}{d}, & |x| \leq d/2 \\ 0, & d/2 \leq |x| \leq a/2 \end{cases}; \\
\dot{E}_1^{(2)}(x) &= \sum_{j=1}^{\infty} \frac{\dot{s}_j - \dot{p}_j}{2} \left[1 - \left(\frac{2x}{d} \right)^2 \right]^{1/2} C_{2j-2}^{3/2} \left(\frac{2x}{d} \right); \\
\dot{E}_2^{(2)}(x) &= \sum_{j=1}^{\infty} \frac{\dot{s}_j + \dot{p}_j}{2} \left[1 - \left(\frac{2x}{d} \right)^2 \right]^{1/2} C_{2j-2}^{3/2} \left(\frac{2x}{d} \right); \\
\dot{E}_1^{(3)}(x) &= \sum_{j=1}^{\infty} \frac{\dot{s}_j - \dot{p}_j}{2} \left[1 - \left(\frac{2x}{d} \right)^2 \right]^{2/3} C_{2j-2}^{11/6} \left(\frac{2x}{d} \right); \\
\dot{E}_2^{(3)}(x) &= \sum_{j=1}^{\infty} \frac{\dot{s}_j + \dot{p}_j}{2} \left[1 - \left(\frac{2x}{d} \right)^2 \right]^{2/3} C_{2j-2}^{11/6} \left(\frac{2x}{d} \right),
\end{aligned}$$

where $\dot{s}_j$ and $\dot{p}_j$ are the complex amplitude coefficients, $C_n^\alpha \left(\frac{2x}{d} \right)$ are Gegenbauer polynomials [18], which can be expressed by the following recurrence relation:

$$\begin{aligned}
C_0^\alpha \left(\frac{2x}{d} \right) &= 1; \quad C_1^\alpha \left(\frac{2x}{d} \right) = 4\alpha \frac{x}{d}; \\
C_{n+1}^\alpha \left(\frac{2x}{d} \right) &= \frac{4(n+\alpha)}{n+1} \frac{x}{d} C_n^\alpha \left(\frac{2x}{d} \right) - \\
&\quad - \frac{n+2\alpha-1}{n+1} C_{n-1}^\alpha \left(\frac{2x}{d} \right), \quad \text{for } n \in \mathbb{N}^*,
\end{aligned}$$

where $\alpha = 3/2$ or $11/6$ depending on the basis function type, respectively.

Therefore, the presented electric field expansions are based on three distinct sets of basis functions. The first set is built on classical trigonometric functions as:

$$R_j^{(1)}(x) = \begin{cases} \sqrt{\frac{2}{d}} \cos \left(\frac{(2j-1)\pi x}{d} \right), & |x| \leq d/2 \\ 0, & d/2 \leq |x| \leq a/2. \end{cases} \quad (1)$$

To properly account for Meixner's edge condition, specialized orthogonal basis functions based on Gegenbauer polynomials are employed. For the case of an infinitely thin metallic edge where the field singularity exponent equals $1/2$, the second set $R_j^{(2)}(x)$ is defined as:

$$R_j^{(2)}(x) = \left[1 - \left(\frac{2x}{d} \right)^2 \right]^{1/2} C_{2j-2}^{3/2} \left(\frac{2x}{d} \right). \quad (2)$$

Alternatively, to model the field behavior near a right-angled metallic wedge (90° sharp corner) where the Meixner singularity exponent shifts to $2/3$, the third set of basis functions $R_j^{(3)}(x)$ is introduced:

$$R_j^{(3)}(x) = \left[1 - \left(\frac{2x}{d} \right)^2 \right]^{2/3} C_{2j-2}^{11/6} \left(\frac{2x}{d} \right). \quad (3)$$

3 Convergence of reflection coefficient solutions

In this section, a comparison of the convergence of calculated values of reflection coefficient $\dot{\Gamma}$ was carried out using the definition of residual error as follows:

$$\delta^{(i)}(N, f) = \frac{|\dot{\Gamma}^{(i)}(N, f) - \dot{\Gamma}_{\text{FEM}}(f)|}{|\dot{\Gamma}_{\text{FEM}}(f)|} \cdot 100\%, \quad (4)$$

where $\dot{\Gamma}^{(i)}(N, f)$ designates the complex reflection coefficient of inductive iris in a rectangular waveguide, which was calculated using first N basis functions of the set $\{R^{(i)}\}$ at the frequency f ; $\dot{\Gamma}_{\text{FEM}}(f)$ stands for the complex reflection coefficient of the inductive iris in a rectangular waveguide, which was calculated using Finite element method (FEM) at the same frequency f .

Fig. 2–4 present results for residual errors obtained by using orthogonal (at the interval $[-d/2; d/2]$) trigonometric basis functions (1), orthogonal basis functions (2) based on Gegenbauer polynomials with a weighting function of power $1/2$, and orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$, respectively. Results were calculated for both inductive irises analyzed in the previous article [12], which were installed into a rectangular waveguide of cross-section $23 \text{ mm} \times 10 \text{ mm}$. In Fig. 2–4 it is seen that convergence is almost the same at different frequencies.

Fig. 2 shows that more than 40 orthogonal trigonometric basis functions (1) are required to ensure a residual error of less than 0.5%. Figs. 3 and 4 demonstrate that using orthogonal basis functions (2), (3), which account for singular field behavior, accelerates convergence several-fold. Specifically, only 10 basis functions (2) or 4 basis functions (3) are needed to achieve the same residual error level. Consequently, in terms of convergence speed, the orthogonal functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$ are the optimal choice.

4 Comparison of basis functions for various thicknesses and widths of the iris window

In order to determine basis functions, which provide better accuracy, a comparison of errors (4) is performed for different sizes of inductive irises. Analyzed rectangular waveguide has a cross-section with inner sizes $23 \text{ mm} \times 10 \text{ mm}$. Figs. 5a–5d show errors of reflection coefficient calculation using 5 basis

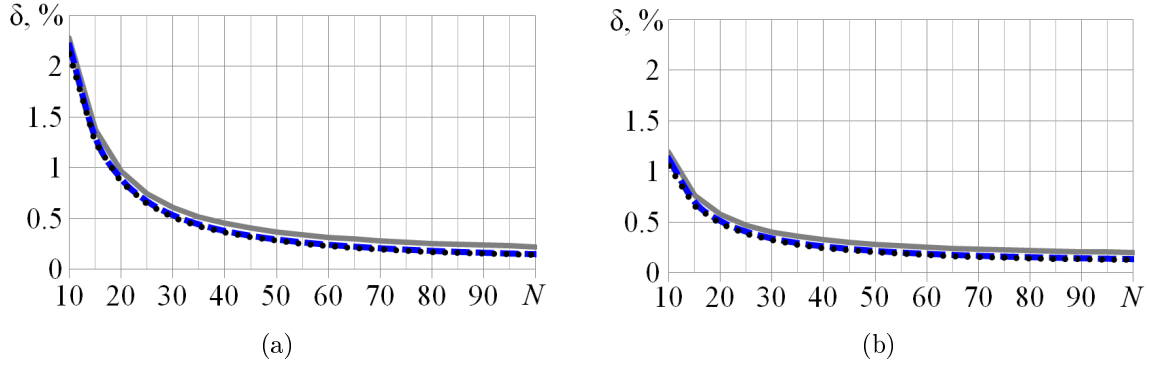


Fig. 2. Residual errors of reflection coefficient calculation by N basis functions (1): a) for iris 1; b) for iris 2. Frequency: — 8 GHz, - - - 10 GHz, 12 GHz

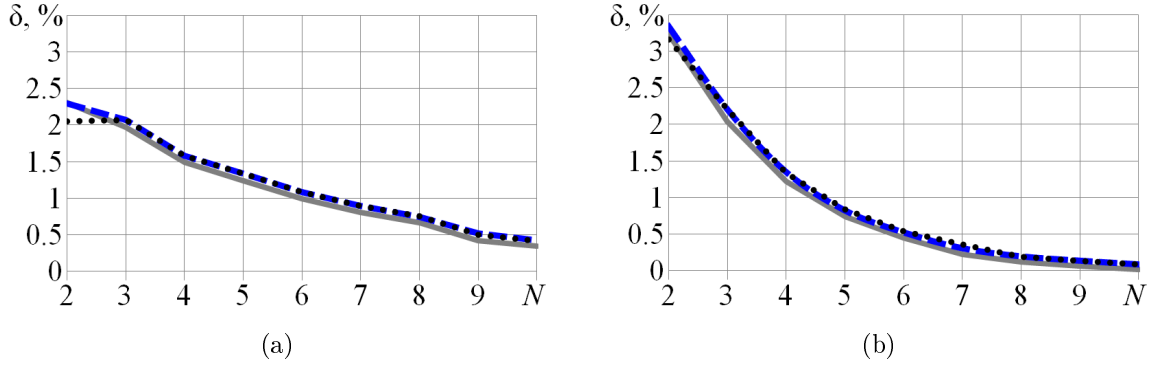


Fig. 3. Residual errors of reflection coefficient calculation by N basis functions (2): a) for iris 1; b) for iris 2. Frequency: — 8 GHz, - - - 10 GHz, 12 GHz

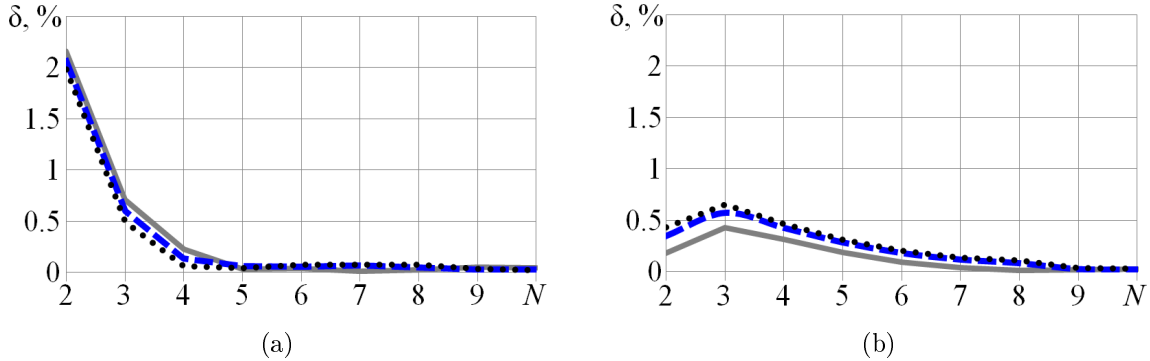


Fig. 4. Residual errors of reflection coefficient calculation by N basis functions (3): a) for iris 1; b) for iris 2. Frequency: — 8 GHz, - - - 10 GHz, 12 GHz

functions (2) and (3) versus the relative iris thickness L/a for the iris window widths $d = 0.6a$, $0.7a$, $0.8a$, $0.9a$, respectively. The relative thickness of an iris L/a was changed in the range from 0.005 to 0.1.

In Fig. 5a, it is seen that for a narrower window of inductive iris ($d = 0.6a$), both types of basis functions (2) and (3) provide sufficiently accurate results with errors less than 0.5 %. For a narrower iris window, more accurate results are provided in the case of using orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power 2/3 for iris thickness L less than $0.03a$, while orthogonal basis functions (2) based on Gegenbauer polynomials with a weighting function of power 1/2 ensure better accuracy for thicker inductive irises with thickness L

more than $0.045a$. It is seen in Fig. 5a that for the iris window $d = 0.6a$ the maximal values of errors occur at iris thicknesses of $L = 0.01a$ and $L = 0.015a$ for basis functions (2) and (3), respectively.

Fig. 5b shows that for a wider iris window with $d = 0.7a$, better accuracy is provided by using orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power 2/3 for iris thickness L less than $0.07a$. On the whole, orthogonal basis functions (3) ensure errors less than 0.3% for iris thickness L in the range from $0.005a$ to $0.1a$. For orthogonal basis functions (2), peak values of errors are more than 3 times higher and reach values of 1.0%. It is seen in Fig. 5b that for the iris window with width $d = 0.7a$, the maximal values of errors occur at iris

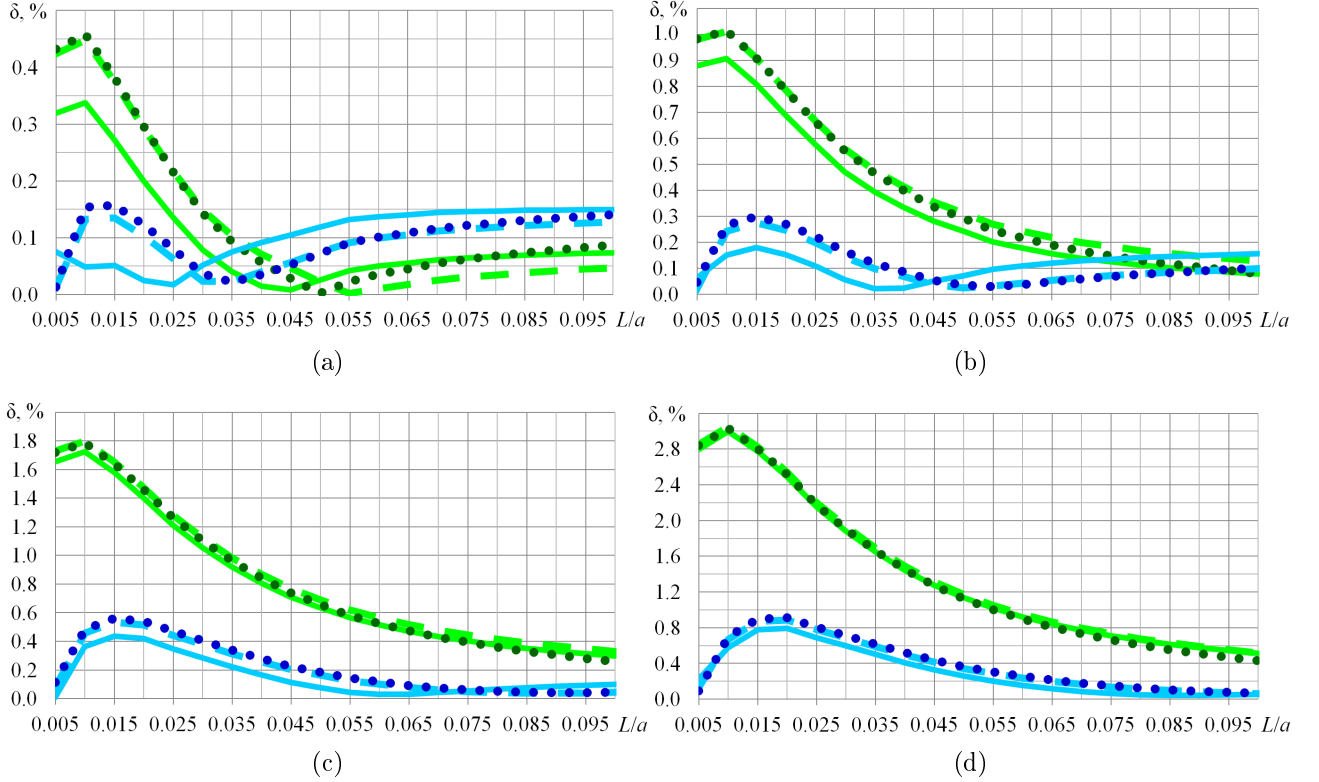


Fig. 5. Errors of reflection coefficient calculation by 5 Basis Functions (BF) (2) and (3) versus relative iris thickness L/a , $a = 23$ mm. Width of inductive iris window: (a) $d = 0.6a$; (b) $d = 0.7a$; (c) $d = 0.8a$; (d) $d = 0.9a$; — BF (2), $f = 8$ GHz; - - BF (2), $f = 10$ GHz; BF (2), $f = 12$ GHz; — BF (3), $f = 8$ GHz; - - BF (3), $f = 10$ GHz; BF (3), $f = 12$ GHz

thicknesses of $L = 0.01a$ and $L = 0.015a$ for basis functions (2) and (3), respectively.

In Fig. 5c, it is seen that for a wider window of inductive iris with $d = 0.8a$, only the set of orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$ provides sufficiently accurate results with errors less than 0.6%. The advantage of orthogonal basis functions (3) over (2) is observed in the whole range of iris thicknesses from 0.005 to 0.1. It is seen in Fig. 5c that for an iris window with the width $d = 0.8a$, the maximal values of errors occur at the iris thicknesses of $L = 0.01a$ and $L = 0.015a$ for basis functions (2) and (3), respectively.

Fig. 5d demonstrated that for a wider window of inductive iris with $d = 0.9a$, the set of orthogonal iris functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$ provides results with errors less than 1.0%. Errors obtained for the set of basis functions (2) are several times higher and reach a peak value of 3.05%. The advantage of orthogonal basis functions (3) over (2) is observed in the whole range of iris thicknesses from 0.005 to 0.1. It is seen in Fig. 5d that for the iris window with $d = 0.9a$, the maximal values of errors occur at iris thicknesses of $L = 0.01a$ and $L = 0.02a$ for basis functions (2) and (3), respectively.

Overall, in terms of minimizing calculation errors, orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$

are the best choice. A multi-fold increase in calculation errors was observed for the first time for wider inductive iris windows compared to narrower ones. It was established that maximal calculation errors occur at inductive iris thicknesses of $L = 0.01a$ and $L = (0.015 - 0.02)a$ for orthogonal basis functions (2) and (3), respectively.

5 Electric field amplitude distributions near the inductive iris

Convergence and accuracy of solutions for reflection coefficient and electromagnetic field in the vicinity of the inductive iris are determined by correctness of obtained distribution of transverse electric field in window of iris and its proximity to real distribution. Let us compare approximations of the transverse electric field's amplitude distribution by the first functions of the sets of basis functions (1)–(3). As a correct distribution for comparison, we will use the one obtained using 10 orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$. The calculations were performed for rectangular waveguide's inner width $a = 23$ mm, operating frequencies of 8, 10, and 12 GHz, values of the inductive iris window's width $d = 0.6a$ and $d = 0.9a$, and the thickness of the iris $L = 0.015a$. The first

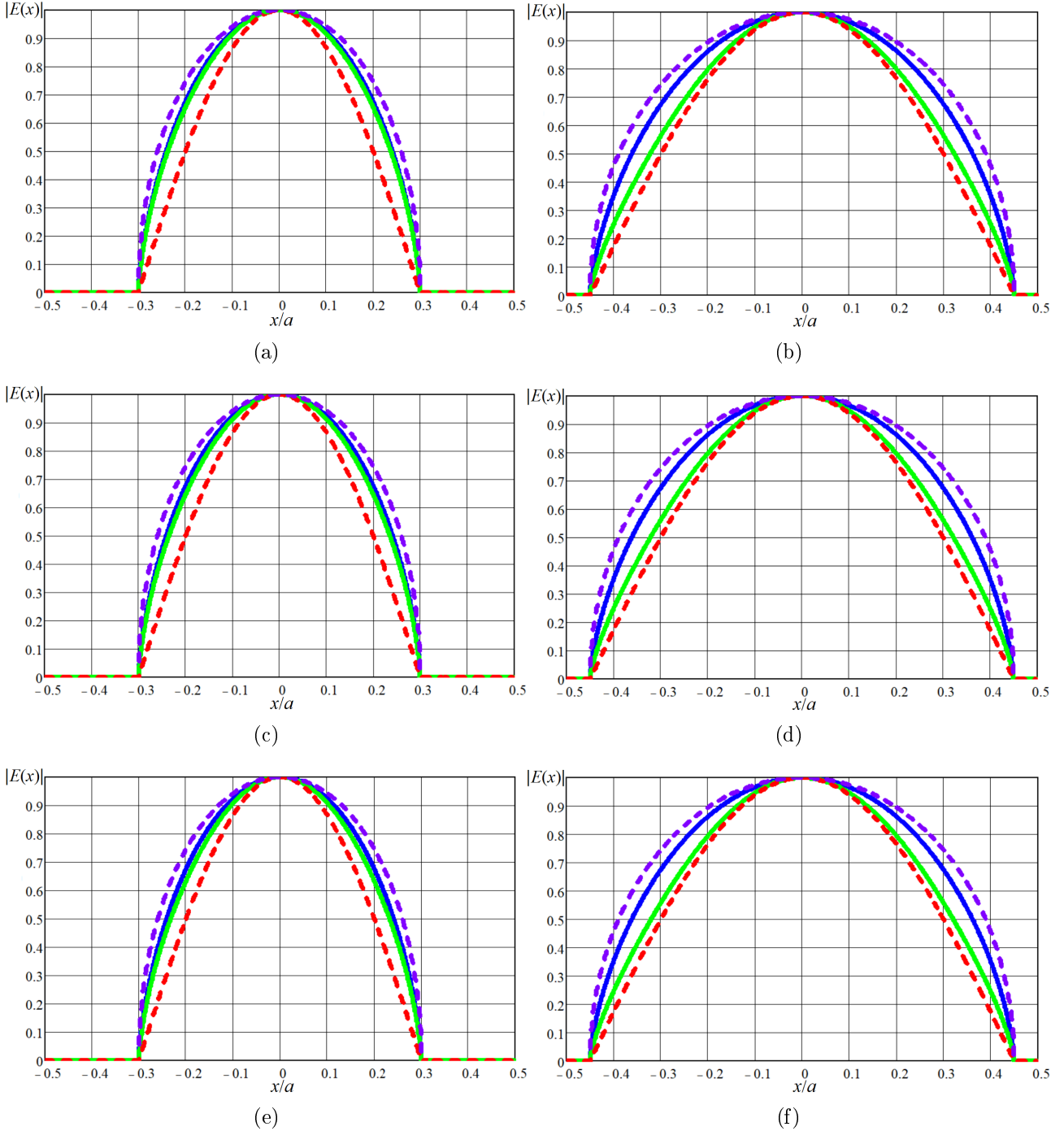


Fig. 6. Normalized electric field amplitude distributions in the window of inductive iris. Sizes of window and frequency: (a) $d = 0.6a$, $f = 8$ GHz; (b) $d = 0.9a$, $f = 8$ GHz; (c) $d = 0.6a$, $f = 10$ GHz; (d) $d = 0.9a$, $f = 10$ GHz; (e) $d = 0.6a$, $f = 12$ GHz; (f) $d = 0.9a$, $f = 12$ GHz. Functions used for decomposition: — 10 functions (3); - - - 1 function (1); - - - 1 function (2); — 1 function (3)

normalized basis functions of considered sets (1)–(3) are as follows:

$$R_1^{(1)} = \begin{cases} \cos(\pi x/d), & |x| \leq d/2 \\ 0, & d/2 \leq |x| \leq a/2 \end{cases};$$

$$R_1^{(2)}(x) = \sqrt{1 - (2x/d)^2};$$

$$R_1^{(3)}(x) = [1 - (2x/d)^2]^{2/3}.$$

Fig. 6 shows the calculated normalized electric field amplitude distribution obtained using 10 functions (3) and 3 distributions given by the first basis function of each set (1)–(3). The real distribution amplitude lies between the cosine-type $R_1^{(1)}(x)$ and square root-type $R_1^{(2)}(x)$ functions. Figs. 6a and 6b show that $R_1^{(3)}(x)$ of the set (3) provides a better initial approximation for a narrower inductive iris window, but a worse one for a wider window where the real distribution is closer

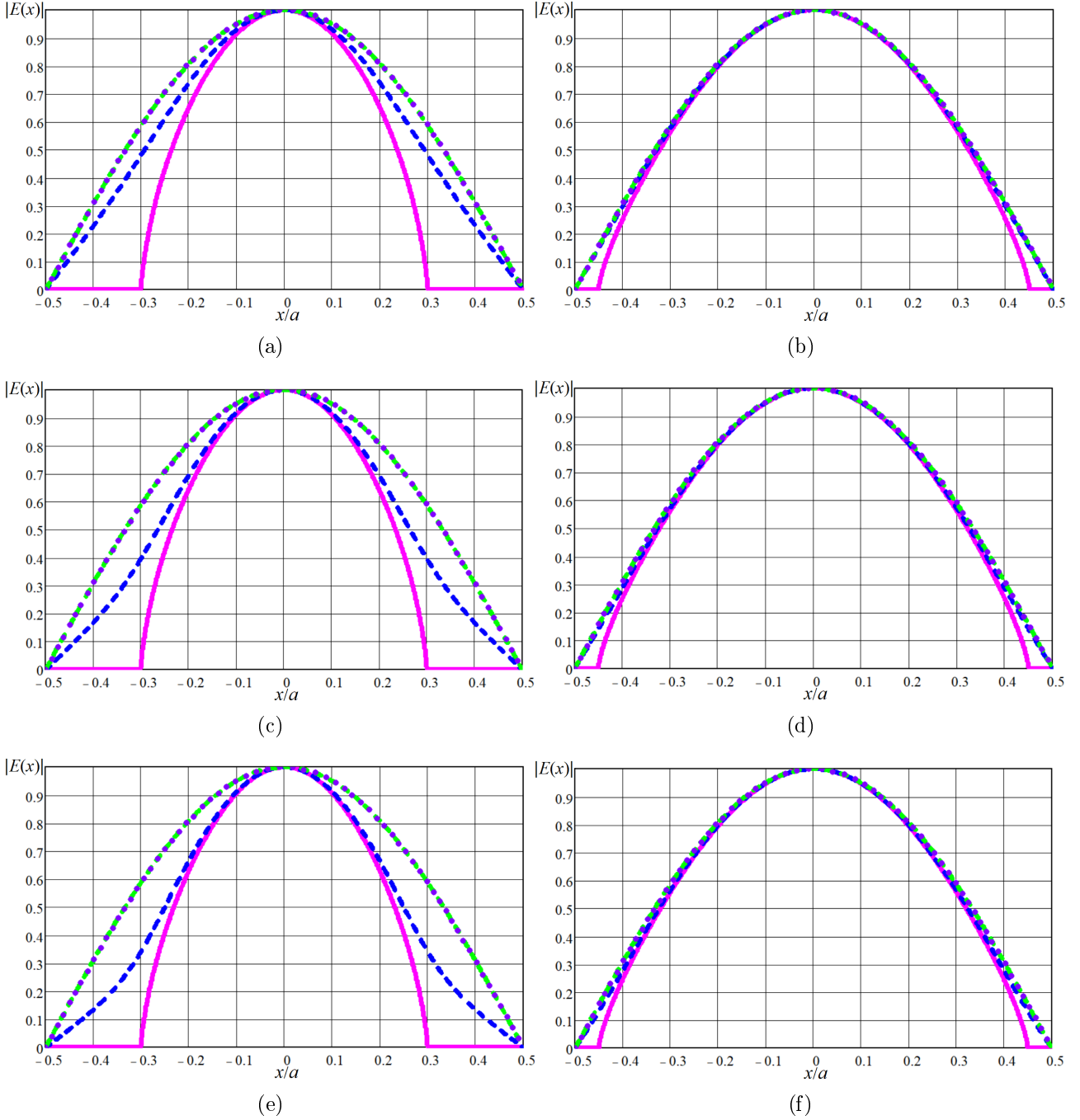


Fig. 7. Normalized electric field amplitude distributions at distances from the inductive iris and in the window. Sizes of window and frequency: (a) $d = 0.6a$, $f = 8$ GHz; (b) $d = 0.9a$, $f = 8$ GHz; (c) $d = 0.6a$, $f = 10$ GHz; (d) $d = 0.9a$, $f = 10$ GHz; (e) $d = 0.6a$, $f = 12$ GHz; (f) $d = 0.9a$, $f = 12$ GHz. Distances from the iris plane: — 0; - - $\lambda_g/16$; - - $\lambda_g/4$; cosine E-field amplitude distribution for the fundamental mode TE_{10} in a rectangular waveguide

to $R_1^{(1)}(x)$. This effect results in much lower values of errors in Fig. 5a compared to their values in Fig. 5d.

In Fig. 6, it is seen that the electric field distribution in the window of the inductive iris is almost independent of the frequency, and becomes slightly narrower with frequency increase.

Fig. 7 presents normalized electric field amplitude distributions at different distances from the inductive iris and in the window. For comparison, a cosine E-field amplitude distribution for the fundamental mode

TE_{10} in a rectangular waveguide is also depicted. In Fig. 7, it is seen that the electric field distribution, which was calculated by the formula (5) from [12] and shown by the dashed green curve, practically coincides with a cosine amplitude distribution shown by the dotted violet curve at a distance of $\lambda_g/4$, where λ_g is the wavelength of TE_{10} mode in the waveguide. This result is important for designing waveguide filters, diplexers and polarizers with irises, because in these devices they are located at distances close to a quarter

of wavelength for compensation of reflections. Obtained coincidence of field distributions means that excited higher-order modes can be neglected at the plane of next adjacent iris location. Consequently, the application of single-mode transfer matrices of waveguide sections between irises must be sufficient for the correct analysis of the whole device. The account of higher-order electromagnetic waves is required only to obtain the correct transfer matrices of the irises themselves.

To confirm relatively large attenuation of the higher-order modes at a distance of $\lambda_g/4$ from the iris, let us calculate the absolute values of the phasors of the electric field for them, as well as for the transmitted fundamental mode TE_{10} . Calculations were made using the developed mathematical model of an inductive iris in a rectangular waveguide with inner width

$a = 23$ mm and electric field decomposition by 10 orthogonal basis functions (3) based on Gegenbauer polynomials with a weighting function of power $2/3$.

The obtained numerical results for relative amplitudes are presented in Table 1, where it can be seen that they weakly depend on the thickness of the iris for a fixed window width and frequency. The most intensive excitation of higher modes occurs at the highest frequency (12 GHz) and for the narrowest considered window of an inductive iris ($d = 0.6a$). In this case, the relative amplitude of the electric field of the fundamental electromagnetic mode exceeds the amplitudes of the electric field of the higher-order modes by more than 28 times. Corresponding ratio of amplitudes for $d > 0.6a$ will be bigger.

Table 1 Relative amplitudes of the electric field of modes at quarter wavelength distance from the inductive iris in a rectangular waveguide of 23 mm $\times$ 10 mm cross-section

Relative window width d/a	Relative iris thickness L/a	Frequency, GHz	Relative amplitude of electric field of the mode TE_{10}	Relative amplitude of electric field of the mode TE_{30}	Relative amplitude of electric field of the mode TE_{50}
0.6	0.01	8	0.79049936	0.00068327	0.00000159
		10	0.91173989	0.01041038	0.00012628
		12	0.95349498	0.03243692	0.00067001
	0.02	8	0.76532807	0.00067826	0.00000142
		10	0.89935410	0.01052508	0.00011446
		12	0.94723948	0.03300922	0.00060700
0.7	0.01	8	0.93198937	0.00049723	0.00000241
		10	0.97499603	0.00693823	0.00017857
		12	0.98718232	0.02122025	0.00095639
	0.02	8	0.92071802	0.00050972	0.00000234
		10	0.97083477	0.00711624	0.00017480
		12	0.98522793	0.02195428	0.00093585
0.8	0.01	8	0.98652896	0.00025778	0.00000176
		10	0.99518945	0.00351452	0.00012823
		12	0.99747912	0.01084353	0.00069600
	0.02	8	0.98353607	0.00027252	0.00000180
		10	0.99416039	0.00372223	0.00013067
		12	0.99697742	0.01148609	0.00070823
0.9	0.01	8	0.99903371	0.00007505	0.00000061
		10	0.99964801	0.00103064	0.00004475
		12	0.99980757	0.00324316	0.00002489
	0.02	8	0.99869106	0.00008380	0.00000066
		10	0.99952621	0.00115156	0.00004480
		12	0.99974323	0.00362582	0.00026789

Therefore, in the design of waveguide devices based on irises with window sizes $d > 0.6a$, higher-order modes excited at any iris can be neglected at the plane of the adjacent iris located at a quarter-wavelength distance. As a result, the single-mode approach is sufficient and significantly simplifies the design of waveguide components, ensuring that higher-order modes fade out to negligibly small values.

Conclusion

Comparative analysis was performed for orthogonal basis functions, including classical trigonometric functions and Gegenbauer polynomials with different weighting functions, within the framework of the hybrid mode-matching and integral equation method. It was demonstrated that achieving a residual error below 0.5 % requires application of over 40 orthogonal trigonometric basis functions, and that incorporating orthogonal basis functions accounting for the singular field behavior accelerates convergence by several times. The fastest convergence of results for reflection coefficient was provided by the orthogonal basis functions based on Gegenbauer polynomials with a weighting function of power 2/3. For them, only 4 basis functions are required to provide the same values of residual error for the reflection coefficient magnitude less than 0.5 %.

In addition, from the point of view of the lowest errors of reflection coefficient calculation, the orthogonal basis functions based on Gegenbauer polynomials with a weighting function of power 2/3 are also the best choice. For the first time, residual errors several times higher were found for wider windows of inductive irises compared to narrower windows. It was established that maximal values of errors occur at inductive iris thicknesses $L = 0.01a$ and $L = (0.015 - 0.02)a$ for orthogonal basis functions based on Gegenbauer polynomials with a weighting function of power 1/2, and orthogonal basis functions based on Gegenbauer polynomials with a weighting function of power 2/3, respectively.

Amplitude distributions of the electric field in the window and near the inductive iris were calculated using the developed mathematical model and plotted for narrower and wider windows. It was showed that at a distance of a quarter wavelength from the inductive iris in a rectangular waveguide relative amplitude of the electric field of the fundamental electromagnetic mode exceeds the amplitudes of the electric field of the higher-order modes by more than 28 times for a window with the width of $0.6a$ and more.

The proposed recommendations on selecting the type of basis functions for the field expansion provide a robust foundation for developing new theoretical models of waveguide filters, diplexers, polarizers, and rotators based on conducting irises in both rectangular and square waveguides.

References

- [1] Campion, J., Glubokov, O., Gomez-Torrent, A., Krivovitca, A., Shah, U., Bolander, L., Li, Y. and Oberhammer, J. (2018). An ultra low-loss silicon-micromachined waveguide filter for D-band telecommunication applications. *2018 IEEE/MTT-S International Microwave Symposium (IMS)*, pp. 583–586. DOI: 10.1109/MWSYM.2018.8439601.
- [2] Mehrabi Gohari, M., Glubokov, O. and Oberhammer, J. (2025). Inline waveguide filters with transmission zeros using frequency-variant couplings. *IEEE Transactions on Microwave Theory and Techniques*, vol. 73, no. 6, pp. 3310–3318. DOI: 10.1109/TMTT.2025.3541149.
- [3] Kirilenko, A., Mospan, L. and Tkachenko, V. (2002). Extracted pole bandpass filters based on the slotted irises. *2002 32nd European Microwave Conference*, pp. 1–4. DOI: 10.1109/EUMA.2002.339448.
- [4] Deng, C., Zhang, R. and Fan, X. (2016). V-band waveguide band-pass filter with tuning screws. *2016 IEEE International Conference on Microwave and Millimeter Wave Technology (ICMMT)*, pp. 416–418. DOI: 10.1109/ICMMT.2016.7761793.
- [5] Zhao, X., Glubokov, O., Campion, J., Gomez-Torrent, A., Krivovitca, A., Shah, U. and Oberhammer, J. (2020). Silicon micromachined D-band diplexer using releasable filling structure technique. *IEEE Transactions on Microwave Theory and Techniques*, vol. 68, no. 8, pp. 3448–3460. DOI: 10.1109/TMTT.2020.3004585.
- [6] Bartlett, C., Glubokov, O., Kamrath, F. and Höft, M. (2023). Highly selective broadband mm-wave diplexer design. *IEEE Microwave and Wireless Technology Letters*, vol. 33, no. 2, pp. 149–152. DOI: 10.1109/LMWC.2022.3205425.
- [7] Virone, G., Tascone, R., Peverini, O. A., Addamo, G. and Orta, R. (2008). Combined-phase-shift waveguide polarizer. *IEEE Microwave and Wireless Components Letters*, vol. 18, no. 8, pp. 509–511. DOI: 10.1109/LMWC.2008.2001005.
- [8] Peverini, O. A., Virone, G., Addamo, G. and Tascone, R. (2011). Development of passive microwave antenna-feed systems for wide-band dual-polarisation receivers. *IET Microwaves, Antennas & Propagation*, vol. 5, no. 8, pp. 1008–1015. DOI: 10.1049/iet-map.2010.0340.
- [9] Kirilenko, A., Mospan, L. and Steshenko, S. (2018). A way to realize a multi-frequency polarization plane rotator. *2018 9th International Conference on Ultrawideband and Ultrashort Impulse Signals (UWBUSIS)*, pp. 218–221. DOI: 10.1109/UWBUSIS.2018.8520246.
- [10] Kirilenko, A. A., Steshenko, S. O., Derkach, V. N. and Ostryzhnyi, Y. M. (2019). A tunable compact polarizer in a circular waveguide. *IEEE Transactions on Microwave Theory and Techniques*, vol. 67, no. 2, pp. 592–596. DOI: 10.1109/TMTT.2018.2881089.
- [11] Kirilenko, A., Steshenko, S. and Ostryzhnyi, Y. (2024). Compact filter-rotator of polarization plane with uniform angular response. *Radioelectronics and Communications Systems*, vol. 67, pp. 258–264. DOI: 10.3103/S0735272724050054.
- [12] Piltyay, S. I. (2026). Electromagnetic analysis of an inductive iris in a rectangular waveguide via a hybrid mode-matching and integral equation technique. *Visnyk NTUU KPI Seriya - Radiotekhnika Radioaparatobuduvannia*, no. 103, pp. 5–15. DOI: 10.64915/RADAP.2026.103.5-15.

- [13] Marchetti, S. and Rozzi, T. (1991). Electric field singularities at sharp edges of planar conductors. *IEEE Transactions on Antennas and Propagation*, vol. 39, no. 9, pp. 1312–1320. DOI: 10.1109/8.99039.
- [14] Krähenbühl, L., Buret, F., Perrussel, R., Voyer, D., Nicolas, P. and Nicolas, L. (2011). Numerical treatment of rounded and sharp corners in the modeling of 2D electrostatic fields. *Journal of Microwaves, Optoelectronics and Electromagnetic Applications*, vol. 10, no. 1, pp. 66–81. DOI: 10.1590/S2179-10742011000100008.
- [15] Meixner, J. (1972). The behavior of electromagnetic fields at edges. *IEEE Transactions on Antennas and Propagation*, vol. 20, no. 4, pp. 442–446. DOI: 10.1109/TAP.1972.1140243.
- [16] Pilyay, S. I. and Dubrovka, F. F. (2013). Eigenmodes analysis of sectoral coaxial ridged waveguides by transverse field-matching technique. Part 1. Theory. *Visnyk NTUU KPI Seriya - Radiotekhnika Radioaparato buduvannya*, no. 54, pp. 13–23. DOI: 10.20535/RADAP.2013.54.13-23.
- [17] Martyniuk, S. Y., Dubrovka, F. F., Zakharchenko, O. S. and Stepanenko, P. Y. (2021). Effective high-precision analysis of thin asymmetric inductive diaphragm in rectangular waveguide using integral equation method. *Radioelectronics and Communications Systems*, vol. 64, pp. 80–91. DOI: 10.3103/S0735272721020035.
- [18] Reimer, M. (2003). *Gegenbauer polynomials*. In *Multivariate polynomial approximation* (pp. 19–20). Birkhäuser, Basel. DOI: 10.1007/978-3-0348-8095-4.
- [19] Boukli-Hacene, N., Sombrin, J. and Papiernik, A. (2003). Approximation by Gegenbauer polynomials in the study of a rectangular ridged waveguide. Application to the analysis of a waveguide septum polarizer. *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 16, no. 4, pp. 299–318. DOI: 10.1002/jnm.500.
- [20] Tribak, A., Mediavilla, A., Cano, J. L., Boussouis, M. and Cepero, K. (2009). Ultra-broadband low axial ratio corrugated quad-ridge polarizer. *European Microwave Conference (EuMC)*, pp. 73–76. DOI: 10.23919/EuMC.2009.5295927.
- [21] Güvenç, M., Şişman, I. and Ergin, A. A. (2023). Design and optimization of a wide-band quad-ridged polarizer for satellite communications. *2023 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (USNC-URSI)*, pp. 1477–1478. DOI: 10.1109/USNC-URSI52151.2023.10237410.
- [22] Deutschmann, B. and Jacob, A. F. (2020). Broadband septum polarizer with triangular common port. *IEEE Transactions on Microwave Theory and Techniques*, vol. 68, no. 2, pp. 693–700. DOI: 10.1109/TMTT.2019.2951138.
- [23] Pilyay, S. I. (2022). Electromagnetic and bandwidth performance optimization of new waveguide polarizers with septum of a stepped-thickness type for satellite systems. *Journal of Electromagnetic Waves and Applications*, vol. 36, no. 9, pp. 1257–1272. DOI: 10.1080/09205063.2022.2040051.
- [24] Güvenç, M., Şişman, I. and Ergin, A. A. (2023). Design, optimization and fabrication of a X-band septum polarizer for satellite communication. *2023 IEEE International Symposium on Antennas and Propagation (USNC-URSI)*, pp. 1347–1348. DOI: 10.1109/USNC-URSI52151.2023.10237446.

Оптимальні базисні функції в електродинамічному аналізі індуктивних діафрагм з урахуванням умови Мейкснера на ребрах

Пільтяй С. І.

Індуктивні діафрагми є базовими компонентами, які широко застосовують у сучасних мікрохвильових фільтрах, дуплексерах, поляризаторах і ротаторах на основі хвильоводів, де знаходження їх точних електродинамічних характеристик є критичним для проектування високоефективних пристроїв. У статті досліджено критерії оптимального вибору базисних функцій для електродинамічного аналізу таких діафрагм за допомогою гібридного методу узгодження мод та інтегральних рівнянь. Дослідження зосереджено на порівняльному аналізі трьох різних типів базисних функцій, які використовують для представлення поля апертури діафрагми: класичних тригонометричних функцій, ортогональних базисних функцій на основі поліномів Гегенбауера з ваговою функцією з показником степеня $1/2$, а також з ваговою функцією з показником степеня $2/3$. Основним внеском цієї роботи є явна демонстрація того, що врахування умови Мейкснера на ребрі шляхом використання поліномів Гегенбауера з ваговою функцією з показником степеня $2/3$ значно підвищує швидкість збіжності та чисельну стійкість розв'язку.

Детально проаналізовано збіжність розв'язків для коефіцієнта відбиття, внаслідок чого виявлено переваги зазначеного набору базисних функцій порівняно з іншими наборами для діафрагм із різними розмірами. Зокрема, встановлено, що для досягнення залишкової похибки розрахунку менше 0,5% достатньо використовувати лише 4 такі базисні функції, тоді як для поліномів Гегенбауера з ваговою функцією з показником степеня $1/2$ необхідно брати 10 функцій, а при використанні тригонометричних розкладів — до 40 функцій. Крім того, у статті досліджено вплив фізичних розмірів вікна діафрагми, таких як товщина та ширина, на ефективність цих базисних функцій. Окрім дослідження збіжності, проаналізовано детальні розподіли амплітуди напруженості електричного поля у вікні діафрагми та прилеглих областях для вузьких і ширших вікон. Ці розподіли підтверджують високу фізичну достовірність моделі, зокрема її здатність точно відображати сингулярності поля на гострих металевих ребрах. Чисельні результати показують, що на відстані чверті довжини хвилі від індуктивної діафрагми в прямокутному хвильоводі відносна амплітуда електричного поля основної електромагнітної моди H_{10} перевищує амплітуди електричного поля загасаючих вищих мод більше ніж у 28 разів. Розроблена математична модель з оптимальними базисними функціями та надані рекомендації можуть широко застосовуватися для ефективного автоматизованого проектування та оптимізації хвильовідних компонентів на основі діафрагм, таких як фільтри, дуплексери, поляризатори та ротатори.

Ключові слова: моделювання електромагнітних полів; метод узгодження мод; інтегральні рівняння; діафрагма; прямокутний хвильовід; розподіл поля; базисні функції; поліноми Гегенбауера; збіжність; залишкова похибка; хвильовідні компоненти